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Graduate School of Education & Information Studies
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Elementary School Mathematics Classrooms

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ELICITING STUDENT THINKING IN

ELEMENTARY SCHOOL MATHEMATICS CLASSROOMS[1]

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Abstract

The importance of student talk in mathematics classrooms figures prominently in curriculum and teaching standards. Student talk is a vehicle for increasing student learning and for helping teachers monitor student understanding and inform student instructional practices. Although researchers have begun to study the moves teachers may make to support students in making their mathematical thinking explicit, sharing out with others and using it as the basis of conversation, much remains to be known about the teacher practices that help students clarify and communicate their mathematical thinking. To learn more about these teacher practices, we look closely at what teachers say and do as they engage with their students in mathematical conversation and how students participate in relation to what teachers say and do. In this report we examine the questions teachers ask and how those questions support students to detail their mathematical thinking. Although all teachers in this study asked students to explain how they solved problems, an important teacher practice for encouraging further student elaboration and giving complete and correct explanations was asking further questions about specific aspects of students' answers or explanations. We describe the variety of teacher questioning practices and the differences in patterns of student participation that emerged.

Introduction

The importance of student talk in mathematics classrooms figures prominently in the *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics, [NCTM], 1991). Student talk is described as a major component of classroom discourse and as a vehicle for increasing student learning: "Students must talk, with one another as well as in response to the teacher …. When students make public conjectures and

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reason with others about mathematics, ideas and knowledge are developed collaboratively, revealing mathematics as constructed by human beings within an intellectual community" (NCTM, 1991, p. 34). Student talk, and how teachers can foster productive talk, appears explicitly or implicitly in nearly all of the NCTM standards for teaching mathematics, for example:

> Standard 2, The Teacher's Role in Discourse, recommends that teachers orchestrate discourse by "posing questions and tasks that elicit, engage and challenge each student's thinking; listening carefully to students' ideas; asking students to clarify and justify their ideas orally and in writing; deciding what to pursue in depth from among the ideas that students bring up during a discussion; deciding when and how to attach mathematical notation and language to students' ideas; deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty; monitoring students' participation in discussions and deciding when and how to encourage each student to participate" (NCTM, 1991, p. 35).
>
> Standard 3, Students' Role in Discourse, calls for the teacher to promote classroom discourse in which "students listen to, respond to, and question the teacher and one another; use of variety of tools to reason, make connections, solve problems, and communicate; initiate problems and questions; make conjectures and present solutions; explore examples and counterexamples to investigate a conjecture; try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers" (NCTM, 1991, p. 45).

Student talk can lead to increased student mathematical knowledge and understanding in two interrelated ways. First, listening to students talk makes it possible for the teachers (and other students) to monitor students' mathematical thinking. Teachers can use information gleaned from student talk to inform their instructional decision-making practices, including problems to pose and follow-up questions to ask (Franke, Fennema, and Carpenter, 1997). Similarly, when students converse with each other, their talk makes it possible for students to gauge each other's strategies and comprehension, providing opportunities for students to help each other build more complete mathematical understanding.

Second, the act of talking can itself help students develop improved understanding. Describing, explaining, and justifying one's thinking all help students internalize principles, construct specific inference rules for solving problems, become aware of misunderstandings and lack of understanding (Chi, 2000; Chi & Bassock, 1989; Chi, Bassock, Lewis, Reimann, & Glaser, 1989; Cooper, 1999), reorganize and clarify material in their own minds, fill in gaps in understanding, internalize and acquire new strategies and knowledge, and develop new perspectives and understanding (Bargh & Schul, 1980; King, 1992; Peterson, Janicki &

Swing, 1981; Rogoff, 1991; Saxe, Gearhart, Note & Paduano, 1993; Valsiner, 1987; Webb, 1991).

In practice, these two mechanisms typically do not occur in isolation but are intertwined. When a student talks about how he or she would solve a problem, others (the teacher, other students) monitor and evaluate what was said and react with questions, suggestions, challenges, or disagreements. These reactions may cause the student to reevaluate his or her thinking and prompt the student to voice revised approaches or strategies. Others react to these revisions, and so on during the course of the dialogue (Yackel, Cobb, & Wood, 1991). This process of negotiating meanings during the course of dialogue, then, reflects a view of mathematics as both a social activity and an individual constructive activity (Lave & Wenger, 1991; Rogoff, 1998; Schoenfeld, 1989; Scribner, 1992).

