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THE EFFECTS OF TEACHER DISCOURSE ON STUDENT BEHAVIOR AND LEARNING IN PEER-DIRECTED GROUPS

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Abstract

Previous research on small-group collaboration identifies several behaviors that significantly predict student learning. These reports focus on student behavior to understand why, for example, large numbers of students are unsuccessful in obtaining explanations or applying help received, leaving unexplored the role that *teachers* play in influencing small-group interaction. We examined the impact of teacher discourse on the behavior and achievement of students in the context of a semester-long program of cooperative learning in four middle school mathematics classrooms. We conclude that student behavior largely mirrored the discourse modeled by and the expectations communicated by teachers. Teachers tended to give unlabeled calculations, procedures, or answers instead of labeled explanations. Teachers often instructed using a recitation approach in which they assumed primary responsibility for solving the problem, having students only provide answers to discrete steps. Finally, teachers rarely encouraged students to verbalize their thinking or to ask questions. Students adopting the role of help-giver showed behavior very similar to that of the teacher: doing most of the work, providing mostly low-level help, and infrequently monitoring other students' level of understanding. The relatively passive behavior of students needing help corresponded to expectations communicated by the teacher about the learner as a fairly passive recipient of the teacher's transmitted knowledge. Finally, we confirmed previous analyses showing that the level of help received from the student or teacher, and the level of student follow-up behavior after receiving help significantly predicted student learning outcomes.

The past 20 years show a tremendous increase in school's use of peer-directed small group work. School districts, state departments of education, national research organizations, and curriculum specialists recommend, sometimes even mandate, the use of peer-based learning (e.g., California Department of Education, 1990; California State Department of Education, 1985, 1992; National Council of Teachers of Mathematics, 1989; National Research Council, 1989, 1995). A main reason for its use, putting students into groups gives them an opportunity to learn from each other. As explained in the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) for Grades 5 to 8 (p. 78), collaborating with peers offers "opportunities to explain, conjecture, and defend one's ideas… [which] can stimulate deeper understanding of concepts and principles." While research on cooperative learning establishes that working collaboratively with others can increase achievement (e.g., Slavin, 1990), it is also clear that not all behavior is equally effective for learning (Bossert, 1988-1989; Webb & Palincsar, 1996). Simply putting students in small groups will not guarantee that they will interact with each other in ways that benefit learning. To promote effective collaborative group work in the classroom, teachers must know which processes promote learning and how to encourage them.

Small-group work provides students opportunities to learn by helping each other. From a theoretical perspective, both the help-giver and the help-receiver stand to benefit from sharing information, especially explanations or detailed descriptions of how to solve problems rather than simply exchanging answers. Giving explanations may promote learning by encouraging the explainer to reorganize and clarify material, recognize misconceptions, fill in gaps in his or her own understanding, internalize and acquire new strategies and knowledge, and develop new perspectives and understanding (Bargh & Schul, 1980; King, 1992; Peterson, Janicki & Swing, 1981; Rogoff, 1991; Saxe, Gearhart, Note & Paduano, 1993; Valsiner, 1987; Webb, 1991). When explaining their problem-solving processes, students think about the salient features of the problem, which develops their problem-solving strategies as well as a metacognitive awareness of what they do and do not understand (Cooper, 1999). Receiving explanations may help students correct misconceptions and strengthen connections between new information and previous learning (Mayer, 1984; Sweller, 1989; Wittrock, 1990), and bridge from the known to the unknown (Rogoff, 1990). Giving and receiving non-elaborated help (answers or calculations), on the other hand, generally results in fewer benefits, perhaps because it involves less cognitive restructuring or clarifying on the part of the help-giver and may not enable help-receivers to correct their misconceptions or lack of understanding. Research on learning in peer-directed small groups empirically confirms the power of giving explanations compared to giving non-elaborated help (Brown & Palincsar, 1989; Fuchs, Fuchs, Hamlett, Phillips, Karns, & Dutka, 1997; King, 1992; Nattiv, 1994; Peterson, Janicki, & Swing, 1981; Slavin, 1987; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990); however, as discussed below, researchers provide mixed evidence about the effectiveness of receiving explanations.

Recognizing the importance of promoting student explaining in cooperative learning settings, Webb and Farivar (1991; see also Farivar & Webb, 1994, 1998) designed and implemented a semester-long program of cooperative learning that focused on building students' explaining skills (described in detail below). Subsequent analyses of student interaction and learning in this program showed that, while important, exchanging explanations was not sufficient for learning, especially among students who needed help. Receiving explanations in peer-directed small groups had a weaker impact than giving explanations. This result aligns with other research showing a weak and inconsistent relationship between receiving explanations and learning outcomes (Hooper, 1992; Nattiv, 1994; Ross & Cousins, 1995a; Webb, 1989, 1991; Webb & Palincsar, 1996).

To understand why receiving explanations did not always improve learning, further analyses examined a set of conditions that Vedder (1985) hypothesizes must be met for the help received to be effective: the student receiving help must have the opportunity to use the explanation to solve the problem or carry out the task for him/herself, and the student must use the opportunity for practice by attempting to apply the explanation received to the problem at hand. Carrying out further activity after receiving explanations may benefit the learner in several ways. First, during the process of using an explanation to try to solve the problem, students may generate self-explanations that help them internalize principles, construct specific inference rules for solving the problem, and repair imperfect mental models (Chi, 2000; Chi & Bassock, 1989; Chi, Bassock, Lewis, Reimann, & Glaser, 1989). Second, attempting to solve problems may help students monitor their own understanding and help them become aware of misunderstandings or lack of understanding (Chi & Bassock, 1989). Not attempting to solve problems after receiving help may leave students with a false sense of competence. Third, making and revealing mistakes while attempting to solve problems may help make *others* aware of a student's misunderstandings or lack of understanding, which can lead the group to provide additional explanations. Without such information, group members may rely on students' own admissions of whether they understand (e.g., "I get it"), which may vary in accuracy (Shavelson, Webb, Stasz, & McArthur, 1988).

To test Vedder's predictions, Webb, Troper, and Fall (1995; see also Webb & Farivar, 1999) analyzed the level of help that students received and the level of activity they engaged in after receiving help. Those analyses confirmed Vedder's prediction: the level of follow-up activity after receiving help most strongly predicted achievement. Moreover, they showed that carrying out high-level follow-up behavior required receiving explanations.

In subsequent analyses designed to understand why some students successfully obtained explanations while others did not, Webb and Mastergeorge (in press) investigated the help-seeking behavior of students who exhibited difficulty carrying out the work. Those analyses were based on Nelson-Le Gall's (1981, 1985, 1992; Nelson-Le Gall, Gumerman & Scott-Jones, 1983) comprehensive, five-step model of children's help-seeking in which the student who has difficulty must realize that s/he needs help, be willing to seek help, identify someone who can provide help, use effective strategies to elicit help (e.g., ask explicit, precise, and direct questions; Peterson, Wilkinson, Spinelli, & Swing, 1984; Wilkinson, 1985; Wilkinson & Spinelli, 1983; Wilkinson & Calculator, 1982a, 1982b), and be prepared to reassess her/his strategies for obtaining help (for an expanded model of adaptive help-seeking behavior in the classroom, see Newman, 1991, 1998). Consistent with Nelson-Le Gall's predictions, students who asked for specific explanations of how to solve problems, instead of requesting calculations or answers, or making nonspecific admissions of confusion, and persisted in seeking those explanations more often obtained high-level help.

The previous sets of analyses, then, identified three categories of student behavior that seem to be critical for promoting learning in cooperative groups: requesting specific explanations and persisting in seeking help, obtaining explanations instead of low-level help, and applying the help received to the problems at hand. The previous analyses also showed the infrequent occurrence of these critical behaviors, despite their explicit inclusion in the cooperative learning preparation activities. Previous reports (especially Webb & Mastergeorge, in press) looked within groups themselves to understand why students did or did not engage in these behaviors.

Although previous research yields much information about which student behaviors impact learning, little research explores the role that the *teachers* play in influencing small-group interaction. The cooperative learning program described above was not introduced in a vacuum, but occurred within an existing classroom context in which teachers (and students) had pre-existing beliefs about teaching and learning and had well-established instructional practices and norms guiding interpersonal exchanges. While the program did address some issues of teacher behavior (especially the importance of relinquishing control over student activities and allowing discussion and collaboration among students), most of the attention centered on the responsibilities and expected behavior of students in the cooperative setting. Nonetheless, teachers maintained a strong presence throughout the cooperative learning program, through their whole-class introductions to the material (and dialogue with the class) and their interactions with specific groups once the daily group work commenced. Consequently, it is likely that teacher discourse, both the explicit encouragement or discouragement of specific behavior and the modeling of behavior, sent signals about expected or desirable behaviors in the classroom and, hence, in small-group interactions.

In the present paper, then, we examine the behavior of the teacher and its possible role in shaping student behavior and, consequently, learning in classrooms using cooperative small-group work. We focus on the behavior modeled by teachers and on the expectations for student behavior that teachers otherwise communicated through their discourse, and investigate how small-group interactions reflect these behaviors and expectations. We analyzed multiple aspects of teacher behavior—help giving, question asking, evaluation of students' responses to teacher questions, monitoring of student progress and comprehension, and explicit statements about desired or undesired student behavior—and explored (a) the extent to which teacher behavior corresponded to, or promoted, the help-giving and help-seeking behaviors found to predict student learning; and (b) the correspondence among teacher and student behavior and learning.

Method

The Cooperative Learning Program

The cooperative learning program had four sequential sets of activities designed to develop students' ability to work effectively in small groups: (a) inclusion activities (also called class-building), (b) activities to develop basic communication skills, (c) activities to develop students' helping skills in work groups, and (d) activities to develop students' ability to give explanations. Due to the large number of activities, we divided them among three curriculum units spread out over the course of a semester. Prior to the first curriculum unit (decimal operations: Phase 1), students carried out activities in (1) and (2). Prior to the second curriculum unit (fractions: Phase 2), students carried out the activities in (3). Prior to the third curriculum unit (percentages: Phase 3), students carried out the activities in (4).

Phase 1

Preliminary activities: class-building. Although the study took place during the spring semester, it quickly became clear that the students knew and interacted with only a few of their classmates, and did not even know all of their classmates' names. Prior to engaging in any group work, then, students carried out a set of inclusion activities to familiarize them with their classmates and to help them feel more comfortable in the classroom. We designed these preliminary activities to lessen feelings of awkwardness once students began working in small groups. Students played games to learn classmates' names (e.g., small groups of students rearranged themselves in front of the class and the rest of the class had to identify each student) and to learn their classmates' interests and aspirations (e.g., each student contributed an item or two to a list, the list was reproduced, and each student circulated among the class to find who fit each description; pairs of students interviewed each other and shared what they learned with the class; small groups of students discussed common interests).

Developing basic communication skills. We designed the second set of activities to develop students' basic communication skills, and to help students learn how to interact with others and to work effectively in small groups. The teacher introduced norms for group behavior, and the class discussed and made charts for posting in the classroom that summarized them (e.g., attentive listening, no put-downs, 12-inch voices—no yelling, equal participation by everyone, zero noise level signal; Gibbs, 1987). Classes also discussed and made charts of social skills to use in small groups: checking for understanding, sharing ideas and information, encouraging, and checking for agreement (Johnson, Johnson, & Holubec, 1988). The class brainstormed ideas for what each behavior "looks like" and "sounds like" and the teacher wrote these ideas on a large sheet of paper. Groups filled out "group processing" sheets to check whether they carried out these skills while working in their groups. Finally, to build team cohesion, each small group chose a group name and created a group sign. This activity reinforced the group's identity and enabled teachers to use the groups' names instead of students' names when calling on them.

These activities provided essential means for teachers to manage classroom noise and to get groups' attention; the activities also established expectations for behavior so that students would feel more comfortable expressing their ideas and opinions both inside their groups and to the whole class. These basic communications and social skills also helped to prevent hostility among students, which would undermine any productive group work, and to prevent groups from dissolving into independent work without student collaboration. Finally, these social skills set the stage for explanation-giving by emphasizing understanding and sharing ideas, rather than focusing on the correct answers.

Phase 2

Developing help-giving and help-receiving skills. We designed the third set of activities to develop students' ability to help each other while working on problems in small groups. To learn the value of two-way communication, students gave directions about drawing figures to another student who either was or was not allowed to ask questions, and then compared and discussed the two experiences. We designed this activity to help students realize the value of active participation compared to only observing others, to reinforce the importance of reciprocal communication, and to show the importance of question asking, as well as question answering. To introduce specific helping skills, the teacher displayed and discussed charts of behaviors for students to engage in when they did not understand how to solve a problem and when they gave help to another student.

Table 1

Chart of Behaviors for Students Who Do Not Understand How to Solve the Problem

Behavior	Example
Problem: Groups design their own restaurant menus with prices for entrees, desserts, and drinks. They each select a meal, and estimate the total cost for their entire group, including 8.5% sales tax and 15% tip.	
1. Recognize that you need help.	"I don't understand how to calculate the sales tax."
2. Decide to get help from another student.	"I'm going to ask someone for help."
3. Choose someone to help you.	"I think Maria could help me."
4. Ask for help.	"Could you help me with the sales tax?"
5. Ask clear and precise questions.	"Our group's bill is $24.00. Why don't we just add $0.85 for the sales tax?"
6. Keep asking until you understand.	"So if the bill was $50.00, are you saying that the sales tax would be 8.5% of $50.00?"

Based upon research on effective help-seeking (Nelson-Le Gall, 1981, 1985; Nelson-Le Gall, Gumerman & Scott-Jones, 1983), the chart of behaviors for students who needed help listed the following steps: recognize that you need help, decide to get help from another student, choose someone to help you, ask for help, ask clear and precise questions, and keep asking until you understand (see Table 1).

To emphasize giving explanations rather than only the answer, and to encourage the active participation of the help-seeker, the chart of behaviors for students who gave help listed the following steps: notice when other students need help; tell other students to ask you if they need help; when someone asks for help, help him or her; be a good listener; give explanations instead of the answer; watch how your teammate solves the problem; give specific feedback on how your teammate solved the problem; check for understanding; praise your teammate for doing a good job (see Table 2).

Table 2

Chart of Behaviors for Students Who <u>Do</u> Understand How to Solve the Problem

Behavior	Example
1. Notice when other students need help.	Look around your group to see if anyone needs help.
2. Tell other students to ask you if they need help.	"If you need help, ask me."
3. When someone asks for help, help him or her.	"Sure I'll help you. What don't you understand?"
4. Be a good listener.	"Let your teammate explain what he or she doesn't understand."
5. Give explanations instead of the answer.	"8.5% is not the same as $0.85. The sales tax is not the same amount of money for every bill. The bigger the bill is, the bigger the tax will be. So here we have to figure out 8.5% of $24.00. 10% of %24.00 is $2.40, so the sales tax will be a little less than that."
6. Watch how your teammate solves the problem.	
7. Give specific feedback on how your teammate solved the problem.	"You multiplied the numbers OK, but you have to be careful of the decimal point. If the bill is $24.00, it doesn't make sense that the sales tax is $204.00."
8. Check for understanding.	"Tell me again why you think the sales tax is $2.04 instead of $204.00."
9. Praise your teammate for doing a good job.	"Good job!" "Nice work!" "You've got it!"

We intended these steps to ensure that help-givers gave students who needed help an opportunity to try to solve problems for themselves, as anecdotal observations in previous studies suggest that groups try to "help" by solving the problems for other students (Shavelson, Webb, Stasz, & McArthur, 1988; Vedder, 1985). (For examples of each step in these charts, see Farivar & Webb, 1994). As a practice activity, students carried out these skills in their small groups while solving novel math problems. Students also completed checklists of these help-giving and help-receiving behaviors after group work to increase their awareness of which skills their groups used and where they needed to improve (see Table 3).

Table 3

Checklist of Help-Seeking and Helping Behavior

"Good Helper" and "Good Help-Receiver" Checklists		
HOW GOOD A HELPER ARE YOU?		
When you are helping do you:	YES	NO
1. Notice when other students need help?	________	________
2. Tell other students to ask you if they need help?	________	________
3. Respond to requests for help?	________	________
4. Listen when you're told the specific kind of help needed?	________	________
5. Give explanations of how or why to do the problem?	________	________
6. Watch how your teammate solves the problem?	________	________
7. Give specific feedback on how your teammate solved the problem?	________	________
8. Check for understanding?	________	________
9. Praise your teammate for doing a good job?	________	________
HOW GOOD ARE YOU AT ASKING FOR HELP?		
When you need help, do you:	YES	NO
1. Recognize that you need help.	________	________
2. Decide to get help from another student.	________	________
3. Choose someone to help you.	________	________
4. Ask for help.	________	________
5. Ask clear and precise questions.	________	________
6. Keep asking until you understand.		

Phase 3

Developing explaining skills. We focused the final set of activities primarily on developing students' ability to give explanations and secondarily on the active participation of the person needing help. In one activity adapted from the study of Swing and Peterson (1982), students performed skits in front of the class to demonstrate "good" helping and "unhelpful" helping. In the skit for good helping, one student explained to another student how to carry out the steps in solving a problem, gave the other student an opportunity to try to solve the problem, corrected the other student's errors with explanations of what she should do and why, asked follow-up questions to make sure that the other student understood, and gave praise for work well done (see Table 4).

Table 4
Helping Skit #1 (Helpful Help)

Robert	I'm having trouble with this one. Maria, will you help me?
Maria	First, I think it would be easier if you write the numbers in a column like this: 3.4 x 2
	First, you multiply the same as you do with whole numbers. What do you get?
Robert	68.
Maria	That's right. Now what do you do?
Robert	Put the decimal point in someplace?
Maria	OK. Now count how many decimal places are in the factors. (Point to the 3.4 and the 2)
Robert	One decimal place.
Maria	Right. So the product has to have the same number of decimal places.
Robert	The answer is 6.8.
Maria	Good job!

In the skit for unhelpful helping, one student gave the other student only the answer, did not describe how to solve the problem, told the other student to hurry up, and told the other student to concentrate on getting the answer rather than on understanding how to solve the problem (see Table 5). The class discussed differences between the skits and how they applied to their own small-group work.

Table 5

Helping Skit #2 (Unhelpful Helping)

Tony	I've finished the first problem. The answer is 6.8.
Raquel	I didn't get that. Are you sure that's right?
Oscar	I got 6.8 too.
Raquel	I'll change my answer then.
Tony	The answer to number 2 is 325.
Oscar	Just a minute. How did you get that?
Tony	Number 3 is 15.6.
Raquel	Slow down!
Tony	If you don't hurry up, you're not going to get finished.
Oscar	Number 4 is 26.

The students also carried out a Pairs-Check (Kagan, 1989) activity. In this activity, groups split into pairs. Members of a pair took turns solving problems while their partner watched and acted as a coach and helper. If a student had difficulty or made an error, the "coach" explained how to solve the problem, monitored how the student reworked the problem, and ensured that the student understood how to solve the problem correctly. After each problem, the two pairs in the group compared answers to make sure that they all solved the problem correctly. They worked as a group to resolve any discrepancies between the work of pairs (see Table 6 for Pairs-Check instructions). We designed this activity to give students practice with all aspects of "teaching," from giving help to monitoring other students' understanding and giving other students opportunities to solve problems for themselves.

Table 6

Instructions for "Pairs-Check" (from Kagan, 1989)

Instructions
You are to work in pairs in your teams. Person 1 in the pair is to do the first problem, while person 2 acts as a coach. Coaches, if you agree that person 1 has done the first problem correctly, give him or her some praise, and then switch roles.
When you have both finished the first two problems, do not continue. You need first to check with the other pair. If you don't agree on the first two problems, figure out what went wrong. When both pairs agree on the first two problems, give a team handshake, and then proceed to the next two problems.
Remember to switch roles after each problem. Person 1 does the odd numbered problems; person 2 does the even numbered problems. After every two problems check with the other pair.

Sample

We implemented the cooperative learning program in 6 seventh-grade general mathematics classes (184 students) at an urban middle school in the Los Angeles metropolitan area. Students had little or no previous experience working collaboratively with other students. Two teachers each taught three classes. All classes had comparable entering student achievement levels and had similar mixes of student gender and ethnic background. All students except those in pre-algebra classes and in remedial or special needs classes took these general math classes, so the classes represented a fairly wide mix of achievement levels. The ethnic breakdown of the sample was 55% Latino, 26% White, 14% African-American, 3% Asian-American, and 2% Middle Eastern or Other. Nearly all students had English proficiency, although many were bilingual. Four of the six classes (two for each teacher) participated in all three phases of the program and serve as the basis for the analyses we present here. The other two classrooms did not receive the full cooperative learning program and served as a comparison group; we do not analyze those classes here (see Webb & Farivar, 1994, for results concerning the contrast between the two comparison classes and the four classes that received the full program). The sample we analyze here consists of the 21 groups that had good quality audiotape data, remained intact, and had non-missing achievement scores (n = 77 in Phase 1, n = 74 in Phase 2, n = 77 in Phase 3).

Procedures

At the beginning of the semester, before any other activity, all classes completed a pretest on general mathematics achievement. Based on pretest scores, ethnic background, and gender, we assigned students to heterogeneous small groups that reflected the mix of backgrounds in the class as closely as possible. We defined three achievement strata in each class based on the pretest scores: high (top 25% of the sample), medium (middle 50%), and low (bottom 25%). We formed groups so that each had one high-achieving student, one low-achieving student, and two medium-achieving students.

Teachers participated in seven full-day workshops to familiarize themselves with the activities they would carry out in their classes during the upcoming curriculum units (three days of training before the unit on decimals, two days before the unit on fractions, and two days before the unit on percents). The training covered issues about group work, the mathematics content to be taught, and how to integrate them. The teachers synchronized their lesson plans so that all classes followed the same schedule of classroom activities (except for the planned variation between conditions on preparation for group work).

At the beginning of each curriculum unit, all classes carried out the communications activities corresponding to that phase of the program. Teachers spread out communications activities over several days. Teachers devoted few class periods entirely to communications activities; most had a mix of these activities and instruction in the mathematics content.

At the beginning of each class period, the teacher introduced the whole class to the material for that day and solved a few example problems with the class. Students then worked within their small groups either on problems assigned in the textbook (general mathematics for Grade 7; Eicholz, O'Daffer, & Fleenor, 1989) or on teacher-prepared activities (e.g., calculating sales tax for meals selected from restaurant menus). The teacher reminded the class about the norms for behavior, and reminded students to consult each other first before asking her for help. The teacher circulated among groups, watching groups work and answering questions where necessary. At the end of each class period, groups turned in their classwork and spent five minutes completing and discussing their checklist of expected group-work behaviors. Periodically, the teacher discussed the groups' experiences in the whole-class setting. The teacher administered a weekly quiz to all students. Students completed a posttest at the end of each unit.

On one day during each curriculum unit, we audiotaped classes working in small groups for the entire class period. We used stereo audiotape recorders with dual input channels so that an observer could identify the student speaking. We audiotaped all classes on several occasions before the study began to familiarize them with the procedures and with the presence of the observers (one per group). During Unit 1 (multiplication with decimals), we audiotaped students trying to determine the costs of long-distance telephone calls (e.g., “What would be the cost of making a 4-minute call to a number with a 755 prefix?” Eicholz, O’Daffer, & Fleenor, 1989, p. 96) using a table with three columns: (a) the prefix of the telephone number, (b) the rate for the first minute of the telephone call for that prefix, and (c) the rate for each additional minute of the telephone call for that prefix. During Unit 2 (fractions), we audiotaped students adding fractions with like or unlike denominators (e.g., 3/4 + 2/3). During Unit 3 (percentages), we audiotaped students converting decimals to percents and percents to decimals (e.g., "Convert 0.36 to a percent," and "In order to find the sale price of a $12 shirt that is on sale for 25% off, convert 25% to a decimal”).

Students completed a pretest at the beginning of each unit and a posttest at the end of each unit. We based the achievement scores used in the analyses presented here on the test problems that correspond to the material covered on the day we audiotaped classes.

The Mathematics Problems

During the three phases, students studied the following curriculum units: Chapter 4, “Multiplication of Decimals,” (Phase 1); Chapter 8, “Addition and Subtraction of Fractions” (Phase 2); and Chapter 12, “Percent” (Phase 3), in a general mathematics textbook for Grade 7 (Eicholz, O’Daffer, & Fleenor, 1989). As we highlight in later sections, teachers closely followed the procedures presented below.

Phase 1. On the day that we videotaped group work, students worked on textbook problems requiring them to determine the costs of long-distance telephone calls using data from a table of telephone rates. The table had three columns: (a) the prefix of the telephone number (the table listed 15 prefixes), (b) the rate for the first minute of the telephone call for that prefix, and (c) the rate for each additional minute of the telephone call for that prefix. For example, for prefix 755, the first minute cost $0.19 and each additional minute cost $0.12. The textbook presented a worked-out example for the problem “What would be the cost of making a 4-minute call to a number with a 755 prefix?” (Eicholz et al., 1989, p. 96):

$ 0.12 ⇓ Rate for additional minutes
x   3 ⇓ Additional minutes

______

0.36
+ 0.19 ⇓ Rate for first minute

_____

$ 0.55 ⇓ Cost of a 4-minute call

Phase 2. A portion of the worked-out example in the textbook for the addition of fractions with unlike denominators had the following steps (although presented in a different format, Eicholz et al., 1989, p. 206):

1. Look at the denominators: 1/4 + 1/8 (Unlike denominators)
2. Find the least common denominator (LCD): The LCD is the least common multiple of 4 and 8. The LCD is 8.
3. Write the equivalent fractions with this denominator: 1/4 + 1/8 = 2/8 + 1/8
4. Add the numerators. Write the sum over the common denominator: 1/4 + 1/8 = 2/8 + 1/8 = 3/8.

Phase 3. Two worked-out examples for the conversion from percent to decimal (Eicholz et al., 1989, p. 298) were the following:

Dividing by 100 shifts the decimal point 2 places to the left. This is a shortcut.

1. 26.7% = 0.267
2. 7.8% = 0.078

Achievement Measures

Pretests. The Phase 1 pretest consisted of 24 items measuring arithmetic and mathematical reasoning skills. The Phase 2 pretest consisted of 17 items concerning the addition and subtraction of fractions. The Phase 3 pretest consisted of 23 items concerning percents and conversion among decimals, fractions, and percents. We used the Phase 1 pretest scores (percent correct) to define three achievement strata:

high (top 25%), medium (middle 50%), and low (bottom 25%). We used 51% and 76% correct on the pretest as the cut points to form the three strata. We formed groups so that each had one high-achieving student, one low-achieving student, and two medium-achieving students.

Posttests. Students completed a posttest at the end of each curriculum unit. For the analyses correlating behavior with student achievement, we focused on the posttest problems that corresponded to the material covered on the days of audio-taping. This made it possible to link students' behavior and performance on exactly the same content. Moreover, we focused on the critical components of those problems, that is, the components that were central to solving the problem and those that caused students the most difficulty.

In Phase 1, one posttest problem paralleled the problem structure of the exercises the students completed while we audio-recorded the groups: "Find the cost of a 10-minute telephone call in which the first minute costs $0.30 and each additional minute costs $0.08." Determining the number of additional minutes in the telephone call (here, 9) constituted the critical component on this problem. Consequently, we used students' accuracy on this component of the problem (scored 1, correct vs. 0, incorrect) as the posttest measure in Phase 1.

In Phase 2, seven posttest problems paralleled those assigned to groups during the day of audio recording (addition of fractions with like and unlike denominators). We used adding the numerators and leaving the denominators unchanged (like denominators) and determining the common denominator and converting the fractions to equivalent fractions with the common denominator (unlike denominators) as the two critical components during our statistical analyses.

In Phase 3, seven posttest problems paralleled group-work problems (converting between decimals and percents). The mean of the scores across these items served as the achievement score used in the analyses.

Coding of Verbal Interaction

Using transcripts of the group-work audiotapes, we identified and categorized student and teacher behavior. We coded students' indication of a need for help, help given by the teachers and students, the activity students carried out after receiving help, non-helping behavior such as comparing answers, and references to classroom norms and expectations for behavior during group work.

The coding differentiated among the different components of each type of problem. For Phase 1 (multiplication with decimals), we used the critical component of determining the number of additional minutes in the telephone call. For Phase 2 (addition and subtraction of fractions), the critical components involved adding the numerators and leaving the denominators the same (adding fractions with like denominators) and finding the common denominator and creating equivalent fractions with the common denominator (adding fractions with unlike denominators). For Phase 3 (converting between percents and decimals), we used the critical component of decimal point placement. Some analyses focus on behavior concerning only the critical components; other analyses sum instances of behavior over all components of the problem.

With one exception, we used the same (or very similar) coding schemes to code teacher-student verbal exchanges and student-student exchanges. One category of interaction appeared for teacher-student exchanges only: a form of recitation that teachers often used (initiation-response-evaluation patterns) which students rarely used with each other. We coded teachers' verbal interaction separately for the whole-class portion of the transcripts (when teachers introduced the class to the day's material and led the class in solving example problems) and the small-group portion (when teachers interacted with particular groups as students worked together to solve problems).

Need for help. Indications of a need for help included general requests for help about how to solve the problem ("How do you do that one?") or general statements of confusion ("I don't get it"), requests for specific explanations ("How did you get 29?", "Why is it over 12?"), and errors ("It's 13 times 30", "I got 6.4"). The variables used in the statistical analyses consisted of a student's frequency of each category of a need for help summed over all problems during group work.

In the analyses linking behavior and student achievement, we included all students who indicated a need for help. The purpose of focusing on these students in the correlational analyses was to confirm previous analyses (using somewhat different coding schemes), which determined the kinds of behavior that best predicted achievement for students who had difficulty with the material (Webb & Farivar, 1999; Webb & Mastergeorge, in press; Webb, Troper, & Fall, 1995).

Help given/received. We defined help given or received as any help specifically directed to a student, whether in response to a question asked or an error made by the target student, or unsolicited by the target student. We coded the help that students received using a five-level rubric. The levels differ in the degree of elaboration and the extent of verbal labeling of the numbers and numerical rules given (see Table 7). We classified one level of help (Level 4) as high because it included at least some explanation of how to obtain the numbers and/or the meaning of the numbers, and included at least some verbal labeling of quantities. Although we refer to Level 4 as high-level help throughout this paper because it constituted the highest level we observed in this study, we acknowledge that it is not high in an absolute sense. In spite of training, Level 4 help remained heavily procedure based, and rarely included any reference to, or explanation of, the underlying structure of the problem or the conceptual reasons for carrying out an arithmetic operation or procedure. The remaining levels of help included no explanation about how to obtain the numbers, nor any indication of the meaning of the numbers, nor any verbal label of any quantity: ranging from a numerical expression or equation without any verbal labeling (Level 3) to no response to a question or a statement of confusion (Level 0).

Follow-up behavior after receiving help. We also examined the nature of work that a student verbalized after receiving help. The work verbalized included acknowledging help (“OK”) or signaling understanding (“OK, I get it”), carrying out work set up by others (“4 times 12 is 48”), and solving a problem without assistance or explaining to another student how to solve the problem. We combined acknowledging help and signaling understanding into a single level because of the ambiguity of many utterances. We could not determine whether statements of “alright” or “OK” signaled understanding of the help received, understanding of how to solve the problem, or merely acknowledgment that the receiver heard the help.

Table 7
Levels of Help

Level	Description	Examples		
		Phase 1	Phase 2	Phase 3
High				
4	Explanation that includes verbal labeling of at least one quantity	19 cents is for the first minute. And then each additional minute is 12 cents. For 5 minutes, the first minute is 19 cents, then the next 4 minutes will be 12.	You see how it has different denominators, so what you have to do is do the common multiples. Go, like, 4 then put 8 then 12 then 16. Then the same for 3, 6, 9, 12. When you do that, the lowest one that you have in common is 12.	You always move the decimal 2 spaces to the right when you're making it into a percent.
Low				
3	Numerical procedure or series of calculations without verbal labeling of any quantity	It's 4 times 12.	3 plus 5 is 8.	And then you count 4 places which gives you 3.
2	Answer to all or part of the problem	29.	11/12	6.4%
1	Non-content response	Do it like she said.	I don't know.	I got something else.
0	No response			

Teacher initiation-response-evaluation (I-R-E) patterns. We analyzed in detail a frequent form of teacher discourse in this study labeled as recitation (Gall, 1984; Nystrand & Gamoran, 1991), initiation-response-evaluation (I-R-E) patterns (Mehan, 1985; Turner et al., 2002), or funnel patterns (Baursfeld, 1988). Teachers often asked a question relating to particular aspects of the problem, received an answer from a student, evaluated the answer, and then quickly went on to the next question. We classified the nature of teacher questions (initiation) in two ways: (a) the nature of the response that the teacher asked the student to provide and (b) the nature of the cognitive processes required of the student to make the response. We categorized required responses as high or low, depending on whether they called for more than a single word, number, or piece of information (see Table 8). We categorized cognitive processes as high, medium, or low, depending on whether the teacher's question required the student to carry out or identify a calculation already set up, or to determine and carry out a not-fully-explicit procedure (see Table 9).