The above description makes it clear that not just any kind of student talk is expected to be productive for supporting or challenging students' thinking. Beyond providing answers, students must describe *how* they would solve problems and *why* they propose certain strategies and approaches. That is, for truly productive dialogue to occur, students must provide evidence of both the extent of their procedural knowledge (also instrumental understanding, Skemp, 1978a,b)— knowledge of "the algorithms, or rules, for completing mathematical tasks" (Hiebert & Lefevre, 1986, p. 6), "step-by-step procedures executed in a specific sequence" (Carpenter, 1986, p. 113), and "action sequences for solving problems" (Rittle-Johnson & Alibali, 1999, p. 175)—and the extent of their conceptual knowledge (also relational understanding, Skemp 1978a,b)—understanding of the underlying concepts or principles and the relationships among them (Hiebert & Lefevre, 1986; Silver, 1986; Rittle-Johnson & Alibali, 1999). Moreover, when describing their thinking, students must be precise and explicit in their talk, especially providing enough detail and making referents clear so that the teacher and fellow classmates can understand their ideas (Sfard & Kieren, 2001; Nathan & Knuth, 2003).

Much of the evidence about the effectiveness of giving explanations for learning comes from the literature on structured collaborative settings inside or outside the classroom (with labels like cooperative learning, collaborative learning, peer-based or peer-directed learning). The most consistent finding in this body of research is that providing explanations is positively related to achievement outcomes, even when controlling for prior achievement, whereas giving only answers is not related or is negatively related to achievement outcomes (Brown & Palincsar, 1989; L. S. Fuchs, D. Fuchs, Hamlett, Phillips, Karns, & Dutka, 1997;

King, 1992; Nattiv, 1994; Peterson et al., 1981; Saxe et al., 1993; Slavin, 1987; Webb, 1991; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990).

Classroom Discourse Practices

Despite the demonstrated importance of students explaining their thinking, "teacher-centered instruction continues to dominate elementary and secondary classrooms" (Cuban, 1993). Kennedy (2004) suggests that, despite numerous reforms, instructional practices remain unaltered. In most classrooms students infrequently ask questions (Graesser & Person, 1994) and teacher talk typically dominates classroom discourse (Cazden, 2001). Classroom discourse is often characterized by forms of instructional discourse described as recitation (Nystrand & Gamoran, 1991), Initiation-Response-Evaluation (I-R-E; Turner et al., 2002), or Initiation-Response-Follow-up (I-R-F; Hicks, 1995-1996; Wells, 1993) in which teachers ask students questions and evaluate their responses in a rapid-fire sequence of questions and answers with little or no wait time (Black, Harrison, Lee, Marshall, & Wiliam, 2002). Moreover, the vast majority of teacher queries consist of short-answer, low-level questions that require students to recall facts, rules, and procedures (Ai, 2002; Graesser & Person, 1994), rather than high-level questions that require students to draw inferences and synthesize ideas (Hiebert & Wearne, 1993; Webb, Nemer, & Ing, 2006; Webb, Ing, Kersting, & Nemer, 2006). Even reform-minded teachers often ask questions that require students to do little more than provide correct answers; their discourse focuses on procedural knowledge and portrays "doing mathematics as a process of memorizing procedures and using these to calculate right answers by plugging in numbers" (Spillane & Zeuli, 1999, p. 14, 17). International comparisons also mirror these findings. The lack of opportunities in U.S. classrooms for students to discuss connections among mathematical ideas and to reason about mathematical concepts constituted one of the most prominent findings of the Third International Mathematics and Science Study (Hiebert et al., 2003; Stigler & Hiebert, 1999). These descriptions of the level of student participation echo those made two or more decades ago (e.g., Cazden, 1986; Doyle, 1985; Gall, 1984; Mehan, 1985).

There are a number of general principles guiding teacher discourse practices in mathematics classrooms. Teachers need to scaffold, monitor, and facilitate discourse around the mathematical ideas in ways that support student learning (Kieran & Dreyfus, 1998). Teachers need to ask questions, engage students with one another, support students in articulating their mathematical thinking and find ways to engage students in comparing ideas or coming to consensus. However, as we watch teachers in classrooms, especially those engaged in attempting to support conversation, we see how differently these discourse practices play out in classrooms and what they mean for student engagement. Teachers ask many kinds of questions in many ways, creating different opportunities that are taken up by students in a variety of ways. We recognize the necessity of describing the general principles, but we also see the importance of making more explicit the particulars around how teachers can engage students in mathematical discourse.

A number of researchers have begun to make explicit the moves teachers may make to support students in making their mathematical thinking explicit, sharing out with others and using it as the basis of conversation. For example, Wood (1998) contrasted discourse patterns in which teachers' questioning *funneled* student responses with discourse patterns in which teachers' questioning *focused* student mathematical thinking. In the former, the teacher's questions funnel the conversation so that the teacher actually does the bulk of the intellectual work. In the latter, the teacher's questions focus student attention on important mathematical ideas but place the responsibility of the intellectual work on students. Sherin (2002) described how the teacher can use a "filtering approach" to focus students' attention on particular mathematical ideas. After soliciting multiple solutions from students during which they listened and evaluated one another's ideas, the teacher intentionally filters the ideas, choosing which ones to pursue with the whole class in order to advance particular instructional goals. Forman and her colleagues investigated the use of revoicing as a pedagogical move to support mathematical conversation and help make explicit students' mathematical thinking (Forman & Ansell, 2002). The teacher can use revoicing as a way to align students to a particular argument or way of thinking about the mathematics. Studies have found that often revoicing supports the development of mathematical ideas (Forman & Ansell, 2002; O'Connor & Michaels, 1993, 1996; Strom, Kemenya, Lehrer & Forman, 2001).

Although work like that of Forman and her colleagues, Wood, and Sherin begins to clarify the particulars of how teachers can support discourse and student learning, much remains to be known about the teacher practices that help students make explicit their mathematical thinking. Almost every (if not every) current "reform" approach to the teaching and learning of mathematics requires that teachers engage in supporting students to make explicit their mathematical thinking. Knowing the general principles helps teachers know how to focus their practice but is not enough to help them know how particular aspects of practice matter for students. Our hope is to be able to support teachers in their effort by learning more about the moves teachers make in helping students make their mathematical thinking explicit and how those play out for students and their learning. To do this, we look closely at what teachers say and do as they engage with their students in mathematical conversation and how students participate in relation to what teachers say and do.

This Study

Our analyses take place within the context of classrooms where teachers had engaged in school-based algebraic reasoning professional development. The focus of this professional development was on engaging students in algebraic reasoning, specifically ideas about the equal sign and number relationships within the context of their teaching practice. Teachers and professional development facilitators collaborated on how they could facilitate conversations that made students' algebraic reasoning ideas explicit and pushed on those ideas. Teachers integrated algebraic ideas into their practice in a variety of ways, engaging students in both whole-class and small-group settings. The ongoing dialogue across the monthly meetings included what teachers were trying out in their classrooms and what they could do next to further support algebraic reasoning. We videotaped and audio-taped these classrooms to document the details of teacher and student interaction around algebraic reasoning. We use this data to look at the relationship between what teachers do and how it is taken up within the context of classroom practice and particular mathematical work. Our goal is to understand the details of practice that support students as they attempt to make their mathematical thinking explicit.

This report focuses on one aspect of teacher practice related to supporting mathematical conversation: the work of supporting students to make their mathematical thinking explicit and public. We examine the questions teachers ask and how those support students to detail their mathematical thinking. Here we show that much is to be learned from attending to the details of teacher practice in relation to students' participation. We find that although the initial questions teachers ask may look the same, how they follow up on those questions can vary tremendously. We show that detailed analyses of the relationship between teacher and student participation can help us as a field understand the key issues to facilitating classroom discourse and better provide support for teachers in their classroom practice.

This report does not compare different teachers' practices and relate them to their students' participation. Although it is tempting to provide that type of analysis, our goal here is to begin to unpack the teacher participation that is important, to name it, to distinguish it from other similar types of moves, and to begin to look at it in relation to student participation. We recognize that we are examining only part of teacher practice related to how teachers get student thinking on the table in a classroom. We do this to try to provide detail about the practices in relation to student thinking and begin to build consensus in the field about the practices. We do not make claims about particular practices that all teachers must use. Here we want to show the range of what we see teachers doing and how students engage around those practices in these classrooms. Instead of making claims about the benefits of particular practices, we are trying to make claims about the type of work we need to do as researchers to better understand mathematical conversations in classrooms that support student learning.

Method

Building on the large scale study of professional development (Jacobs, Franke, Carpenter, Levi, & Battey, 2007), we selected teachers who had been engaged in the work for over a year so that we could examine more closely their classroom practice related to algebraic thinking. Here we focus on three classrooms and the ways the teachers supported students to share their mathematical thinking. We videotaped and audio-taped in these classrooms in ways that allowed for a close analyses of the relationship between teacher practice and student participation.

Sample

Three elementary school classrooms, the teachers (two second-grade, one third-grade) and their students from a large urban school district in Southern California are the focus of this study. These classrooms are from schools that serve predominantly African American and Latino students and are similar in terms of their academic performance, percentage of students receiving free or reduced lunch and percentage of students designated as English language learners (California Department of Education, 2006). These teachers were part of a large-scale study focused on supporting teachers to engage with students in algebraic thinking (see Carpenter, Franke, & Levi, 2003; Jacobs et al., 2007). The teachers participated in at least 1 year of on-site professional development and a large-scale study of algebraic thinking professional development.