Table 8

Examples of Teacher Initiation-Response-Evaluation Patterns: Student Response Required

Student Response Required[1]	Phase 1	Phase 2	Phase 3
Low			
Piece of information but not a problem step	School's phone number. Now what, what is this stand for? (*area code*)	n/a	n/a
One-word answer	Where is the prefix? (*Here)*	What do you call these? (*The denominator*)	How much did we say the starter jacket was going to cost? (*75*)
Yes/no answer	Did you multiply it times 13? (*Yeah*)	Alright, now can that be reduced? (*Yeah*)	See how I line up my decimal points, also? (*Yes*)
Single number	So you have to multiply 9 times 13. What do you get? (*117*)	What is 4/12 plus 9/12? (*13/12*)	36 times 25. (*900*)
High			
Single arithmetic sequence	So you have to multiply, what? (*54 times 38*)	Now what do I add? (*5 and 3*)	What do I subtract? (*13 dollars and 38 cents*)
Multiple arithmetic sequences	Ho w are we going to figure it out? (*You add the cost of the first minute to the 3 minutes times 12 cents.*)	Tell me what you have to do when you add fractions. (*You have to see if the numbers on the bottom are equal, then you have to add the top numbers.*)	n/a
Step of the problem described in general terms	What do you do first? (*See how much the first minute is.*)	What do you do with that? (*Reduce it.*)	How do I change that to a decimal? (*Divide it by 100.*)
Multiple steps of the problem described in general terms	n/a	What do we know about this problem? (*Adding two fractions with common denominators)*	n/a
Explanation of why a procedure is used	Where did you get 12? (*I subtracted 1.*)	What's the problem of that adding this one? (*You don't have a common denominator.*)	Why is it supposed to be 2.75? (*Because there are 4 decimal places.*)

Note. [1]Responses are ranked from low to high in each category.

Table 9

Examples of Teacher Initiation-Response-Evaluation Patterns: Cognitive Processes Required of Student

	Phase 1	Phase 2	Phase 3
Low			
Look up problem number in book	What is number 1? (*5 minutes dial-direct call to Washington D.C.*)	Can someone else read number 3? (*1/6 plus 2/3*)	How much is the discount? (*37%*)
Look up number in a table	How much is the first minute? (*22 cents*)	n/a	n/a
Recall/identify piece of information	Where is the prefix? (*There*)	What do you call these? (*The denominator*)	What's another word for that? (*Subtraction*)
Read answer off her/his paper	What did you get for number 10? ($1.17)	What did you get? (*9/8*)	What did you get for 20 bucks? (*The picture frame*)
Medium			
Recall problem step when problem already solved in class or group	I have the cost here for the first minute, and there's the other 3 minutes. And then how are we going to figure it out? (*You add the first minute to the 36.*)	You divide 3 into 12 and you get 4. So that means that 3 times 4 equals 12. And then what do you have to do? (*Multiply the 1*)	2 times 5. Carry the 1. And the next step? (*Add*)
Carry out calculation that teacher provides numerically	13 cents each. So you have to multiply 9 times 13. What do you get? (*$1.17*)	4 times what is 12? (*3*)	So you take this price and subtract it from the original. (*$37.77*)
arry out calculation that teacher provides in general terms	And how many minutes are left over? (*9*)	What is the equivalent fraction of 1/3? (*2/6*)	How do you round that off? (*$15.00*)
High			
Determine and carry out arithmetic procedure: all numbers already given explicitly	If there were 8 minutes total, and 1 minute is for the first minute, how many minutes is it going to be for the rest of it? (*8 – 1 = 7 minutes*)	You have to multiply times 2. 3 times 2, and 2 times 2. What's the answer going to be? (*3 X 2 = 6. 2 X 2 = 4*)	You went here and moved [the decimal point] two places to the left. How about 85%? (*0.85*)

Table 9 continued:

Determine and carry out multiple arithmetic procedures: all numbers already given explicitly	12 cents each. Now, this phone call was 4 minutes long. So how are we going to figure out how much that [the 3 additional minutes] costs? (*4 minutes total – 1 minute (of different initial cost) = 3 minutes. 3 minutes X $0.12 each = $0.36*)	How about if I wanted to add 1/4 plus 1/8? Ok? Here is 1/4 up here, and here's 1/8 down here. How are we going to add those two together? (*1/4 = 2/8. 2/8 + 1/8 = 3/8*)	n/a
Determine and carry out arithmetic procedure: some quantities given in general terms but not specific numbers	How many minutes are left if we already took care of one minute? (*Look up total number of minutes in call: XX minutes. XX minutes – 1 minute = YY minutes*)	5/12, and what will 1/4 be equal to? (*Common denominator = 12. _ = 3/12*)	How many places here [does the decimal point go] to the right? (*2 places)*
Determine and carry out multiple arithmetic procedures: some quantities given in general terms but not specific numbers	8 cents. Let's say, I talked for 3 minutes. How much will it cost? (*13 + (3-1)*8 = 37 cents)*	n/a	n/a
Determine and carry out arithmetic procedures: some quantities not given in general or specific terms	n/a	n/a	4 over 10. Tell me how you got this. *([converting 2/5 to 4/10] I divided 5 [into] 10, is 2. 2 times 2 is 4)*
Solve entire problem: no quantities stated	So if you are going to call directly to Washington D.C. how much would that be?	What's it going to be?	10%, does anybody remember what that would be if I changed it to a decimal?
Explain solution process	Then where did you get 12?	30, how did you get that?	How did you do that?

We categorized teachers' evaluation of students responses as high, medium, or low, depending on whether the teacher provided any information about the accuracy of a student's response, or requested or provided any elaboration of the student's response (see Tables 10 and 11 for correct and incorrect student responses, respectively). We should again note that we designated "high" level behaviors relative to the discourse we observed in this study, not in an absolute sense.

Monitoring of student progress or understanding. We also categorized instances of teachers and students monitoring student progress or understanding. For teachers, we considered questions that probed student thinking rather than requesting answers, and requests for answers that occurred outside of I-R-E episodes. Teacher-monitoring questions included asking students if they needed help, asking students to identify what they did or did not understand (or asking students questions to try to identify their source of confusion), asking students to explain how to solve the problem or how they obtained their answer (before any student had provided an answer, or in response to correct or incorrect answers), asking students whether they understood her explanations, and asking which answer a student or group had obtained (outside of I-R-E episodes). Student-monitoring behavior included comparing answers ("What did you get for #6?"), asking another student if s/he needed help ("Do you want us to help you?"), asking another student if s/he understood ("Do you get this?"), asking another student to explain how s/he obtained the answer or solved the problem ("You tell us how to do it"), and asking another student to check the result for accuracy ("Let me see how you did it, anyhow. I want to see if you did it right").

References to desired or undesired student behavior. Finally, we categorized statements made by teachers or students that explicitly referred to desired or undesired student behavior. Teachers referred to the importance of the following behaviors or directed students to engage in them: working together and waiting for each other ("See if you can figure this out. Talk to your group"), helping each other generally ("You are supposed to help each other out here"), helping each other in specific ways ("You have to explain this to her," "Show him what to do. Don't let him copy"), checking with each other or checking for agreement ("Be sure you check with each other"), and checking for understanding ("Make sure everybody understands how to do it").

Table 10

Examples of Teacher Initiation-Response-Evaluation Patterns: Teacher's Evaluation of Student's Correct Responses

	Phase 1	Phase 2	Phase 3
Low			
No response, or goes immediately to the next question	T: 9 times 1, how much? (*S: 9*) T: Plus 2?	T: Alright, now can that be reduced? (*S: Yeah.*) T: What do you divide by?	T: How many places do you move the decimal? (*S: 2*) T: So then what would it be?
Repeats question even when student's answer is correct	T: All right, you could say 7-7-2 prefix. 10 minute this time, long call. All right? Find out what the first minute is...how much? (*S: 22 cents*) T: How much is the first minute? Anybody.	T: Ok, we could go down to 2/4. But can we do it lower than that. (*S: _*) T: Is there some thing else that's lower than 2/4?	T: How much did we say the starter jacket was going to cost? (*S: 75*) T: How much?
Tells student, "I didn't ask you, I asked…."	n/a	T: I asked Lupe.	n/a.
Medium			
Gives a one-word evaluation	Alright.	Yes.	Good.
Repeats answer by a student	22 cents.	1/12 and 1 whole, alright.	2.
High			
Asks student to explain how s/he got answer	n/a	It would be 6. How do you know?	Why is it supposed to be 2.75?
Asks student to give a more complete answer	Ok, 69 what?	What's smaller than 6?	2 places, which direction?
Rephrases student's answer in a more complete way	Ok, the 12 times 4 is for the 4 additional minutes. The 19 cents is for the first minute.	You can't divide them both by the same number any more except one.	Move the decimal 4 places to the left.
Gives explanation for why student's answer is correct	See where my decimal point is? 29 cents.	It's already to it's lowest term.	77%. That means you moved it over once, you gained a dot, and you moved it over again, you gained another dot, like that.

Table 11

Examples of Teacher Initiation-Response-Evaluation Patterns: Teacher's Evaluation of Student's Incorrect Responses

	Phase 1	Phase 2	Phase 3
Low			
Says "no"	No. Can't be.	No.	No.
Repeats question/calls on another student	T: One minute cost 22 cents, and how many minutes are left to figure out the cost of? (*S: Is it 9?*) T: How many minutes are left if we already took care of one minute?	T: We are finding an equivalent fraction of 1/4 that has a denominator of 12. Ok? (*S: 1*) T: We want equivalent fraction.	T: First we need to change the percent to a decimal. Abel, do you want to tell everybody how to do that? (*S: I don't know.*) T: Here's 10%. How do I change that to a decimal?
Asks the question again but in a different way	T: And they cost 12 cents each. How much is that? T: How many are 6 dozen eggs?	T: 6 what? How many 12th are left over? (*S: 1*) T: How many 12th are left?	T: So 10%, what that would be if I changed it to a decimal? (*S: ten one hundreds*) T: A decimal not a fraction.
No response (ignores)			
Medium			
Gives correct answer herself to what she had asked but no reason for why the student's answer was wrong	T: So how much is the first 3 minutes? (*S: First 3 minutes?*) T: 54.	T: This is sixteen, times what is sixteen? (*S: 0*) T: How about one.	T: Which way did the decimal place go? (*S: Here*) T: It moves to the right 2 places.
High			
Asks student how s/he got answer	T: How do you get 6?	T: 30, how did you get that?	T: Ah, Where did you multiply?
Breaks the problem down further by asking another question	T: Elva, how much is the first minute? (S: I don't know.) T: Where is the prefix?	T: All you have to do is just add the numerator here, because the denominators are the same. What is 3 plus 5?	n/a
Gives correct answer to her question and gives correct numerical procedure for finding answer	3? 3 minutes, plus 2 more minutes, makes 5 minutes.	You got 1/2 and 1/2, but you ended up with 2/4	I think you multiplied by a whole number. Multiply by the decimal this time.
Gives explanation for why answer was incorrect	T: Not away from the money. You have to take away from the time.	Ok, we could go down to 2/4. But we can reduce it lower than that.	Well, you have to move the decimal 2 places to the right, and stop.

Teachers also referred to expected behaviors that appeared on classroom charts or were discussed by the class ("I'm going to come around and check to see if you are working together, encouraging each other, and checking for agreement," "If they explain it, that means they don't give them the answer, right?")

Students made references to working together or waiting for each other ("Wait, we have to wait for her"), helping each other ("Do you need some help?"), checking for agreement, ("We checked for agreement"), checking for understanding ("We are checking for understanding in this group"), the importance of doing the work or understanding it ("I want to know how to do it, so I have to ask," and "Don't copy me. Do your own work!"), and the acceptability of copying or depending on others to do the work ("Just copy it," and "Tell me the answer").

Results and Discussion

In the first two sections, we review the results concerning the major behaviors affecting students who needed help: help-seeking behavior, help received, and follow-up behavior after receiving help. In the first section, we present the frequency of student behavior across the three phases for those students who made an indication of a need for help on critical components of the problems. In the second section, we summarize the relationships between student behavior and posttest performance. In these two sections, we replicate the analyses reported previously (refs) using the new, slightly modified coding systems and establish the basis of comparison with teacher behavior. The remaining sections present information about teacher behavior (both when interacting with the whole class and when interacting with small groups) and its correspondence with student behavior.

Frequency of Behavior during Group Work: Students Who Needed Help on Critical Components of the Problems

Table 12 presents the means for the major behavior variables that we coded for students who indicated a need for help with critical components of the problems in each phase. For Phase 1, Table 12 presents the results for students who had difficulty with the component of the problem that concerned identifying the number of "additional minutes" in the telephone call. For Phase 2, Table 12 presents the results for the students who had difficulty determining the denominator when adding fractions with like denominators, or who had difficulty determining the common denominator and/or converting equivalent fractions when the fractions had unlike denominators. For Phase 3, Table 12 presents the results for the students who had difficulty determining how to convert decimals to percents, or percents to decimals.

In Table 12 we present the following major findings. First, in all three phases of the study, students asked fewer specific questions than general questions and the mean frequency of asking specific questions was uniformly low. Second, students received less high-level help (Level 4: explanations with at least one verbal label of a number) than all other levels of help combined. Students did not often receive high-level help from other students or from the teacher; most help consisted of unlabeled numerical procedures or calculations, or the answer to part or all of the problem. When students needed help, they received high-level help slightly less often from another student (the percentage of high-level help ranged from 12% to 32% across the three phases) than from the teacher (the percentage of high-level help ranged from 17% to 40% across the three phases). However, this difference resulted from the fact that students sometimes failed to respond to their teammates' indications of a need for help, whereas teachers never ignored a student who indicated a need for help. When we exclude instances of "no response," the percentage of high-level help received from another student (ranging from 13% to 40% across the three phases) was very similar to that of teachers given above. Third, students more often carried out low-level follow-up behavior after receiving help (acknowledging the help received, signaling understanding, carrying out work set up by others) than engaging in higher-level behaviors such as explaining or solving problems without assistance.

In Table 12 we show the average level of behavior among students who indicated a need for help. In Table 13 we provide information on how many students experienced different types of behaviors, giving the number and percent of students who fell into different categories according to the maximum level of behavior. As we point out in Table 13, except for Phase 1, a minority of students asked specific questions, received high-level help, or carried out behavior at the highest level after receiving help. In Phase 1, more than half of students asked specific questions or received help at the highest level. The decreasing incidence of these desirable behaviors in the later phases of the study runs counter to the increasing stress on these behaviors and increasing practice activities designed to encourage these behaviors over the course of the program.

Table 12

Mean Frequency of Behavior of Students Who Indicated a Need for Help on Critical Components of the Problems

	Phase 1 (n = 46)	Phase 2 Like Denom. (n = 17)	Phase 2 Unlike Denom. (n = 25)	Phase 3 (n = 47)
Indicates a need for help				
General questions	3.07	1.18	1.08	1.51
Specific questions	.93	.29	.28	.19
Errors	1.28	.94	.68	1.45
Receives help:				
From a student				
0 No response	.91	.29	.20	.47
1 Non-content response	.13	.18	.36	.30
2 Answer to part or all of the problem	.39	.76	.32	.62
3 Procedures or calculations without verbal labeling	1.61	.71	.92	.11
4 Explanation with verbal labeling	1.06	.53	.24	.70
From the teacher				
0 No response	0	0	0	0
1 Non-content response	0	0	.16	.11
2 Answer to part or all of the problem	.17	.06	.04	.15
3 Procedures or calculations without verbal labeling	.17	.12	.04	.06
4 Explanation with verbal labeling	.07	.12	.12	.13
From a student or the teacher				
0 No response	.91	.29	.20	.47
1 Non-content response	.13	.18	.52	.40
2 Answer to part or all of the problem	.57	.82	.36	.77
3 Procedures or calculations without verbal labeling	1.78	.82	.96	.17
4 Explanation with verbal labeling	1.13	.65	.36	.83
Follow-up behavior after receiving help				
1 Acknowledges help/signals understanding	1.93	.35	.88	.72
2 Carries out work set up by others	.76	.06	.44	.43
3 Solves problem without assistance or gives explanation	.98	.59	.28	.96

Table 13

Maximum Level of Behavior among Students Needing Help on the Critical Components of the Problems

	Phase 1 (n = 46)		Phase 2				Phase 3 (n = 47)	
			Like Denom. (n = 17)		Unlike Denom. (n = 25)			
	n[1]	%[2]	n	%	n	%	n	%
Did not ask specific questions	22	48	13	76	21	84	39	83
Asked specific questions	24	52	4	24	4	16	8	17
Maximum level of help received from student or the teacher								
0 No response	5	11	5	29	6	24	7	15
1 Non-content response	1	2	0	0	2	8	8	17
2 Answer to part or all of the problem	3	7	1	6	3	12	10	21
3 Procedures or calculations without verbal labeling	7	15	3	18	9	36	3	6
4 Explanation with verbal labeling	30	65	8	47	5	20	19	40
Maximum level of follow-up behavior								
0 None	10	22	7	41	11	44	13	28
1 Acknowledges help/signals understanding	8	17	3	18	5	20	7	15
2 Carries out work set up by others	10	22	1	6	6	24	7	15
3 Solves problem without assistance or gives explanation	18	39	6	35	3	12	20	43

Note. [1]Number of students, [2]Percent of students.