Prior to the algebraic thinking professional development and the large scale study, the district administrators and teachers recognized the value of engaging in algebraic reasoning in elementary school, and long-term plans for overall school improvement were underway. The district, in its second year of new leadership when the study began, had a history of poor performance and a long-standing sense from outside that it would never do well. According to the state's ranking system and standardized test scores, it was one of the lowest performing school districts in California. As in many urban school districts, hiring and retaining qualified teachers was a struggle. Although the district was making progress, at the beginning of the study, only 57% of the teachers in the sample held credentials and 30% of the teachers were in their first or second year of teaching. The community served by this district had shifted from being predominately African American to being predominately Latino, and at the time of our work, the schools served students of whom 99% were minority, 52% were classified as English Language Learners, and 93% received free or reduced-cost lunch.

Professional Development Program

Participating teachers engaged in professional development to explore the development of students' algebraic reasoning and, in particular, how that reasoning could support students' understanding of arithmetic. The professional development included both school-based workgroup meetings and on-site support. Regular workgroup meetings provided opportunities to engage teachers, mathematics coaches, and professional development facilitators in ongoing learning and to create a community of learners in which all participants supported one another's learning (for further description see Jacobs et al., 2007).

The professional development content, drawn from *Thinking Mathematically: Integrating Arithmetic and Algebra in the Elementary School* (Carpenter et al., 2003), highlighted relational thinking, including (a) understanding the equal sign as an indicator of a relation; (b) using number relations to simplify calculations; and (c) generating, representing, and justifying conjectures about fundamental properties of number operations.

Students' mathematical thinking served as the focus for our interactions in professional development. Our conversations were informed by research on children's algebraic reasoning. We specifically worked to focus teachers' attention on what their students could do while they engaged in algebraic reasoning rather than on what they could not do. Our goal was to help teachers create, for themselves, organized ways of understanding and connecting student responses. As teachers noticed more solutions over the year, we continued to support them in ways that helped them develop notions of themselves as individuals and members of communities who could detail solutions, organize knowledge of students' thinking, and use that information to guide instruction (Franke, Carpenter, Levi, & Fennema,, 2001; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998).

The algebraic-reasoning work depended not only on teachers' encouraging students to solve problems in their own ways but also on teachers' engaging their students in conversations to help them explicate their thinking and debate their reasons for thinking as they did. Teachers needed to see their students' participation in these types of mathematical conversations as feasible and the conversations as valuable, and they had to learn to lead them. For many teachers, thinking about how to both seed and orchestrate conversations was challenging. Within the workgroups, then, teachers were able to gain a sense of possibility—a vision for what was possible and details of supporting practices. Throughout the year we used true or false number sentences and open number sentences as contexts in which teachers could base attempts to seed and orchestrate conversations with students (Carpenter et al., 2003; Davis, 1964). We addressed the range of student responses to both particular number sentences and sequences of number sentences, and we explored ways to orchestrate discussions around those tasks.

Procedures

On two occasions within a 1-week period, we videotaped each teacher's class using two cameras and six audio setups. Each video camera had two audio feeds connected to flat microphones (four flat microphones in all), so that four pairs of students could be recorded simultaneously. Each flat microphone was positioned between members of a pair. Two pairs were audio-taped only. To capture the complete conversation, three microphones were used for each pair: an individual lapel microphone for each student and a flat microphone positioned between the students in the pair (each attached to a different audio recorder). The recording from the flat microphone was the primary source of the conversation in the pair; the recordings from the individual microphones were used to identify the speaker and to fill in gaps in the conversation.

Classrooms were taped as teachers taught topics related to equality and relational thinking. Teachers were asked to cover those topics but were not directed further about the particular problems to present or how to structure instruction. Teachers were also asked to not hesitate to have students to talk, to her or to each other. A common practice in these classrooms was "pairshare" during which pairs of students worked together to solve and discuss problems assigned by the teacher. The structure of the class for all teachers was to introduce a problem, ask pairs to work together to solve the problem and share their thinking, and then bring the class together for selected students to share their answers and strategies with the whole class (usually at the board).

We captured all teacher-student talk during whole-class portions of the class and individual student talk during pairshare for at least 12 of the 20 students in each class. We made comprehensive transcripts of each class session consisting of verbatim records of teacher and student talk, annotated to include details of their nonverbal participation. We also collected student written work, took field notes during class sessions, administered student achievement measures (written tests and individual interviews), and surveyed teachers about their classroom practice over the course of the year.

Classwork Problems

Teachers were asked to cover equality and relational thinking on the days that we observed their classes, topics that were central to the professional development program on algebraic thinking. The following are sample problems: (a) $50 + 50 = 25 + \square + 50$, and (b) $11 + 2 = 5 + 8$ (true or false?).

Coding of Student and Teacher Participation

The coding scheme used to analyze this data highlights the practices teachers used to elicit individual student thinking and stimulate mathematical discussion. This scheme to capture student and teacher participation is a result of several iterations, growing both from the literature about mathematical talk and through our review and discussion of the data.