Behavior Predicting Posttest Performance Among Students Who Indicated a Need for Help on Critical Components of the Problems

In Table 14, we present partial correlations (controlling for pretest scores) between the behavior variables and posttest scores for the students who exhibited a need for help during the class period. Several behavior variables significantly correlated with posttest scores. The frequency of asking general questions related negatively to posttest scores in Phase 1. In Phase 1, students who asked a greater number of general questions obtained lower posttest scores than students who asked fewer general questions. The frequency of asking specific questions related positively to posttest scores in Phases 1 and 3. Students who asked a greater number of specific questions obtained higher posttest scores than students who asked fewer specific questions. The frequency of errors did not relate to posttest performance. The maximum level of help received from either a student or from the teacher related positively to posttest scores in all three phases, although only for fractions with like denominators in Phase 2. Students who received high-level help performed better on the posttest than students who did not. Finally, the maximum level of follow-up behavior related positively to posttest scores throughout the three phases. Students who solved at least one problem correctly without assistance or explained how to solve the problem obtained higher posttest scores than students who carried out less active work after receiving help (carrying out work set up by others, acknowledging help, or not responding to help received).

Table 14

Partial Correlations[1] Between Behavior and Posttest Scores Among Students Needing Help on the Critical Components of the Problems

	Phase 1	Phase 2		Phase 3
		Like Denom.	Unlike Denom.	
Indications of a need for help[2]				
General questions	-.30*	.09	-.38+	-.16
Specific questions	.46**	.06	-.13	.32*
Errors	.21	.07	.11	-.13
Maximum level of help received from student or the teacher	.32*	.55*	.15	.39**
Maximum level of follow-up behavior	.53**	.61**	.58**	.30*

Note. [1]Controlling for pretest scores; [2]Frequency of occurrence.
+p = .06; * p < .05; **p < .01

In Table 15 we present the combined effects of the two behavior variables that showed the most consistent relationships with posttest scores: receiving help and follow-up behavior after receiving help. In Table 15 we list the mean posttest performance for students in four categories according to whether they received help at the highest level and whether they carried out follow-up behavior at the highest level. Students who received the highest level of help and carried out the highest level of follow-up behavior produced uniformly high posttest scores (means ranging from .81 to 1.00, where 1.00 is a perfect score). Students who neither received high-level help nor carried out high-level follow-up behavior tended to have the lowest posttest scores. Students who either received high-level help or who carried out high-level follow-up behavior (but not both) tended to receive scores between the highest and lowest categories. Students who only carried out high-level follow-up behavior tended to score higher than students who only received high-level help.

Table 15

Posttest Performance for Different Patterns of Behavior Among Students Who Exhibited Difficulty During Group Work

		Phase 1		Phase 2				Phase 3	
				Like Denom.		Unlike Denom.			
Received highest level help (labeled explanations)	Carried out highest level follow-up behavior (applied help received)	n	M	n	M	n	M	n	M
No	No	13	.08	6	.28	18	.36	20	.53
Yes	No	15	.00	5	.47	4	.41	7	.71
No	Yes	3	.00	3	.78	2	1.00	8	.66
Yes	Yes	15	.87	3	1.00	1	1.00	12	.81

In detailed qualitative analyses of students' experiences in Phase 1, Webb and Mastergeorge (in press) showed that persistence in help seeking and the nature of students' questions, as well as the quality of explanations offered, distinguished the experiences of students in the four categories in Table 15. First, students who both received the highest level of help and used it to solve group-work problems without assistance (the highest level of follow-up behavior, see Row 4 in Table 15) were most likely to ask specific questions that pinpointed the area of uncertainty and persisted in seeking help until they obtained understandable explanations. In the present analyses, the differences in help seeking appeared in Phases 1 and 3. Students in the fourth category in Table 15 were most likely to ask specific questions: in Phase 1, 67% of the students in this category asked at least one specific question compared to 33% to 40% of students in the other three categories; and in Phase 3, 42% of the students in this category asked at least one specific question compared to 0% to 25% of students in the other three categories. In Phase 2, however, this pattern did not emerge. Second, students who neither received high-level help nor carried out high-level follow-up behavior—the first row in Table 15—tended to ask the few specific questions. The percentage of students in this category who asked any specific question was 38% in Phase 1, 17% and 11% in Phase 2, and 0% in Phase 3. When students did not ask specific questions, groups had difficulty determining students' areas of uncertainty, which may have prevented them from formulating relevant and targeted explanations. Furthermore, groups may have inferred that students who only asked general questions or who only asked for answers lacked motivation or could not carry out the task (see Webb & Mastergeorge, in press).

Even when students did receive explanations, those who carried out high-level follow-up behavior were more likely to perform well on the posttest than those who did not (compare the second and fourth rows in Table 15). Webb and Mastergeorge (in press) described two reasons why students who received explanations may not have gone on to solve problems for themselves. First, students may not have understood the explanations offered, which were often unclear, incomplete, sometimes not relevant to the help-seeker's area of confusion, and heavily focused on procedures without accompanying justification for why to use the procedures. Thus, they did not persist in seeking understandable explanations, or their teammates either could not or would not provide understandable explanations, or both. Second, students may have erroneously assumed that they understood the explanations. Without attempting to solve problems after receiving help, many of these students may not have realized that they still had lingering uncertainty. The high frequency of acknowledging help/signaling understanding previously shown in Table 12, coupled with the low frequency of carrying out unassisted problem solving, suggests that students may often have overestimated their level of understanding. The infrequency with which students asked each other to make their thinking explicit, as well as the superficial nature of comprehension monitoring, may have exacerbated this problem. In the sections below, we probe in depth the nature of students' and teachers' monitoring of student comprehension, as well as students' roles as both learners and help-givers.

Teacher Modeling of Help-Giving Behavior

The data we present in Table 12 shows that students who had difficulty with critical components of the group-work problems rarely received high-level help from either their teammates or from the teacher. To give an overall picture of the level of work carried out in groups, in Table 16 we break down all help given by students or by the teacher. This table includes (a) help given to all students whether or not they indicated a need for it, and (b) help across all components of the problem, not only the specific components used in Table 1. When interacting with small groups, teachers gave relatively few explanations with verbal labels (the highest level of help coded in this study; ranging from 17% of instances of help in Phase 1 to 24% in Phase 2). Most of their help consisted of answers, unlabeled calculations, or procedures. When interacting with the whole class, teachers gave proportionately more high-level help, but this still comprised a minority of their help giving (ranging from 36% of instances of help in Phase 2 to 44% in Phase 3). As in interactions with small groups, most teacher help consisted of answers or unlabeled calculations or procedures. Given the level of help that teachers modeled most frequently, it comes as no surprise that students also tended to give help that consisted of answers or unlabeled calculations or procedures (see Table 16).

Although the importance of explaining stands out as the primary focus of the cooperative learning program and every phase of the program emphasized giving explanations, it remains possible that the students and the teachers did not understand the difference between labeled explanations and unlabeled descriptions of numerical procedures. The classroom activities most often admonished students against giving only the answer, rather than specifically addressing the need to verbally label the numbers used in the explanations. The following excerpts make very clear the importance of verbally labeling the numbers used in help. These excerpts involve the same student and demonstrate how a lack of verbal labeling caused a major misunderstanding that the help-seeking student never overcame in Phase 1.

In the first problem (Problem A: Finding the cost of a 9-minute call in which the first minute cost $0.19 and each additional minute cost $0.12), Student 1 did not understand the procedure that other students used to solve the problem: (a) subtract one minute (the first minute) from the total number of minutes in the phone call, (b) multiply the result (the number of additional minutes after the first minute) by the cost of each additional minute, (c) add the cost of the first minute (line 1). In response to her question about what the other students subtracted, she received only unlabeled numerical procedures about what to subtract, namely, the unlabeled procedure "take one off from the 9" (line 2), which in labeled terms would have been "subtract one *minute* from the *total number of minutes in the call, 9 minutes*".

Table 16
Levels of Help Given by Students and Teachers (All Components of the Problems)

	Phase 1		Phase 2		Phase 3	
	M	%	M	%	M	%
Student help[1]						
0 No response	.66	13	.30	10	.57	20
1 Non-content response	.66	13	.53	17	.48	17
2 Answer to part or all of the problem	1.04	21	1.01	32	.92	32
3 Procedures or calculations without verbal labeling	1.90	38	.82	26	.48	17
4 Explanation with verbal labeling	.69	14	.46	15	.45	16
Teacher help (interacting with students in small groups)[1]						
0 No response	.10	6	.07	7	.00	0
1 Non-content response	.30	18	.08	9	.09	16
2 Answer to part or all of the problem	.78	48	.30	32	.25	44
3 Procedures or calculations without verbal labeling	.48	29	.26	28	.13	23
4 Explanation with verbal labeling	.27	17	.23	24	.10	18
Teacher help (interacting with the whole class)[2]						
0 No response	.00	0	.00	0	1.00	5
1 Non-content response	.25	2	.50	2	.25	1
2 Answer to part or all of the problem	3.00	28	7.75	32	7.75	38
3 Procedures or calculations without verbal labeling	3.25	30	7.25	30	2.50	12
4 Explanation with verbal labeling	4.25	40	8.75	36	9.00	44

Note. [1]n = 77 students in Phase 1; n = 74 students in Phase 2; n = 77 students in Phase 3.
[2]n = 4 classes in each phase.

Problem A

1	Student 1	I don't know. I don't understand it… What'd you take off?
2	Student 2	You take one off from the 9, and then times the 9, I mean 8.

Student 1 misunderstood what the "one" represented and in the next problem subtracted "one" from the cost of each additional minute instead of subtracting one from the total number of minutes in the call (lines 3, 5, 7).

Problem B

3	Student 1	I got a dollar 54. Why is it a dollar 52?
4	Teacher	Well, you multiplied the wrong thing. I don't know where you even got the 12.
5	Student 1	I got 11 right here. Then I get, that's what I heard, they were saying to take off one.
6	Teacher	Not away from the *money.* You have to take away from the *time.*
7	Student 1	Then I take away from that, and then…
8	Teacher	OK, now you know what to do. Alright?

Although the teacher corrected the student's error (lines 4 and 6), she did not explain what quantity "the time" represented, namely the total number of minutes in the telephone call, nor did she explain why Student 1 should subtract from "the time" instead of from "the money" (line 6). That is, the teacher did not clarify the structure of the telephone call as consisting of two subgroups of minutes: the first minute with one cost, and the remaining minutes (one minute less than the total number of minutes) with a different per-minute cost. After the teacher voiced her correction, the student repeated what the teacher said, but in an unlabeled form ("take away from that," line 7). The teacher interpreted the student's correct, albeit unlabeled, statement as indicating that the student "knew what to do" (line 8). That is, from the student's correct statement, the teacher appeared to make the assessment that the student could now determine and carry out the correct procedure (an inference that teachers appeared to make repeatedly during instruction, we recount in later sections). The teacher erred in her assessment, however, and the student could not solve any subsequent group-work problem and never did learn how to solve the problem. Interestingly, the student herself recognized her confusion, repeatedly insisting on subsequent problems that she "Didn't understand it" and "Didn't know what to do." Despite her explicit statements of confusion, this student continued to receive only unlabeled numerical procedures from her teammates, and received no further help from the teacher.

In Phase 2, teachers often referred to numerators or denominators as "it", "this" or "them" without specific labels as in: "This [the denominator] stays the same [when adding fractions with like denominators]," "Are they [the numbers given by a student] the smallest common [denominator]?" "You have to change them [the unlike denominators] to the common multiples." Moreover, teachers described unlabeled procedures while referring to ambiguous numbers. For example, for the problem 1/3 + _ , the teacher asked the question, "What are the multiples of three?" without clarifying whether she was referring to the denominator of the first fraction or the denominator of the second. Students also left off referents in their questions and suggestions. For example, when adding 5/8 + 1/3, a student asked "Do we always times it [the smaller denominator] by the highest [the larger denominator]?" The lack of labeling confused many students, as we show in a later example in which a student erroneously applied this procedure (multiplying the unlike denominators) to the *numerators* of fractions with like denominators.

In Phase 3, help-givers created ambiguities by referring to "moving it [the decimal point]" without specifying the number of spaces, the direction of the move, or both. When working in groups, students often suggested moving the decimal point in the wrong direction (converting 3.00 to 0.03%, or converting 1.36 to .0136%) or the wrong number of spaces (converting 1.36 to 13.6%). In addition to their incorrect suggestions, they often admitted uncertainty ("I got 0.136 but I know that's wrong").

Consistent with the procedural focus of the textbook examples and instructional approach, teacher gave procedurally based descriptions of how to solve the problems. In all of the teachers' interactions with small groups, only once did a teacher offer an explanation that referred to the conceptual basis for a problem. This occurred in a Phase 1 group calculating the costs of telephone calls in which the first three minutes had a certain cost depending on whether the call was "dial-direct" or "operator assisted." Not only did the teacher clarify the difference between the two, and point to the structure of the problem with a particular cost for the beginning of the call and a cost for each additional minute (lines 3, 5), she explained the basis for the different cost of the two call types (line 5).

1	Teacher	What's it say, number 1?
2	Student	5 minutes dial-direct call to Washington D.C.
3	Teacher	Yeah, do you know what that means, dial-direct? Who dials?
4	Student	You do.
5	Teacher	You do, right. Otherwise, this is when you call the operator and she helps you. It costs a lot more for her to do that. Look, [it] costs you 50 cents for the 3 minutes, [and] 2 dollars and 5 cents to have her do it. There's a lot difference. But each additional minute is the same. So if you are going to call directly to Washington D.C., all right?

We do not know whether this kind of explanation, with explicit attention to the structure of the phone call, would help students learn how to solve the problems. The student in this case had already solved all of the previous problems correctly and had no difficulty with anything other than a lack of familiarity with the terms "dial-direct" and "operator assisted." No other teacher explanation explicitly mentioned the structure of the phone call as having one minute (or three minutes) at the beginning of the call that cost more than each of the remaining minutes, nor did the teachers mention a reason for this call structure. In Phase 2, no teacher ever explained why fractions had to have common denominators before adding them. Similarly, in Phase 3 teachers never explained why the decimal point moved in the conversion between decimals and percents. In fact, when a student made the connection between a percent and decimal in Phase 3 by showing the equivalence of the percent, fraction, and decimal ("ten over, ten one hundreds"), the teacher cut short the suggestion ("[Give] a decimal, not a fraction"). Thus, the teacher passed up the opportunity introduced by the student to clarify the equivalence of the different forms of number.

The teachers' focus on procedures in their help giving implies a goal of developing students' procedural knowledge, also called instrumental understanding (Skemp, 1978), which characteristically includes knowledge of "the algorithms, or rules, for completing mathematical tasks" (Hiebert & Lefevre, 1986, p. 6); "step-by-step procedures executed in a specific sequence" (Carpenter, 1986, p. 113); and "action sequences for solving problems" (Rittle-Johnson & Alibali, 1999, p. 175). Without addressing the underlying structure of problems and the reasons behind applying certain procedures, students rarely develop "relational understanding," Skemp's (1978, p. 9) term for performing procedures with understanding: "knowing both what to do and why." Researchers also refer to understanding the underlying concepts or principles and the relationships among them as "conceptual knowledge" (Hiebert & Lefevre, 1986; Rittle-Johnson & Alibali, 1999; Silver, 1986;). Teachers' sole focus on procedural aspects of the problems may limit students' progress in learning mathematics. It is well documented that students with relational understanding can better transfer their knowledge to novel problems than can students with instrumental understanding (Silver, 1986).

Teacher and Student Discourse: Initiation-Response-Evaluation Patterns

More often than the teachers gave help to students, they modeled a form of instructional discourse described as initiation-response-evaluation (I-R-E; Mehan, 1985; Turner et al., 2002). That is, the teacher asked a question relating to some aspect of the problem, received an answer from a student, and then evaluated the answer. Tables 17 to 20 provide information about the nature of the response that the teacher asked students to provide, and the cognitive processes required of the student to answer the teacher's question, and the nature of the teacher's evaluation of student responses to her questions. Information about teacher interaction with the whole class appears in Tables 17 and 18; information about teacher interaction with small groups appears in Tables 19 and 20.

Teacher interaction with the whole class. Consistent with much previous research on classroom interactions (e.g., Cazden, 2001), I-R-E episodes dominated teacher discourse when interacting with the whole-class. The frequencies of teacher questions and evaluations of students' responses were very large (Table 17), indicating a rapid-fire sequence of questions/answers with little or no wait time (see also Black, Harrison, Lee, Marshall, & Wiliam, 2002).

Table 17

Teacher Initiation-Response-Evaluation Patterns with Whole Class: Mean Frequency of Required Student Responses

	Phase 1		Phase 2		Phase 3	
	M	%	M	%	M	%
Response that teacher asks student to provide						
Low	**28.00**	**88**	**52.25**	**77**	**16.50**	**69**
Piece of information but not a problem step	0.25	1	.00	0	.25	1
One-word answer	6.00	19	3.75	5	2.75	11
Yes/no answer	1.75	5	7.25	11	1.00	4
Single number	20.00	63	41.25	60	12.25	51
High	**4.00**	**12**	**16.00**	**23**	**7.50**	**31**
Single arithmetic sequence	.00	0	3.00	4	1.75	7
Multiple arithmetic sequences	1.50	5	2.00	3	.00	0
Step of the problem described in general terms	2.25	7	8.25	12	5.50	23
Multiple steps of the problem described in general terms	.00	0	.25	0	.00	0
Explanation of why a procedure is used	.25	1	2.50	4	.25	1
Cognitive processes required of student						
Low	**16.75**	**54**	**10.75**	**16**	**2.75**	**11**
Look up problem number in book	1.75	6	1.50	2	.25	1
Look up number in a table	11.00	35	.00	0	.00	0
Recall/identify piece of information	4.00	13	7.50	11	2.50	10
Read answer off her/his paper	.00	0	1.75	3	.00	0
Medium	**10.25**	**33**	**45.00**	**66**	**16.25**	**68**
Recall problem step in problem already solved	2.00	6	8.00	12	6.25	26
Carry out calculation that teacher provides numerically	4.75	15	26.00	38	8.50	35
Carry out calculation that teacher provides in general terms	3.50	11	11.00	16	1.50	6
High	**4.25**	**14**	**12.50**	**18**	**5.00**	**21**
Determine and carry out arithmetic procedure: all numbers already given explicitly	1.25	4	3.75	5	1.75	7
Determine and carry out multiple arithmetic procedures: all numbers already given explicitly	.25	1	.00	0	.00	0
Determine and carry out arithmetic procedure: some quantities given in general terms but not specific numbers	1.00	3	6.00	9	2.75	11
Determine and carry out multiple arithmetic procedures: some quantities given in general terms but not specific numbers	.25	0	.00	0	.00	0
Solve entire problem: no quantities stated	.00	0	.75	1	.25	1
Explain solution process	1.50	5	1.75	3	.25	1

Note. n = 4 classes in each phase.

Table 18

Teacher Initiation-Response-Evaluation Patterns with Whole Class: Mean Frequency of Teachers' Evaluation of Students' Responses

	Phase 1		Phase 2		Phase 3	
	M	%	M	%	M	%
Student's response is correct						
Low	**4.75**	**13**	**18.75**	**23**	**4.75**	**16**
Tells student to be quiet	.00	0	3.50	4	.25	1
Repeats question without acknowledging student's response	2.25	6	4.75	6	1.75	6
No response; immediately asks another question	2.50	7	10.50	13	2.75	9
Medium	**26.25**	**70**	**51.75**	**61**	**19.75**	**68**
Repeats student's answer	16.25	44	27.75	32	12.50	43
Gives one-word evaluation	10.00	27	24.00	28	7.25	25
High	**6.25**	**17**	**14.50**	**17**	**4.50**	**16**
Gives explanation for why response is correct	.25	1	.75	1	.50	2
Adds detail to student's response	4.75	13	9.00	11	3.50	12
Asks student to add detail to her/his response	1.25	3	1.25	1	.25	1
Asks student to explain how she/he obtained answer	.00	0	3.50	4	.25	1
Student's response is incorrect						
Low	**7.50**	**71**	**13.00**	**73**	**7.75**	**78**
Ignores student's answer	.75	7	2.00	11	.50	5
Repeats question in a different way	2/50	24	2.50	14	1.75	18
Calls on a different student	3.25	31	5.25	30	3.75	38
Says "no"	1.00	10	3.25	18	1.75	18
Medium	**.50**	**5**	**1.25**	**7**	**.50**	**5**
Provides correct answer with no additional information	.50	5	1.25	7	.50	5
High	**2.50**	**24**	**3.50**	**20**	**1.75**	**18**
Provides correct answer and numerical procedure for obtaining answer	1.00	10	.00	0	.25	3
Explains why student's answer is incorrect	1.00	10	2.75	15	1.50	15
Asks student to explain how she/he obtained answer	.50	5	.75	4	.00	0

Note: n = 4 classes in each phase.

As we show in Table 17, the majority of teachers' leading questions requested low-level information, usually a single-number answer to one step of the problem. Teachers infrequently asked students to give an elaborated response. While some of the teachers' questions required elaborated responses consisting of arithmetic sequences ("…now what do I add?"), more questions required descriptions of a step of the problem in general terms (e.g., "What do you do first?" or "What do you do with that?"). Teachers very rarely requested that a student explain *why* she used a procedure. Across the three phases, teachers slightly increased the number of times they requested "high-level" responses (more than a single number or answer). This increase resulted from the increasing frequency of teachers asking students to describe a step of the problem in general terms. However, requesting high-level responses remained relatively infrequent in all phases.

Teachers' questions generally required students to engage in only low- or medium-level cognitive processes in order to formulate a response (see Table 17). In Phase 1, teachers often asked students to carry out fairly low-level processes, such as looking up a number in a textbook table or recalling a piece of information. In Phases 2 and 3, teachers often required students to carry out a calculation that the teacher had already set up, either in explicit numerical terms ("…So you have to multiply 9 times 13. What do you get?") or in general terms ("Alright, this [the common denominator] is 6...1/3 equals how many sixths?"). Teachers infrequently asked students questions that required them to *determine* and carry out procedures for themselves. Instead, when teachers required students to carry out steps in the problem, they usually provided the quantities to use, either as explicitly named numbers ("If there were 8 minutes total, and 1 minute is for the first minute, how many minutes is it going to be for the rest of it?") or in general terms ("How many minutes are left if we already took care of one minute?"). Rarely did teachers ask students to solve problems or explain how to solve them without having already set up the procedure or naming the quantities to be used.

As we present in Table 17, teachers responded minimally to students' responses. When a student gave a correct answer, teachers usually repeated the answer or gave a one-word evaluation, or both. In a substantial proportion of instances, the teacher did not respond at all to the answer but immediately asked another question. In some cases, the teacher added detail to a student's response. Rarely did the teacher ask the student to explain how she obtained the answer. Except for a few, seemingly rhetorical (given the lack of wait time), questions at the end of her evaluation ("alright?"), the teacher never inquired whether *other* students understood the answer or invited other students to ask questions about the student's work. When a student answered incorrectly, the teacher was more likely to explain why the answer was incorrect or give the correct procedure, or to ask the student to explain how s/he obtained the answer (a tendency shown by teachers in other studies, see Moyer & Milewicz, 2002), but these behaviors still occurred infrequently. More often, the teacher did not try to identify what procedure the student used to obtain the incorrect answer or what confusion formed the basis for the incorrect procedure, but instead repeated her question or rephrased it, or simply called on a different student, sometimes without any explicit acknowledgment of the incorrect answer. These instances represent missed opportunities for the teacher to diagnose and clarify sources of student misunderstanding.

The following examples of I-R-E teaching episodes from the three phases show more clearly how teachers used their questions to move the class through the problem-solving procedures. The following excerpt exemplifies a typical initiation-response-evaluation sequence that the teacher engaged in with the class. The problem required students to calculate the cost of an 8-minute call in which the first minute cost 13 cents and each additional minute cost 8 cents.

1	Teacher	Alright, I would like everybody to start with number 2. Who could read it? Ah, back at the end in the corner. Page 96, number 2.
2	Student 3	8-minute call to [prefix] 726.
3	Teacher	8 minutes. How much for the first minute?
4	Student 4	13 cents.
5	Teacher	13 cents. How many minutes do I have left?
6	Student 5	6. [incorrect]
7	Teacher	How do you get 6?
8	Student 6	Ah.
9	Teacher	How many?
10	Student 7	7.
11	Teacher	How much for each minute?

12	Student 8	13 cents. [incorrect]
13	Teacher	It's also 13?
14	Student 9	8.
15	Student 10	Yeah, 8 cents.
16	Teacher	13 for the first minute, alright, and 8 cents for each additional. How much is that? 7 times 8 cents. And the total cost will be?
17	Student 11	69.
18	Teacher	Alright, please write it on your paper. You have to write this: 8 minutes, the first minute cost 13 cents. Then you have 7 minutes left, which cost 8 cents each. Alright, any question about number 2?

In this excerpt, the teacher set up nearly each step in the problem (separating the total number of minutes in the call into two subgroups consisting of the first minute and the additional seven minutes, lines 3, 5; calling attention to the different costs for the first minute and each additional minute, lines 3, 11; providing the arithmetic procedure for determining the cost of the seven additional minutes, lines 16, 18). The only step she did not explicitly identify was adding the cost of the first minute to the cost of the seven additional minutes. Although she identified most of the steps in the procedure for solving the problem and gave a well-labeled and fairly complete overall summary (line 18), students had only a minor role to play in the teaching episode. The teacher expected the students to provide the correct answers for the arithmetic procedures that she herself set up and, therefore, the students had no responsibility for determining the procedures themselves. Moreover, the teacher did not explain why students' answers were correct or incorrect, nor did she ask students to justify their answers or provide any information about their thinking processes or strategies for obtaining their answers. Notably, she did not probe the reasons for the two incorrect answers given by students (lines 6, 12). Although she initially asked the student how he obtained his incorrect result (line 6), she did not persist when the student did not volunteer what he had done, and almost immediately called on a different student. Furthermore, except for a seemingly rhetorical question at the end ("Alright, any question about number 2?"), she did not invite students to ask questions or seek help in any way, further confirming that the students' role was limited to providing the correct numbers in response to her questions.

Here we provide two examples for Phase 2: one for the addition of fractions with like denominators and another for the addition of fractions with unlike denominators. In the following excerpt from a whole-class introduction to the addition of fractions with like denominators, the teacher asked a few open-ended questions requiring students to generate at least part of the procedure (adding the numerators). Significantly, however, she provided the correct denominator herself without explanation and without asking students to provide it, thus providing the answer to the most difficult part of the problem (on the posttest, a substantial proportion of students added the denominators).

1	Teacher	4/9 plus 2/9. Tell me what we have to do when we add fractions?
2	Student 13	The numbers on the bottom are equal.
3	Teacher	Okay what do we call these?
4	Student 14	The denominator.
5	Teacher	Good. The denominator. They have to be the same, don't they. Alright, what do I have to do with this problem, then?
6	Student 15	You have to add the top numbers.
7	Teacher	We call that the?
8	Student 16	Numerator.
9	Teacher	Good. 4 plus 2? Anybody? [inaudible response, probably 6] over 9. Is this the final answer? What must I do? Someone tell me.
10	Student 17	Reduce it.
11	Teacher	Okay, reduce it and divide by? And the answer would be?
12	Student 16	3/3. [incorrect]
13	Teacher	2/3. What do we do when we add fractions with a common what?
14	Student 14	Denominator.

15	Teacher	Alright and then we add the numerators and then what do we have to do? Reduce to the lowest term? How about number two? Could someone read it for me?

In this excerpt, the teacher allowed students to call attention to the important features of the problem: recognizing the two fractions' equal denominators (line 2) and that they could add the numerators directly (line 6). More often, however, her questions asked students to provide low-level responses such as giving the correct label for the numerator or denominator (lines 4, 8, 14) or carrying out an operation already given explicitly or suggested implicitly (lines 9, 11). Significantly, the teacher did not explain the necessity of equal denominators (line 5), why they could add the numerators without modification to the fractions (line 9), or why the denominator of the sum of the two fractions remained unchanged (line 9), nor did she invite students to provide explanations. Finally, she did not probe the student's thinking underlying the incorrect answer to one of her questions (line 12).

In the following excerpt for the addition of fractions with unlike denominators, the teacher asked more open-ended questions, requiring students to recognize the difference in the denominators and to determine the numerator of the equivalent fraction. In addition, the teacher elaborated on the answer that the student gave for the common denominator, showing the calculations that yielded the correct answer. However, she continued to take responsibility for breaking the problem into its component steps (find the common denominator, line 7; convert to equivalent fractions with the common denominator, line 8; add the fractions, line 11) and asking students to address only one part of the problem at a time.

1	Teacher	1/6 plus 2/3. I see some trouble. What is it?
2	Student 18	The denominators are not the same.
3	Teacher	The denominators are not the same.
4	Student 19	You pick a number. You find the common numbers that they have.
5	Teacher	Alright is that [called] the least common denominator?
6	Student 20	Yeah. And it would be six.
7	Teacher	Six goes into six and three goes into six. Remember what we did, the factors? Alright what do we have to do now?

8	Student 21	So we go three times two equals six. Now we go two times two equals four.
9	Teacher	Now these are called equivalent fractions. Alright, this stays the same. What do I do now?
10	Student 21	Now you add them.
11	Teacher	Now you add. Four plus one over six, can I reduce that?
12	Student 22	No.
13	Teacher	Number three [next problem].

As in the other excerpts, the teacher did not provide students any opportunity to ask questions about the procedures, about her descriptions of them, or the reasons for carrying them out. We do not know whether she assumed that all students understood how to carry out the procedures or understood her explanations (generalizing from the few who responded correctly to her questions).

In the following excerpt from Phase 3, the teacher took full responsibility for setting up the procedure for converting a percent to a decimal, and only asked the students to verbalize the final placement of the decimal point (line 2). The teacher herself gave the correct answer, however (line 3). As before, the teacher did not explain the procedure nor did she investigate whether students understood what to do.

1	Teacher	…I'm gonna write down a percent which we will turn into a decimal, and I wrote it big for a special reason. Because- oh, I forgot to give these away. I was going to give away some dimes. Ok, 28%. I want to turn this into a decimal, ok? And this is, this is my way—a short cut way of figuring out what's going to happen to the decimal point because we know it's going to change. Take the two dots out of the percent sign and I move the decimal point in 28 over 2 places. So I'm taking away the percent sign and I'm moving one here, I'm moving one here, so where does the decimal point end up being? Eric?
2	Student	In front of the 2.

3	Teacher	In front of the 2. Like this. 0.28. Ok? The decimal, the percent sign disappears and the two dots move the decimal point over 2 places like that.

Teacher interaction with small groups. Teachers often carried out I-R-E patterns with small groups. In many ways, the patterns mirrored those found in teachers' interaction with the whole class (Tables 19 and 20). Teachers usually asked questions that required students to give non-elaborated answers, most often a single number. Again, teachers' questions usually required students to utilize only low- or medium-level cognitive processes. Teachers working with small groups less-frequently required students to engage in high-level cognitive processes than they did in whole-class interaction. When working with small groups, teachers almost always set up the steps for solving the problem and simply asked students for the result of a specific calculation. Rarely did the teacher ask students to determine or carry out any procedure for themselves. Teachers' responses to students' answers remained similar to their responses in whole-class interaction, especially when students answered correctly. When students answered incorrectly, however, the teacher more often provided high-level feedback when interacting with small groups, especially in Phases 2 and 3. This results from the higher incidence of the teacher explaining why the student's answer was incorrect and asking the student how s/he obtained the incorrect answer (see Table 20).