Identifying Segments

Classroom interactions were broken-up into several episodes for analysis. The structure of these episodes generally followed this standard format—the teacher posed a problem, students discussed the solution within a smaller group composed of two or four students, then the teacher led a discussion of the problem with the entire class. We used these natural divisions to segment the classroom interaction into whole class and pairshare. In this report we focus only on the whole class portion of the classroom interaction.

Some of the whole-class discussions for a single problem were quite lengthy and several students were given a chance to share their thinking. To ensure that we captured the full depth of the discussions and maintained coding in context, we further broke the whole-class episodes into segments. We defined a segment as an extended interaction or discussion between the teacher and an individual student, in which that student had at least two conversational turns. Although some of the segments consisted of just two turns, others were longer. The segments began when the teacher called on the student and ended either when the teacher called on another student. During segments, teachers sometimes directed the discussion toward the whole class instead of an individual student. If the teacher returned to the original student after a whole-class choral discussion, the segment ended when the interaction with the original student ended. If the teacher did not return to the original student after a whole-class choral discussion, the segment ended when the interaction with the whole class began. After segments were identified, student and teacher participation within each segment were coded.

Student Participation

Using transcripts of all class talk (notated to include important nonverbal interaction) and videotapes, we coded student participation during whole-class interaction with the teacher for each mathematics problem according to these two categories: (a) accuracy of answer given (correct, incorrect, or none); and (b) nature of explanation given (correct and complete; ambiguous or incomplete; or incorrect)[2].

For each problem, we used dichotomous scoring for the accuracy of the answer given (or implied) and the nature of each explanation provided (see Table 1 for examples). Students provided different types of correct and complete explanations from computational explanations to relational thinking explanations. In this report we focused on whether the explanations provided were correct and complete, ambiguous or incomplete, or incorrect. We also coded whether students provided additional or further elaboration on their explanation after teachers asked questions.

Our coding scheme allowed for the possibility that the same student might provide multiple answers or explanations during a particular segment. For example, a student who provided a correct answer and both an incomplete or ambiguous explanation and a correct explanation was coded as offering both types of explanations in connection with a correct answer. In this scenario, the student might first provide an incorrect explanation and after interacting with the teacher, provide further explanation that eventually led to a correct, complete explanation. This coding scheme allowed us to determine whether the initial explanation students provided was correct, incorrect, incomplete or ambiguous and whether students eventually provided a correct and complete explanation after prompting from the teacher.

[2] There were numerous ways students participated in these classrooms which we did not focus on in this report.

Table 1

Examples of Student Explanations

Types of explanation	Problem and example explanation
Correct and complete	
Computational	Problem: 20 + 10 = 10 + ?
	Twenty plus ten is thirty, so … The equal sign means that you have to be the same, it has to be the same, so if there's a ten here, then a twenty has to be there. Twenty plus ten is thirty, ten plus twenty is thirty.
Relational	Problem: 11 + 2 = 10 + ?
	'Cause eleven is higher than ten, verdad, is higher than ten, y this one [referring to the two], this one's lower, this one's [referring to the three the student had written in the blank] gotta be higher than this one.
Ambiguous or incomplete	Problem: 100 + ? = 100 + 50
	The fifty will go right there because it has to be the same number.
Incorrect	Problem: 4 + 9 = 5 x 3 – 2 (True or False?)
	I thought it was false because four plus nine is thirteen; and five times three is fifteen. Those two do not match.

Teacher Participation

There were numerous strategies teachers used to help make student thinking explicit. For example, teachers asked questions, revoiced or repeated student answers or explanations, described strategies they thought students used to solve particular problems and highlighted mathematical ideas in student explanations. These teacher moves often occurred in various combinations and frequencies within particular segments. In this report, we focused on the nature of questions teachers asked (see Table 2 for examples) by considering the following types of teacher questions:

1. general questions,
2. specific questions,
3. probing sequence of specific questions,
4. bundles of questions, and
5. leading questions.

Teachers asked general questions that were not related to anything specific that a student said. Students often provided an answer or some explanation and the teacher asked the student a general question to prompt for further explanation or clarification. These types of questions were not related to anything specific that a student said.

If the teacher asked a question about something specific that a student said, this was considered a specific question. This differs from the general question because this type of question highlights a specific concept or strategy that the student said or wrote on the board. This specific question might also be used to direct the student to recognize a certain aspect of the problem or to prompt for further explanation or clarification.

When teachers asked a series of more than two related questions about something specific that a student said, this was defined as a probing sequence of specific questions. One purpose of this probing sequence of questions might be to clarify or unpack a student's thinking or explanation. A sequence of specific questions implies that there are multiple specific questions in a segment.

There were other types of questions used by teachers to pursue student thinking that were not necessarily general or specific. We grouped these questions into "other questions." This other category included teachers asking bundles of questions or leading questions. Bundles of questions were instances in which the teacher asked more than two questions and did not provide the student any opportunity to answer any of the questions. Leading questions were also a series of questions. However, unlike the probing sequence of specific questions or bundles of

questions, this series of questions provided opportunities for students to respond by guiding students to particular answers or explanations.

Table 2

Examples of Teacher Participation

Type of question	Dialogue
Asks general question	Problem: 375 = ? + (3 * 10) Teacher: I'm just a little unsure of how you came up with 345. Can you show me what you did?
Asks specific question	Problem: 100 + ? = 100 + 50 Student: The 50 will go right there because it has to be the same number. Teacher: What has to be the same number?
Asks probing sequence of questions	Problem: 20 + 10 = 10 + ? Student: We didn't put a 20 in the box. We erased the box. Teacher: Okay. Student: And then we put a K. Teacher: Okay. Student: And we knew that 10 equals 10, so then we put 20 plus 20 so K equals 20. And then we thought that 20 plus 10 equals 30 and this has to be 30 so we put a 20 because 20, 20 and 10, 10. So K equals 20. Teacher: Okay, I have a question about the K. What does that mean? The K? Student: I told (name of student) that I wanted to put a letter. Teacher: A letter? Okay. So you could put any letter in there and it wouldn't matter? Student: Yeah. Teacher: So I could put the letter C? Student: Yeah. Teacher: And it would still be 20? Student: Yeah.
Asks bundle of questions	Problem: a + b = b + a Teacher: What do you mean by B has a partner and A has a partner? Can you come up to the board and draw that for us? What do you mean that B has a partner and A has a partner? Here. Let's see. Why don't you use this one? What does that mean? If you want to take any notes on your paper, you may.
Asks leading questions	Problem: a = b + b Teacher: What if we add these two numbers together? What's three hundred plus three hundred?

Our coding scheme allowed for the possibility that these types of questions occurred in combination within the same segment. However, after coding the types of questions that occurred within each segment, we characterized the segment into discrete categories. In other words, if there were multiple types of questions within the same segment, the segment was classified into only one type of question. These discrete categories allowed for a systematic analysis framework across all segments and made it possible for us to analyze the question types that defined each segment rather than analyzing all possible combinations of questions that occurred within particular segments. Our decision rules for this classification are as follows:

- If the segment included a probing sequence of specific questions, it was classified into this category even if there were general question or specific questions that occurred within the same segment.
- If the segment included a general question but not a probing sequence of specific questions it was classified into the general question category.
- If the segment included a specific question but did not include a series of specific questions or a general question, it was classified into the specific question category.
- If the sequence included leading questions that were not specific or general or were not a series of specific questions, it was classified into the leading question category.
- If the sequence includes only a bundle of questions and none of the other types of questions, the segment was categorized into the bundle of questions category.

This decision to characterize the segments into discrete categories for the purpose of these analyses allowed for an exploration of the detailed and nuanced ways that teacher questions teachers used and how they played out in terms of eliciting student explanations. The results section presents the analyses using these discrete teacher questioning coding categories.

Results

Description of Teachers' Questioning Practices

Teachers' directives to students to share their thinking. During whole-class instruction, teachers frequently directed students to share their thinking. In all segments except one (98%), teachers asked the target students to explain their thinking. In 91% of the segments, the teacher explicitly prompted an explanation by requesting a student explanation at the outset of the segment (73%) or by asking students to explain how they got their answer when they didn't immediately volunteer an explanation (33%). In 76% of segments, the teacher also asked the student to elaborate further on the student's explanation (these teacher practices are described in detail in the following sections). Teachers also frequently made remarks reminding students to give other students a chance to explain ("Give [Student] a chance [to explain]"), to listen to each others' explanations ("Let's listen", "I like the way the people at table two are giving them their full attention", "I like the way [Student] is paying close attention to what [Students] are about to share"), and to understand each others' thinking ("Let's understand [Student's] thinking"). In all segments except two (97%), the target student did provide an explanation (or multiple explanations). Clearly, in these classrooms, students were expected to give explanations about their thinking, and did so.

Teacher questions to prompt *further* student explaining. In the remaining sections, we analyze the *kinds* of questions teachers asked to prompt students to clarify or elaborate on their explanations. Table 3 summarizes teacher behavior in response to student explanations about how they solved the problem. As can be seen in Table 3, teachers asked questions about student explanations in the majority of segments (76%). Moreover, they asked an assortment of questions, often in combination, including single general or specific questions, probing sequences of questions, bundles of questions, and leading questions (see Table 1 for examples), with no one type of question predominating.