As demonstrated by the nature of teacher discourse within I-R-E episodes, the vast majority of teacher questions required non-elaborated responses and low-level thinking from students. This instructional pattern provided teachers with information about whether students could carry out prescribed operations or calculations, rather than information about whether students could determine and carry out for themselves the procedures to solve a problem. Moreover, the teacher seemed to assume that if a few students could answer correctly, the rest of class could too.

Student interaction with other students. During Phase 2, several students utilized initiation-response-evaluation patterns when helping other students. These episodes mimicked teachers' taking responsibility for setting up the structure of the problem and breaking it into its component parts, and asking questions requiring the target student to carry out low-level thinking to provide single numbers in response. In the following excerpt involving a student who did not know how to reduce fractions, for example, the student helper (S24) asked mostly simple calculation questions: “12 goes into 18 how many times?” (line 4), “How many are left over?” (line 6), “How many times does 6 go into 12?” (line 12).

Table 19

Teacher Initiation-Response-Evaluation Patterns with Small Groups: Mean Frequency of Required Student Responses

	Phase 1		Phase 2		Phase 3	
	M	%	M	%	M	%
Response that teacher asks student to provide						
Low	**1.40**	**81**	**.60**	**83**	**.63**	**74**
Piece of information but not a problem step	.00	0	.00	0	.12	14
One-word answer	.17	10	.00	0	.01	1
Yes/no answer	.04	2	.15	21	.05	6
Single number	1.19	69	.45	63	.45	53
High	**.32**	**19**	**.12**	**17**	**.22**	**26**
Single arithmetic sequence	.10	6	.03	4	.03	4
Multiple arithmetic sequences	.04	2	.00	0	.00	0
Step of the problem described in general terms	.17	10	.09	13	.19	22
Explanation of why a procedure is used	.01	1	.00	0	.00	0
Cognitive processes required of student						
Low	**.83**	**49**	**.09**	**14**	**.41**	**43**
Look up problem number in book	.14	8	.00	0	.16	17
Look up number in a table	.47	28	.00	0	.00	0
Recall/identify piece of information	.21	13	.05	8	.25	26
Read answer off her/his paper	.01	1	.04	6	.00	9
Medium	**.79**	**47**	**.47**	**72**	**.52**	**55**
Recall problem step in problem already solved	.04	2	.09	14	.17	18
Carry out calculation that teacher provides numerically	.32	19	.18	28	.22	23
Carry out calculation that teacher provides in general terms	.43	26	.20	31	.13	14
High	**.06**	**4**	**.09**	**14**	**.02**	**2**
Determine and carry out arithmetic procedure: all numbers already given explicitly	.05	3	.08	12	.01	1
Determine and carry out arithmetic procedure: some quantities given in general terms but not specific numbers	.01	1	.01	2	.01	1

Note. n = 77 students in Phase 1; n = 74 students in Phase 2; n = 77 students in Phase 3.

Table 20

Teacher Initiation-Response-Evaluation Patterns with Small Groups: Mean Frequency of Teachers' Evaluation of Students' Responses

	Phase 1		Phase 2		Phase 3	
	M	%	M	%	M	%
Student's response is correct						
Low	**.18**	**14**	**.12**	**32**	**.22**	**22**
Tells student to be quiet	.00	0	.01	3	.00	0
Repeats question without acknowledging student's response	.00	02	.00	0	.05	5
No response; immediately asks another question	.18	1420	.11	30	.17	17
Medium	**.92**	**71**	**.23**	**62**	**.63**	**62**
Repeats student's answer	.32	25	.11	30	.31	31
Gives one-word evaluation	.60	47	.12	32	.32	32
High	**.19**	**15**	**.02**	**5**	**.17**	**17**
Adds detail to student's response	.18	14	.01	3	.14	14
Asks student to add detail to her/his response	.01	1	.00	0	.03	3
Asks student to explain how she/he obtained answer	.00	0	.01	3	.00	0
Student's response is incorrect						
Low	**.55**	**65**	**.21**	**46**	**.10**	**40**
Ignores student's answer	.05	6	.05	11	.00	0
Repeats question in a different way	.09	10	.07	15	.03	12
Calls on a different student	.23	27	.08	17	.03	12
Says "no"	.18	21	.01	2	.04	16
Medium	**.12**	**14**	**.05**	**11**	**.03**	**12**
Provides correct answer with no additional information	.12	14	.05	11	.03	12
High	**.18**	**21**	**.20**	**43**	**.12**	**48**
Provides correct answer and numerical procedure or obtaining answer	.03	4	.00	2	.03	12
Explains why student's answer is incorrect	.10	12	.14	30	.06	24
Asks a more specific question	.01	1	.01	2	.00	0
Asks student to explain how she/he obtained answer	.04	5	.05	11	.03	12

Note. n = 77 students in Phase 1; n = 74 students in Phase 2; n = 77 students in Phase 3.

1	Student 23	I got 18/12.
2	Student 24	Okay 18/12. Okay what do you have to do then? Okay when the top number is bigger than the bottom number you always have to divide the top number by the bottom number. Did you understand that?
3	Student 23	Yeah.
4	Student 24	When the top number is bigger than the bottom number you have to go, like, 12 goes into 18 how many times?
5	Student 23	Oh yeah. One.
6	Student 24	And how many are left over?
7	Student 23	Six.
8	Student 24	Okay and what would be the denominator?
9	Student 23	12.
10	Student 24	Yes, so it is 1 and a 6. Then you have to reduce that. And when you reduce that then what do you get?
11	Student 23	Um 3/6?
12	Student 24	When you reduce 1 and 6/12 … 1 and 6/12, what would it be? Okay, the one stays the same, the whole number always stays the same when you are doing this. So you just have to lower 6/12 into its lowest terms. What is the biggest number that, how many times does 6 go into 12?
13	Student 23	Twice.
14	Student 24	What would that be? 1 and…?
15	Student 23	1 and 1/2.
16	Student 24	Okay, when you finish number five [next problem] tell us.

More so than in the teacher's I-R-E episodes, the helper student embedded multiple explanations into the sequence of questions and answers (lines 2, 4, 12). Further, the helper student made two attempts to monitor the help-seeking student's understanding that went beyond the teacher's monitoring (for similar findings of more in-depth I-R-E behavior between students than between the teacher and the classroom, see Graesser, Bowers, Hacker, & Person, 1997). First, Student 24 asked Student 23 whether he understood her explanation (line 2). Second, Student 24 directed Student 23 to attempt the next problem (line 16), presumably to check whether Student 23 could solve the problem without assistance. Student 24 did, in fact, check Student 23's work on several subsequent problems, reminding him to reduce his fractions (which he did correctly). However, the bulk of this I-R-E excerpt highly resembles the structure of the teacher's I-R-E episodes, with Student 24 formulating the steps in the problem and asking questions that called for low-level responses, and placing Student 23 in a passive responder role requiring only single-number responses. Student 23 could not reduce any fraction correctly on the posttest, showing that he may followed the prescribed procedures during group work but did not learn how to carry them out for himself.

In summary, through their I-R-E discourse patterns with the whole class and with small groups, teachers modeled three types of behavior. First, they modeled mostly low-level, non-elaborated discourse. In both their questions of students and their evaluation of students' answers, they focused on answers and truncated descriptions of procedures, rather than eliciting explanations of how to solve problems or even detailed procedural descriptions. This may have encouraged students to give answers and arithmetic procedures when they adopted a teacher role, and to accept other students' answers to problems without requesting or providing elaboration on how to solve problems. Second, teachers modeled interactions that defined the role of the "learner" as a passive responder to questions and the role of the "teacher" as the person who assumes the major responsibility for solving the problem. These roles implicitly called for students who understood how to solve the problem to play an active role (solving problems, giving help) and students who had difficulty understanding how to solve the problem to play a passive role (depending on others to show them what to do, and carrying out work set up by others rather than testing their own understanding by trying to solve problems without assistance). Third, teachers modeled little monitoring of student comprehension, but instead appeared to draw inferences about student knowledge from the accuracy of the responses to low-level questions, an issue that we explore in more detail below.

Teacher and Student Monitoring of Student Progress/Understanding.

Teacher monitoring of student progress/understanding. As we described in the preceding section, in the context of I-R-E episodes teachers often used low-level questions and low-level responses to assess the level of student knowledge. Only rarely did teachers inquire about the student thinking underlying their responses. This section focuses on any behavior of the teacher (whether inside or outside I-R-E episodes) that probed student thinking beyond asking for answers or simple procedures.

Table 21

Mean Frequency of Teachers' Monitoring of Student Progress or Understanding

	Phase 1	Phase 2	Phase 3
Interacting with the whole class[1]			
Asks students if they need help or have questions	.50	.25	.25
Asks students to identify what they do or do not understand; or asks question to identify source of student confusion	.00	.25	.50
Asks students to explain how to solve the problem or how they obtained their answer	1.75	6.75	1.25
Before anyone had given answer	.50	1.75	.00
In response to correct student answer	.75	3.75	1.25
In response to incorrect student answer	.25	1.25	.00
Asks students whether they understood her explanation	.00	.00	.75
Interacting with small groups[2]			
Asks students if they need help	.05	.04	.01
Asks students to identify what they do or do not understand; or asks question to identify source of student confusion	.00	.00	.00
Asks students what answer they obtained	.19	.31	.13
Asks students to explain how to solve the problem or how they obtained their answer	.11	.12	.06
Before anyone had given answer	.00	.00	.01
In response to correct student answer	.03	.05	.01
In response to incorrect student answer	.08	.07	.05
Asks students whether they understood her explanation	.01	.00	.00

Note. [1]n = 4 classes in each phase; [2]n = 77 students in Phase 1; n = 74 students in Phase 2; n = 77 students in Phase 3.

We observed six ways in which teachers monitored student progress or probed student thinking that went beyond evaluating answers to simple questions: asking students whether they needed help or had any questions, asking questions to identify the source of student confusion, asking students which answers they obtained, asking students to explain how to solve the problem or how they obtained their answer, asking students whether they understood her explanation, and directing students to apply the explanation given. Table 21 presents mean frequencies of those teacher behaviors. Compared to the frequency of teacher helping behavior (see Table 16) and teacher I-R-E behavior (Tables 17 to 20), teacher infrequently monitored student progress. Although still infrequent, teachers most often monitored student understanding by asking students what answers they obtained and how they obtained their answers.

Teacher questioning of correct responses. Teachers sometimes asked students to explain how they obtained their correct answer or how they would solve the problem (or part of it). As the following very typical example from Phase 1 demonstrates, however, when teachers asked for an explanation, they accepted students simply providing an unlabeled numerical procedure. When a student provided the numerical calculations for obtaining the cost of a 5-minute call in which the first minute cost 19 cents and each additional minute cost 12 cents (line 2), the teacher did not ask why the student proposed his calculations, nor did she provide any elaboration other than to add partial labels for two of his numbers (line 3, we consider the labels “additional minutes” and “first minute” partial because she omitted the words *cost of* in each instance).

1	Teacher	If you have 67 cents, you have the correct answer. Who would like to explain it? All right, your group, please. Nice and loudly. Not too loud because of the microphone now.
2	Student	Ah, 12 times 4 and then you get the answer, and you add 19 cents.
3	Teacher	Ok, the 12 times 4 is for the 4 additional minutes. The 19 cents is for the first minute. Good.

The following excerpt (for a 4-minute call in which the first minute cost 19 cents and each additional minute cost 12 cents) shows a rare instance of the teacher questioning why the student proposed a certain calculation ("Why'd you get that?" line 3). However, in most respects, her behavior resembled the teacher's interaction in the previous excerpt. She did not press the student to provide more than a numerical procedure and she, herself, rephrased what the student said, adding labels to her numbers to provide a more complete description (lines 3, 5).

1	Teacher	Ok, you said 3 12. What do you mean by 3 12?
2	Student	3 12's. Like 3 12s. Yeah.
3	Teacher	Ok, so you are saying multiply 12 cents by 3. Why'd you get that?
4	Student	Because 19 is one, and then there is four, 3 more other ones.
5	Teacher	Ok, there are 4 minutes total. One of the minutes costs 19 cents, and the other 3 cost 12 cents each. So we are going to multiply 3 times 12 to find how much those 3 minutes cost.

The instances in which teachers asked students how they obtained correct answers in Phase 2 similarly focused on numerical procedures. In the first very typical excerpt below (adding 2/5 + 3/10), when the student described his procedure, the teacher asked him to explain how he obtained the fraction 4/10 that he used as an intermediate step (line 2). When the student said the he could not explain it (line 3), the teacher herself supplied the answer, thereby putting words into his mouth (lines 4, 5). When he continued to provide numerical calculations (line 7), the teacher did not press him to explain why he chose the procedure, nor did she provide any explanation herself or even add labels to his numbers (line 8).

1	Student	Denominator. And I did it by, multiplying it by, by yeah, dividing 5 with 10, which equals 10. Which equals 2, but I got, wait, that's not how I did it. Oh, I just write my answer, I got 4 over 10, 3 over 10, equals 7/10.
2	Teacher	All right, 4 over 10. Tell me how you got this?
3	Student	Can't explain it.
4	Teacher	You decided 10 was the...least common multiple?
5	Student	Yeah.
6	Teacher	Alright. How did you get 4 over 10 then?
7	Student	I divided 5 with 10, is 2. 2 times 2 is 4.
8	Teacher	Like this.
9	Student	Yeah.

In the following excerpt from Phase 2 (adding 1/6 + 2/3), the teacher first asked why she could not add the numerators (when the denominators of the fractions were different, line 1). She accepted the student's statement of fact, that the fractions had different denominators (line 2, 3), but did not press the student to explain, nor did she herself explain, why fractions need a common denominator in order to be added. Strikingly, her response to the student who proposed a procedure for finding the least common denominator (line 4, 5) suggests that she did not want to find out what the student thought or what procedure he used, but instead desired a response that corresponded to the procedure she was about to demonstrate.

1	Teacher	What['s] different in problem number 3? We could add these two [the numerators in the previous problem], but why can't we add in number 3?
2	Student 25	You don't have common denominator.
3	Teacher	That's right. They don't have the same denominator. So for these kind of fractions we have to find the common

		denominator. To do that, I'm going to write this vertically 1/6 plus 2/3, to help me. I want you to copy this down like that. Now we have to find a common denominator for both. How do we find the least common denominator?
4	Student 26	What you do is multiply 3 or divide 6 (unclear).
5	Teacher	I'm not sure. You might be right about that, but I want to show you a different process, ok? Hold on for that one. Ok, what we do to find the least common denominator is to find all the multiples, 6 and 3, then find the smallest one they both have in common. [another student's name], can you name me the first 2 multiples of 6?

In nearly all of the instances of the teacher asking students to explain how they solved (or would solve) the problem in Phase 3, the focus was strictly on the procedure for moving the decimal point, as shown in the following excerpt. Although the teacher asked students to "explain," she accepted as complete descriptions of where to move the decimal point and never asked students to explain why they should move the decimal point in a particular direction or a certain number of spaces, nor did she provide an explanation.

1	Student 27	Uh, um, 100, 10 percent is the whole thing, then um, the 100 has 2 like zeros, so the 10 has to go from 2 zeros.
2	Teacher	Two places. Somebody else explain how we can do this?
3	Student 28	Put it in, into decimal.
4	Teacher	Take the percentage sign…
5	Student 28	Change it to a decimal and put it in front of the 10.
6	Teacher	Put it in front here?
7	Student 29	A percent she moved to the right.
8	Teacher	Move it to the right. If I change percent to decimal where do I move it?
9	Students	To the left.

Teacher questioning of incorrect answers. Teachers sometimes asked students how they arrived at their incorrect answers. On some occasions, this teacher monitoring of student work seemed to help the student learn the procedures. In the following excerpt from Phase 1, the student trying to determine the cost of a 30-minute call, in which the first minute cost 22 cents and each additional minute cost 13 cents, made the error of applying the additional-minute cost to all of the minutes in the phone call, including the first minute (line 2). The teacher realized his error and proceeded to help him. She told him the correct procedure for finding the number of additional minutes ("take one minute away from this," line 7), but he did not understand her explanation (lines 8, 10) and asked her twice to clarify. In her first two attempts to explain, she did not label all of the quantities in her explanations, which may have prevented the student from understanding. In her first explanation (line 7), she used the word "this" instead of the full label *the total number of minutes*. In her second attempt (line 9), she used "first one" instead of *first minute* and "how many are left" instead of *how many minutes are left.* In her third attempt (lines 11, 13), she used complete labels for the quantities (*cost* of *the first minute*, line 11; *how many minutes are left*, line 13), at which point the student stopped asking questions about what she meant. After that, the student answered her questions correctly, and the teacher ceased her intervention as soon as he voiced the correct final answer. This student did learn the correct procedure for solving the problem. He solved subsequent group-work problems correctly, and performed well on the posttest. Other students, as we describe below, did not fare as well.

1	Teacher	How did you get this?
2	Student	30 times 13 and then I wrote the problem.
3	Teacher	Where did you get the 30, though?
4	Student	Supposed to be 22 times 13?
5	Teacher	No. How much is the first, first minute?
6	Student	22 cents.
7	Teacher	Alright, so I have to take one minute away from this now.
8	Student	Why?
9	Teacher	'Cause the first one cost 22 cents. How many are left?

10	Student	What do you mean?
11	Teacher	How about writing that this way. For 30 minutes, ok? Use the first minute. How much does it cost? [teacher writes on student's paper]
12	Student	22 cents.
13	Teacher	Now tell me how many minutes are left?
14	Student	29.
15	Teacher	And how much do they cost?
16	Student	13 cents.
17	Teacher	Each.
18	Student	Each.
19	Teacher	You have to multiply that. … You can do that right there. On the paper, what did you get?
21	Student	It's 377 plus 22. … It's 399.

In the excerpt above, the teacher asked two questions in which she inquired about the student's thinking, as opposed to requesting answers to calculations or steps in the problem. Upon noticing his incorrect answer to the problem, she first asked a general question about how he obtained his answer (line 1). Then, after the student verbalized an error (line 2), she asked a specific question about how he obtained the incorrect number (30) he used in his calculation (line 3; the student failed to separate the call into the first minute and the additional minutes, instead treating the call as an undifferentiated set of minutes with a single per-minute cost). The teacher almost certainly knew at this point what error the student made and was encouraging him to explain his thinking, one of the only occurrences of this kind in the study. Had the student described his thought processes, it may have presented an opportunity for the teacher to explain the structure of the call. Interestingly, the student did not take the teacher's question at face value and explain what his calculations, but instead seemed to interpret her question as a challenge to discover the correct answer, as shown by his substitution of a different (also incorrect) number for his previous, incorrect one (line 4). Rather than explain his thinking, he tried to guess what number he should have used. The teacher did not persist in trying to uncover the student's thinking, however, but instead resorted to the I-R-E pattern of discourse described earlier. It is likely that the student's interruptions of the teacher's questioning to ask for clarification (lines 8, 10), which forced the teacher to ask more explicitly labeled questions, accounted for his success.

At other times, teacher monitoring and help giving failed to increase student understanding. In the excerpt below, the teacher noticed another student with an incorrect answer. The problem asked for the cost of a 7-minute call in which the first minute cost 19 cents and each additional minute cost 12 cents. In contrast to the student in the above excerpt, it is likely that this student had no understanding of what to do and merely multiplied the cost of the first minute, 19, by the cost of each additional minute, 12, making an arithmetic error to arrive at 128 instead of 228 (line 1).

Noticing the student's incorrect answer, the teacher asked how he obtained it (line 1), as in the previous example. The student gave only a partial answer, saying that he multiplied (line 4), but without giving the specific numbers. The teacher did not persist in her questioning and try to identify the numbers, however. Instead, she started an I-R-E pattern (beginning on line 5) similar to the one above. She made several attempts at questions designed to have him verbalize the number of additional minutes in the phone call, but the student's erroneous responses (lines 8, 12, 16, 18) made it clear that he did not understand her question. Because the student gave erroneous answers even when the teacher fully labeled the quantities in the procedure ("7 minutes total," line 13; how many minutes were left after taking away the first minute, lines 15, 17), it is likely that the student's confusion went further than not understanding which quantities she referred to. The teacher did not ask how he came up with his incorrect responses, however, and re-started an I-R-E pattern (line 19), at which point the student started giving correct answers to her questions (lines 20, 22, 24, 26, 28). As soon as he verbalized the correct result (line 28), the teacher moved on to another group. Although the teacher directed this student to produce the correct calculations for this problem, his future attempts demonstrate that he never learned how to solve the problem.

1	Teacher	Ah-ha. How do you get, wait a minute, how do you get 128?
2	Student	I, uh.
3	Teacher	Speak to me!
4	Student	I multiplied.

5	Teacher	The first one is 19 cents and then what?
6	Student	And that's, each additional minute was 12 cents.
7	Teacher	And how many additional minutes are there?
8	Student	12.
9	Teacher	No. Can't be.
10	Student	It says, each additional minute...
11	Teacher	It cost 12 cents but how many minutes were there?
12	Student	There are 6, I mean 7 minutes.
13	Teacher	Total. 7 minutes total. First was 19, then you have to multiply 12 times what?
14	Student	7.
15	Teacher	You already took away the first minute. How many do you have left?
16	Student	12.
17	Teacher	No. Minutes. Minutes.
18	Student	Oh. So you have no minutes.
19	Teacher	7 minutes, the first minute is how much?
20	Student	First minute is 19 cents.
21	Teacher	OK, how many minutes do I have left now?
22	Student	6.
23	Teacher	And they cost 12 cents each. How much is that?
24	Student	6 times 12?
25	Teacher	Multiply it out.
26	Student	72.
27	Teacher	72, then you have to add it up with 19.
28	Student	So I add 72 with 19 cents.
29	Teacher	That's right.

It is striking that the teacher employed very similar patterns of help and comprehension monitoring for the two students, despite their vastly different level of understanding. The student in the first excerpt had a basic understanding that the cost of the call was a function of the number of minutes in the call and the per-minute cost, but didn't realize that he had to take into account the different per-minute costs in the. In contrast, the student in the second excerpt did not seem to grasp even the fundamental notion that cost depended on the number of minutes and the cost per minute. It is almost as though the teacher had a certain script to follow, regardless of the nature of a student's misunderstanding. We do not know whether the teacher could have deviated from this script if she knew the depth of the second student's lack of understanding, or would have done so in the highly demanding situation of monitoring the progress of eight groups simultaneously.

Similar patterns of teacher behavior appear in later phases. The following excerpts from Phase 2 show the teacher asking multiple questions in an attempt to identify the error the student made, but also showed the teacher moving directly to the correct procedure instead of explaining why the student's approach was incorrect. In the excerpt below, the student tried to add the fractions 3/16 and 5/16. She erroneously applied part of a procedure previously discussed for finding the common denominator in the context of adding fractions with unlike denominators (multiplying the denominators) to the numerators in the current problem. She multiplied the numerators and used the resulting product as the numerator for each fraction (line 1). Hence, she obtained the intermediate incorrect result of 15/16 + 15/16, which she correctly added to produce the answer 30/16 (lines 3, 5). The teacher repeatedly asked the student what she had done (lines 4, 6, 8, 10), but did not seem to fully grasp the student's calculations. The teacher gave up trying to understand the student's procedure (line 12) and simply told the student to add the numerators "because the denominators are the same" (line 16). The student dutifully carried out the calculation (although her posttest performance showed that she never learned the procedure) and the teacher moved on to another group.

1	Student	I messed up. 3/16. 5/16 times. Yeah that is right. 15. 15.
		[intervening discussion among other students]
2	Teacher	Alright, now if that, what did you get for number one?
3	Student	I got something totally different. 30/16.

4	Teacher	How many?
5	Student	30/16.
6	Teacher	30, how did you get that?
7	Student	Fifteen and fifteen.
8	Teacher	Fifteen and fifteen, how did you get fifteen and fifteen?
9	Student	Okay, 'cause 5 times 3 is closest to 16 which is 15 and ?
10	Teacher	Where did you get that from?
11	Student	I don't know. Timesing that one was closest...
12	Teacher	This is sixteen, times what is sixteen?
13	Student	Nothing.
14	Teacher	How about one.
15	Student	Oh!
16	Teacher	And this stays the same, 3 times 1. See this right here. All you have to do is just add the numerators here, because the denominators are the same. What is 3 plus 5?
17	Student	Oh, 3 plus 5 is 8.
18	Teacher	Okay, write it down.

In the following Phase 2 excerpt, the student tried to reduce the fraction 8/16 to "its lowest terms" and erroneously divided the numerator by two twice and divided the denominator by two once (lines 1, 7). The teacher asked enough questions to ascertain what the student had done (lines 2, 4) and told her both that her approach was wrong ("You can't do that," line 8) and the correct procedure to use ("You have to divide them both [the numerator and the denominator] by the same number," line 6). By asking why her procedure was incorrect (line 9), the student presented the teacher with an opportunity to explain. The teacher, however, bypassed that opportunity and instead proceeded to direct the student to divide the numerator and denominator each by eight (lines 10, 12), and did not wait to see whether the student carried through.

1	Student	Ms. [teacher's name], right here I can take a short cut? I can just go to 2/8?
2	Teacher	Ok, what did you divide by to get that?
3	Student	Huh?
4	Teacher	What did you divide? You had to divide by something. Did you divide by 4 and 4 (unclear)...Is 16 divided by 4...8? Huh-uh.
5	Student	What do you mean 16 divided by 4, oh, 2?
6	Teacher	You have to divide them both by the same number.
7	Student	Ok. But, is what I do, half of 8 is 4, then half of 4 is 2.
8	Teacher	Oh, but you can't do that.
9	Student	I can't?
10	Teacher	Well, why don't you divide by 8? Does 8 go into 8? Does 8 go into 16?
11	Student	Yes.
12	Teacher	Ok, try that.

This student did not fully understand the concept of equivalent fractions and the teacher did not address her misunderstanding. This student maintained her lack of understanding and showed numerous errors on the posttest problems involving reducing fractions, including dividing the numerator and denominator by different numbers.

We give no examples for Phase 3 because all instances of teacher monitoring of incorrect work concerned multiplication errors rather than mistakes about converting between percents and decimals.

Student monitoring of each other's work. Table 22 shows the frequency of student monitoring of each other's work, thinking, and need for help. More common than all other instances of monitoring combined, students most frequently compared answers as their form of monitoring understanding. Discrepant answers often prompted discussion, as we describe below, but agreements almost always generated no discussion at all.

Table 22

Mean Frequency of Students' Monitoring of Each Other's Work, Thinking, and Need for Help

	Phase 1	Phase 2	Phase 3
Students compare answers	2.16	1.93	1.53
Student in helper role			
Asks another student if s/he needs help	.31	.39	.13
Asks another student if s/he understands	.39	.22	.18
Asks another student to explain how s/he obtained (or would obtain) the answer	.01	.08	.08
Asks to see another student's work (to check accuracy)	.03	.19	.04
Student in help-seeking role			
Asks another student to explain how s/he solved the problem or obtained final answer (general question)	.49	.55	.60
Asks another student to explain how s/he solved a specific part of the problem or obtained specific partial result (specific question)	.60	.28	.04

Students rarely attempted to monitor each other's comprehension. Students who knew how to solve the problems (as evidenced by their correct answers or procedures) were labeled as students in a helper role in Table 22. These students sometimes asked other students if they needed help (or declared that they or another student would or should help) or if others understood, but they rarely tried to probe other students' thinking or level of understanding. On a few occasions, one student asked another student to explain how s/he obtained the (incorrect) answer or asked to see another student's work to diagnose the error.

Students' monitoring behavior strongly resembled teachers' monitoring behavior in many respects, especially in not uncovering the misconceptions underlying errors, as seen in the following example from Phase 2. As in a previous example, Student 31 apparently used multiplication involving the numerators when adding fractions with like denominators (7/12 + 11/12). Student 30 noticed and corrected Student 31's error, described the correct procedure (lines 3, 5), and posed the correct calculation for Student 31 to carry out (line 5). Although Student 30 had asked why Student 31 applied multiplication (line 1), Student 30 did not give Student 31 an opportunity to explain, and even cut her off when she tried to express her confusion (line 2). Thus, it appears that Student 30's question constituted a challenge rather than a true request for information. Interestingly, Student 31 did remember the rule to add the numerators, but never learned how to add fractions. On the posttest, she added the numerators for all fraction problems, whether the denominators were like or unlike, and added the denominators, again regardless of whether they were like or unlike.

1	Student 30	Do number 4. Do it neatly. Why are you doing times? It's not times. It's adding. But it's not times. And you put times.
2	Student 31	I know. But how am I supposed to...
3	Student 30	See? When they're the same. What is this? Denominator, right? The bottom is the denominator?
4	Student 31	Yeah.
5	Student 30	Ok. If the same denominator is on the bottom? Ok. You go like this. You go...One like that, right? So you just do it. But if it isn't, that's another thing. See you add 7 plus 11. What is 7 plus 11?
6	Student 31	7 plus 11, ah, 18.

Other students did appear to ask questions in which they wanted to know why a student made an error, but then they did not follow up when the student failed to respond. In the following example, Student 32 added the denominators as well as the numerators when adding the fractions 7/12 and 11/12, obtaining 18/24, which she then reduced to 9/12. Student 33 noticed her error, told her the correct procedure (line 2), and asked how she obtained her incorrect result (line 4). When Student 32 admitted that she did not know—or possibly did not understand—Student 33 accepted her response, did not press her further, and proceeded to work with another student.

1	Student 32	Yeah, it's 9 … [half of] 18 is 9 and that [the denominator] is 24.
2	Student 33	You don't add the denominators. Those stay the same.

3	Student 32	No uh.
4	Student 33	Then, how did you get 24?
5	Student 32	Because I don't know.

In a few instances, students asked other students for their answers to make sure they were correct ("When you get the answer, tell me what it is," and "Reduce it and tell me what you got") or to describe a procedure to make sure they knew what to do, as in the following instance from Phase 3:

1	Student 1	Ok, what does that equal then? How do you, how do you, you guys, how do you turn 10 into a decimal?
2	Student 2	You move the thing over. Point 10, zero point 10. See?
3	Student 1	Ok, go, you do yours.

In most cases, the students having difficulty asked other students to explain how they solved the problem (labeled as students in a help-seeking role in Table 22). They either did not know how to solve the problem, asking a general question about what another student had done ("How did you get that?"), or their work differed from that of another student and they asked a specific question about it ("Why did you put 2 there?") While these questions usually generated a response about how to solve the problem (although often only numerical procedures or calculations rather than labeled explanations), students asked these questions to obtain help, not to monitor other students' level of understanding.

In summary, neither teachers nor students spent much time monitoring student work, progress, or comprehension. Moreover, teachers' and students' monitoring attempts focused on numerical procedures rather than uncovering students' underlying conceptions or misconceptions. The procedural focus of the monitoring observed in this study may, in part, be due to the different possible interpretations of the word "understanding." Teachers acknowledged the goal of developing student understanding and students knew that they needed to "check for understanding," but they may have had in mind instrumental understanding or procedural knowledge, rather than relational understanding or conceptual knowledge (Carpenter, 1986; Hiebert & Lefevre, 1986; Rittle-Johnson & Alibali , 1999; Skemp, 1978).

The monitoring behavior demonstrated by these teachers and students strongly parallels other research showing the extreme difficulty of error diagnosis, even for skilled tutors, and that uncovering the causes of errors occurs infrequently (Graesser et al., 1997). Moreover, both teachers and students employed monitoring questions similar to those found by other researchers investigating the questions used by teachers to assess student understanding (e.g., Black & Wiliam, 1998). In a study of 48 preservice teachers' questioning strategies during diagnostic mathematics interviews with elementary school students, for example, Moyer and Milewicz (2002) found that preservice teachers often accepted answers without any follow up, asked leading questions that directed the child's response, or abandoned questioning to teach the concept instead. Even when probing students' responses, many questions did not acknowledge or take into account the particulars of a child's response.