Table 3

Nature of Teacher Questions Following Student Explanations

Teacher questioning of student explanation	Number of segments
Teacher asked question(s) about student explanation	50 (76%)
Probing sequence(s) of specific questions	7
And general question(s)	3
And specific question(s)	1
And leading question(s)	1
And specific question(s)	2
General question(s)	6
And specific question(s)	3
And leading question(s)	1
Specific question(s)	15
And leading question(s)	3
Non-probing bundle of questions(s)	2
Leading question(s)	6
Teacher did not ask question(s) about student explanation[a]	16 (24%)
Total	66 (100%)

[a]Teachers' responses to student explanations were not coded as questions unless the teachers gave the student an opportunity to respond; see coding description in method section.

As will be described in detail in later sections, teachers' questions were posed in response to what students said, such as requests for students to clarify ambiguous explanations, questions directed toward uncovering the reasoning underlying errors students made, or requests for further elaboration of correct problem-solving strategies. Teachers' questions were clearly not part of a premeditated script, such as always responding to a student's explanation with a generic request for repetition or further elaboration. Furthermore, teachers' questions appeared to be tied to multiple and changing goals within interactions, such as trying to understand students' thinking when it was not clearly stated, helping students to understand how to solve a problem in the face of misconceptions, or highlighting features of a strategy or mathematical idea for the benefit of other students in the class.

Teacher questioning and students' initial explanations. We next explored whether teachers responded differently depending on the nature of students' initial explanations. To do this, we sorted students' explanations into two categories according to their accuracy and completeness: (a) correct and complete, (b) ambiguous, incomplete, or incorrect. Table 4 shows that whether teachers asked *any* question about a student explanation did not depend much on whether the initial explanation was correct and complete or not. Teachers asked about both correct and complete explanations and ambiguous, incorrect or incomplete explanations. Teachers asked questions about correct explanations in 18 out of 27 cases (67%) and asked questions about ambiguous, incorrect or incomplete explanations in 32 out of 39 cases (82%).

We also explored whether the *nature* of teachers' questions depended on whether the initial explanation was correct and complete or not. To simplify the presentation of results, because some segments had multiple questioning types (see Table 3), we categorized segments into discrete groupings according to the predominant teacher questioning type. For the remainder of analysis, then we use the following resulting classification: (a) segments with probing sequences of specific questions (possibly in conjunction with other questioning types), (b) segments with general questions (and other questioning types *except* probing sequences), (c) segments with specific questions (and other questioning types *except* probing sequences and general questions), and (d) segments with other questions (either bundles of unrelated questions, or leading questions, but not probing sequences, general questions, or specific questions). As can be seen in Table 4, teachers used nearly all questioning types when students' initial explanations were correct and complete as well as when explanations were ambiguous, incorrect, or incomplete.

Table 4

Relationship Between Specific Kinds of Teacher Questioning and Nature of Students' Initial Explanations

Nature of teacher questioning of student explanation in the segment	Number of segments	Initial student explanation: Correct and complete	Initial student explanation: Ambiguous, incorrect, or incomplete
Teacher asked question(s) about student explanation	50	18	32
Probing sequence of specific questions	14	5	9
General question[a]	10	3	7
Specific question[b]	18	9	9
Other questions[c]	8	1	7
Bundle of questions	2	0	2
Leading question(s)	6	1	5
Teacher asked no question about student explanation	16	9	7
Total	66	27	39

[a]Segment does not include probing sequence of specific questions.
[b]Segment does not include probing sequence of specific questions or general questions.
[c]Segment does include probing sequence of specific questions, single specific or general questions.

Teacher Questioning and Students' Explanations.

The impact of teacher questioning on student behavior—in terms of students elaborating further on their explanations, and whether students provided a correct and complete explanation of how to solve the problem—varied dramatically from segment to segment in these classrooms. Not all questioning resulted in students elaborating their thinking, nor did it always result in students providing correct and complete explanations. This section probes the relationship between teachers' questioning practices and students' explanations of their problem-solving strategies. First, we examine whether students elaborated on their explanations during a segment. Second, we consider whether students provided a correct and complete explanation during a segment.

Teacher questioning and student elaboration of explanations. Table 5 gives the number of segments in which students either did or did not elaborate on their explanation according to whether the teacher asked questions about a student's explanation. As can be seen in Table 5, whether students elaborated on their explanations depended greatly on whether teachers asked questions about students' explanations. When teachers did not ask follow-up questions, students tended not to provide elaboration. Of the 16 segments with no teacher follow-up questions, none showed student elaboration. When teachers did ask follow-up questions, students were much more likely to provide elaboration. Of the 50 segments with teacher follow-up questions, 36 (72%) yielded student elaboration. This difference is statistically significant $\chi^2(1, N = 66) = 31.65, p < .001$. However, it should also be noted that a sizeable minority of segments in which the teacher asked follow-up questions (14, 28%), students provided no elaboration of their explanation. This result shows that teacher follow-up questions are not a guarantee of further student elaboration of their thinking. A major purpose of the rest of this report is to unpack when follow-up questioning leads to further student elaboration and when it does not.