Expectations for Student Behavior Verbalized by Teachers and Students

Teacher statements. In addition to modeling behavior, teachers communicated their expectations about student behavior through explicit statements. As we point out in Table 23, teachers actively encouraged group work. Teachers most often encouraged students to work together and to help each other. In Phases 1 and 2, teachers sometimes used specific language when directing students to help each other ("How did you get it? Tell them," "That's right. That's how you do it. Perfect. Ok, you have to explain this to her," "Can you help him and see that he understands it?"), but more often gave general directives ("You are supposed to help each other out here."). In Phase 3, teachers posed all directives about helping in general terms. Only once did a teacher ever refer to the importance of student understanding, other than reading the behaviors listed on the classroom charts. Nor did teachers mention the responsibilities of the student receiving the help. The closest reference to the importance of engaging in constructive activity was a single admonition made in Phase 1: "Next time you do this, show him what to do. Don't let him copy." The student behavior observed in this study corresponded closely to the expected behavior signaled by the teacher. Students usually worked together and helped each other, although not at a high level.

Student statements. As shown in Table 23, students also made statements that indicated recognition of their responsibility to work together and help each other. Infrequently, however, did students refer to their need to check for "agreement" or "understanding." Somewhat more often they referred to the task of individual students to do the work and not depend on others. Equally often, however, they allowed or encouraged each other to copy work or answers. Similar to teachers' statements, students' statements showed that they understood in general terms the importance of working together, helping each other, and at least sometimes, doing the work rather than depending on others. We found less evidence that students understood specific responsibilities, such as giving explanations instead of answers and checking each other's understanding. The student behavior we outlined in previous sections corresponds with their explicit statements about expectations and emphasis.

Conclusions

Consistent with previous analyses of student behavior and achievement in this cooperative learning program, students who learned how to solve the problem engaged in the following behaviors: asking each other specific (rather than general) questions, providing explanations that included verbal labels of the quantities in the problems, and using the help received to try to solve problems without assistance. However, these behaviors occurred relatively infrequently, even given the intensive preparation for group work used in this study. Most groups carried out only low-level discussions, with the exchange of unlabeled numerical calculations, procedures, and answers predominating. Students rarely probed each other's thinking or monitored each other's comprehension. Most importantly, students who had difficulty played a much less active role than students who understood (or believed that they understood) how to solve the problems. More often than not, the former students did not describe what they did or did not understand, and passively accepted help given by their teammates without applying it. Despite the accumulation of activities designed to increase students' helping skills, the frequencies of student helping behavior remained quite stable over the course of the semester.

In large part, student interaction in this study mirrored the discourse modeled by the teachers and the expectations communicated during their whole-class introductions and their interactions with small groups. Teachers tended to give unlabeled calculations, procedures, or answers instead of labeled explanations. They often used a recitation approach to instruction in which the teacher assumed most of the responsibility for solving the problem and mainly asked students to supply correct answers to discrete steps in the problem. Finally, teachers rarely encouraged students to verbalize their thinking or problem-solving strategies, or to ask questions. Students adopting the role of help-giver showed behavior very similar to that of the teacher: doing most of the work, providing mostly low-level help, and infrequently monitoring other students' problem-solving strategies and level of understanding. The relatively passive behavior of students needing help, especially in accepting help from their group mates with little more than acknowledgment, corresponded to expectations communicated by the teacher about the learner as a fairly passive recipient of the teacher's transmitted knowledge.

The patterns of teacher behavior observed in this study—especially the high incidence of initiation-response-evaluation recitation patterns—parallel the teacher behavior observed in this country and around the world (Cazden, 1985, 2001; Doyle, 1985; Gall, 1984; Mehan, 1985). Such behavior even occurs among reform-minded teachers. In a study of 25 reform-minded teachers, Spillane and Zeuli (1999, pp. 14, 17) found that all but 4 used procedure-bounded discourse such as asking "questions that required students to do little more than supply the right answer," focusing "students on procedural knowledge," and portraying "doing mathematics as a process of memorizing procedures and using these to calculate right answers by plugging in numbers." International comparisons also mirror these findings. One of the most prominent findings of the Third International Mathematics and Science Study (Hiebert et al., 2003), for example, was the lack of opportunities in U.S. classrooms for students to discuss connections among mathematical ideas and to reason about mathematical concepts.

The lack of focus on teacher discourse in this study's program of cooperative learning likely compounded the fact that teachers probably engaged in low-level discourse as an accustomed practice. Aside from instructing teachers to encourage students to carry out the prescribed behaviors, to direct students to work with each other and consult the teacher only when the group reached an impasse, to refrain from helping students and groups until they had exhausted their own resources, and to redirect students' attention to each other whenever possible, we did not direct teachers to engage with students or groups in any particular way.

The nature of the task and the procedural focus of the textbook may have exacerbated the high frequency of low-level discourse we observed. As we described earlier, assigned problems required few steps, had single correct answers and well-defined procedures for obtaining them, and the textbook focused on procedures for carrying out problems, rather than conceptual understanding. Recognizing that such tasks may limit the meaningfulness of classroom conversation and learning, calls for mathematics reform emphasizing the use of real-world and meaningful problems to raise the level of discourse and thinking in classrooms (e.g., National Council of Teachers of Mathematics, 1989). In a similar vein, Cohen (1994) showed that ill-structured problems without clear-cut answers or procedures, and group tasks that require resources and skills possessed by different group members may increase the level of discussion in cooperative groups. The use of such tasks does not guarantee high-level discourse, however. In the Spillane and Zeuli (1999, p. 15) study of 25 reform-minded teachers cited above, 10 teachers used procedure-bounded discourse even with conceptually-oriented tasks "designed to help students explore principled mathematical knowledge" and offered students opportunities to explore "concepts and to appreciate doing mathematics as conjecturing, reasoning about solutions and methods, and justifying ideas." Similarly, in an international comparison of mathematics instruction, Smith (2000) found that U.S. teachers using mathematically rich problems tended to break them down into single steps and reduced them to simple arithmetic problems.

The low-level discourse observed here may also result from the program's emphasis on some behaviors and not others. The program had several activities concerning help seeking, but the focus on the kind of questions students should ask varied. While the chart of help-seeking behaviors and group processing checklists encouraged students to "ask clear and precise questions," the skit of "helpful" helping behavior modeled general questions such as "I'm having trouble with this one. Maria, will you help me?" Other activities stressed the importance of asking questions but did not specify the type of question; for example, one activity required students to give directions about drawing figures to another student, either with questions allowed or without questions allowed, to demonstrate the importance of asking questions.

The cooperative learning program placed most of its emphasis on giving explanations instead of the answer. The charts developed by the class and posted in the classroom had examples of giving explanations and giving specific feedback to teammates; further, groups attended to this behavior on their group processing checklists. The role-playing skits contrasted giving only answers with providing explanations or descriptions. Other activities also stressed giving explanations, such as Pairs-Check (Kagan, 1989) in which students took turns solving problems while their partner served as coach and helper. The coach held responsibility for explaining how to solve problems when the other member of the pair made errors, for monitoring how the other student reworked the problem, and for ensuring that the student understood how to solve it. Pairs of students then compared answers and needed to resolve any discrepancies. All of these activities focused on the contrast between answers and more elaborated descriptions. However, students did not receive explicit admonishment to provide verbal labels for their numbers, and descriptions of numerical procedures (help we coded as Level 3) usually counted as explanations.

Finally, the preparation activities gave scant attention to carrying out follow-up behavior after receiving help or monitoring student work after providing help. The Pairs-Check procedure described above had the greatest focus on these responsibilities, but even then students spent most of the time describing procedures to other students rather than applying them after receiving help or monitoring each other's comprehension or knowledge.

Although the results of this study demonstrate that a program of cooperative learning must focus explicitly on the responsibilities of help-seekers and help-givers for asking specific questions and monitoring one's own and each other's progress and understanding, it is unlikely that the current program could have implemented further activities. We implemented the current program in classrooms in which teachers lectured and asked occasional recall questions of students, and in which students otherwise worked on seatwork individually without talking among themselves. The implementation of small-group work stood out as a radical departure from teachers' and students' accustomed practices, and involved dramatic shifts in beliefs about teaching and learning (especially whether students could teach each other and could learn without a great deal of teacher guidance), as well as big changes in classroom organization. Moreover, given that at the start of the second semester, many classmates did not know each other's names nor have any history interacting with each other inside or outside of class, a substantial portion of the program had to focus on inclusion and class building to help students feel included and comfortable working with each other. All of this had to occur before beginning to seriously discuss such issues as giving explanations rather than answers. The program was very full and additional activities would not logistically fit into the semester.

Over a longer time period, or in classrooms with experience in small-group collaboration or in which students regularly to shared their strategies for solving problems, we could implement more focused activities. Other researchers have developed effective methods for increasing the level of discourse in cooperative groups that often produce greater student learning as well. These include (a) providing instruction on explaining skills (Fuchs, Fuchs, Kazdan, & Allen, 1999; Gillies & Ashman, 1996, 1998; Swing & Peterson, 1982); (b) assigning students to play roles of summarizer (also called learning leader or recaller) and listener (also called active listener, learning listener, or listener/facilitator; Hythecker, Dansereau, & Rocklin, 1988; O'Donnell, 1999; Yager, Johnson, & Johnson, 1985), often incorporated into scripts for groups to follow (O'Donnell, 1999) to encourage students to give justifications and explanations of their solution methods; (c) requiring students to ask each other specific high-level questions about the material (often called reciprocal questioning, Fantuzzo, Riggio, Connelly, & Dimeff, 1989; King, 1989, 1990, 1992, 1999); (d) providing students specific prompts to encourage them to give elaborated explanations, explain material in their own words, and explain why they believe their answers are correct or incorrect (Coleman, 1998; Palincsar, Anderson, and David, 1993); (e) offering specific instruction in giving conceptual rather than algorithmic explanations (Fuchs, Fuchs, Hamlett, Phillips, Karns, & Dutka (1997); and (f) using specific metacognitive prompts to promote comprehension monitoring and explanations of student reasoning (Mevarech and Kramarski, 1997).

What can teachers do to raise the level of discourse among students—especially encouraging students to share their thinking—and raise the level of constructive activity of group members? It is useful to revisit some of the research introduced early in this paper for additional details about specific strategies researchers developed and implemented. In Palincsar and Brown's (1989; see also Palincsar, 1986; Palincsar & Brown, 1984) form of instruction called reciprocal teaching, teachers helped students carry out certain strategies designed to improve comprehension: generating questions about what they read, clarifying what they do not understand, summarizing what they read, and generating predictions. The researchers developed these strategies to be more accessible to students than general directives "to monitor comprehension" or, as in the present study, "check for understanding" (Brown, Campione, Webber, & McGilly, 1992). For effective implementation of these strategies, teachers must initially take the leadership in explaining the strategies and in modeling their use in making sense of material.

Then teachers ask students to demonstrate the strategies, but give them considerable support. For example, to help a student to generate questions to ask other students, the teacher might probe what information the student gleans from the material, and help the student phrase a specific question using that information, or suggest another way to phrase the question. The teacher gradually assumes the less active role of coach, giving students feedback and encouragement them.

Also in the context of classrooms using small-group work, Yackel, Cobb, and Wood (1991; see also Wood, Cobb, & Yackel, 1995) discussed two general strategies that teachers in their studies used to guide the mutual construction of norms for expected student behavior such as explaining their solution methods to other students and trying to understand other students' problem-solving approaches. First, the teacher used specific situations that arose spontaneously during group work as a springboard for whole-class discussions about the obligations and expectations of the students in these situations. For example, the teacher asked specific groups to explain what transpired during small-group work (e.g., one student solved all of the problems by himself) and describe what behaviors students should, resultantly, engage in (e. g., explain how he solved the problems). Second, the teacher invented specific situations while discussing students' obligations with the whole class. For example, the teacher encouraged the class to verbalize the responsibilities of students (both the speaker and the listeners) when, for example, students divide up the work and do different problems without consulting with each other about how they solved them. The teacher in Yackel et al.'s (1991) study also intervened with small groups directly to renegotiate obligations of students, such as ensuring that students listen to each other's methods. In those situations, she explicitly described her expectations for how the students should behave and why.

Hogan and Pressley (1997) describe similar behavior among teachers trying to promote student learning through thoughtful dialogue, both with students in small groups and with whole classrooms. By asking questions to focus and monitor student thinking, posing hints and suggestions about considering more aspects of the problem, and providing encouragement to think about the problem, teachers help students to "articulate, generate, and refine" their thinking (Hogan & Pressley, 1997, p. 81). The researchers describe this process as instructional scaffolding, in which the teacher provides the right amount of support to enable students to make progress without actually doing the work for them (Wood, Bruner, & Ross, 1976; see also the work of Palincsar & Brown, 1989, described above). Recognizing that teachers cannot interact with all students in the class, even when organized in groups, Hogan and Pressley (1997) describe whole-class scaffolding in which the teacher encourages students to articulate, clarify, and explain their ideas; and elaborate on ideas and connect them to those of other students. To accomplish this, teachers carry out such behaviors as encouraging students to resolve conflicts and disagreements, inviting students to react to each others' ideas, asking for elaboration or clarification of what a student said, and turning questions back to the asker. Presumably, students will transfer these expectations for thinking and verbalization to their interaction with peers in small groups.

Teacher interaction with students in small groups such as that described in the Wood et al. (1991) and Hogan and Pressley (1997) studies requires very close monitoring of group interaction. In the present study, teachers' monitoring of group work focused on whether students worked together, generally cooperated and gave help if asked, and sought help from their teammates before going to the teacher. Unless students requested help, teachers generally did not intervene when the group actively worked together on the task, partly to allow the teacher to observe all groups in the classroom. Teachers would have to pay much closer attention to group dialogue in order to intervene and redirect groups in which students copy work from each other or feed each other numbers and answers to write on their papers, or describe numerical procedures and calculations without explaining their thinking and methods.

The close monitoring of group work described above represents a radical departure from the teachers' behavior in this study. As discussed by Franke, Fennema, and Carpenter (1997), major changes in teacher practices require large shifts in their beliefs about learning (students can learn in many ways, not only from direct teacher instruction) and teaching (information about students' thinking should drive instructional decisions). Moreover, these changes occur over time as teachers work to transform their teaching. Wood, Cobb, and Yackel (1991a, 1991b, 1995) describe year-long teaching experiments in which teachers changed their beliefs about teaching mathematics as they tried to reconcile the conflicts between their accustomed practices (e.g., guiding students through procedures step-by-step as outlined in the textbook's teacher manual) and the experiments' use of instructional activities (e.g., open-ended problems) and settings (e.g., pair collaboration) designed to promote students' construction of their own mathematical learning. The process of teacher change occurred gradually as teachers reflected on their own practices and on the learning of their students.

The challenges faced by one particular teacher (Wood et al., 1991), as she tried to move from teacher-centered to student-centered instruction, highlight the issues raised in this study. The description of her teaching at the start of the study strongly resembles the behavior of the teachers in the present study (Wood et al., 1991, p. 601):

> Initially, she would explain and demonstrate the methods she intended the students to use. Then, she would ask questions to evaluate whether the pupils had understood. If they did not, she would then correct their responses by directing them through the intended procedure in a step-by-step manner until they produced the accepted answer.

Her instructional style, imposing her methods and procedures on students, conflicted with her goal for change, to encourage students to use their own approaches to solve problems. Resolving this conflict required her to change her beliefs about teaching and learning, especially that students could learn by working out disagreements among themselves without her interference, help, or direction to particular methods. As she started listening to students and valuing their thinking, they "responded with their own explanations, which were more complex than the teacher had anticipated" (Wood et al., 1991, p. 601). Her relinquishing control over students' methods and allowing them to create their own meaningful approaches went hand-in-hand with her changing beliefs about the role and responsibility of the teacher (transmit knowledge versus guide students' development of knowledge), a process that occurred over the course of a school year.

The high correspondence between teacher and student behavior we found in this study underscores the fact that promoting high-level discourse in collaborative groups may require a focus on teacher behavior as well as student behavior. Moreover, the research on changing teacher practices and beliefs discussed above, which correspond closely to changing conceptions of learning and teaching that occurred over the past several decades, especially that learning is a constructive and active, rather than a passive, process (Shuell, 1996), suggests that we cannot consider teacher behavior separately from teacher beliefs. To raise the level of discourse in classrooms using collaborative learning, then, may require comprehensive attention to multiple aspects of the classroom context, not just an emphasis on desirable student behavior. Stigler and Hiebert's (1997, p. 19) emphasize a similar notion, posing a warning against simple solutions for improving teaching that "focus on individual features of teaching, such as … asking higher-order questions or forming cooperative groups," which probably will not produce comprehensive changes in teaching and learning. Future research must investigate how changes in the classroom context, including teachers' beliefs, instructional practice, and communication about expected student behavior, may influence the quality of small-group functioning and, consequently, student learning.

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