Table 5 also shows that student elaboration was not restricted only to incorrect or incomplete explanations. Students elaborated on correct explanations (e.g., providing additional detail) as well as on incorrect, incomplete, or ambiguous explanations. For example, in 14 of the 36 (39%) segments in which students elaborated on their explanation, students had initially given a correct and complete explanation.

Table 5

Relationship Between Teacher Questioning and Students' elaboration on Their Initial Explanations

Teacher questioning category and initial student explanation	Number of segments	Student elaborated on initial explanation	
		Yes	No
Teacher asked question(s) about student explanation	50	36	14
Correct and complete	18	14	4
Ambiguous, incomplete, incorrect, or no explanation	32	22	10
Teacher asked no question about student explanation	16	0	16
Correct and complete	9	0	9
Ambiguous, incomplete, incorrect, or no explanation	7	0	7
Total	66	36	30

Table 6

Relationship Between Specific Kinds of Teacher Questioning and Students' Elaboration of Their Explanations

Nature of teacher questioning of student explanation	Number of segments	Student elaborated on initial explanation	
		Yes	No
Segment included probing sequence of specific questions	14	14	0
Segment included general question	10	8	2
Segment included specific question	18	12	6
Other	8	2	6
Bundle of questions	2	0	2
Leading questions	6	2	4
Total	50	36	14

Table 6 gives the number of segments in which students elaborated on their initial explanations according to the type of teacher questioning. When teachers asked sequences of specific questions (alone or in conjunction with other questioning types), the target students provided elaboration of their explanations. When the teacher asked general questions or specific questions (but not probing sequences), the target students often provided elaboration of their explanation. Other types of teacher questioning (bundles of questions, leading questions) did not often lead to students' elaboration of their explanations.

Teacher questioning and students' success in giving correct and complete explanations. Table 7 gives the number of segments in which students either did or did not give correct and complete explanations according to whether the teacher asked questions about a student's explanation. Our particular focus in this table is whether students who gave initial explanations that were not correct succeeded in giving correct and complete explanations later during the segment during the context of interaction with the teacher. As can be seen in Table 7, although few students succeeded in giving correct, complete explanations after they had provided ambiguous, incorrect, or incomplete explanations, whether students succeeded in giving a correct and complete explanation after having provided an explanation that was not correct or complete depended on whether teachers asked questions about students' explanations. Of the 32 segments with teacher follow-up questions after an initially ambiguous, incorrect, or incomplete explanation, 8 (25%) had a correct, complete explanation given by the target students. When teachers did not ask students follow-up questions about their ambiguous, incorrect, or incomplete explanations (7 segments), *none* of the target students in those segments produced a correct, complete student explanation $\chi^2(1, N = 39) = 3.59$, $p = .06$.

Table 7 also shows that the few segments in which target students did provide correct and complete explanations after having initially provided explanations that were not correct or complete mostly involved probing sequences of teacher questions, and sometimes involved general and specific questions. Segments that only involved bundled or leading teacher questions did not produce correct and complete explanations on the part of the target student.

In examining Table 7, we found that in the 31 cases where the target student in the segment did not provide a correct and complete explanation, more than half of the time a complete and correct explanation was provided by another student, the teacher or the class. In 17 of the 31 cases a correct and complete explanation was given as a part of the public discourse. For example, in the segments with general and specific questions 12 of the 14 segments had complete explanations provided by others. When teachers asked leading questions, in contrast, of the five segments in which the target student did not give a correct explanation, only one had a complete and correct explanation given by someone and in this one case it was the teacher that provided the complete explanation. Even when teachers asked no questions three of the seven segments in which the target student did not give a correct explanation had complete explanations provided by someone else (in two of the cases it was the teacher who provided them). In 14 of the 31 cases, no correct and complete explanation was ever provided. In summary, in the majority of whole-class segments (79%, 52 of 66), a correct and complete explanation was given by the target student in the segment, either initially or in the context of teacher-student interaction, or by another student or by the teacher. In only a minority of segments (21%) was an ambiguous, incorrect, or incomplete explanation left uncorrected.

Table 7

Relationship Between Specific Kinds of Teacher Follow-up and Accuracy of Students' Explanations

Nature of teacher questioning of student explanation	Total number of segments	Initial student explanation was correct	Initial student explanation was not correct: Student gave correct explanation later in segment	Initial student explanation was not correct: Student did not give correct explanation, but a correct explanation was given by the teacher or another student	Initial student explanation was not correct: No correct explanation was given
Teacher asked question(s) about student explanation	50	18	8	14	10
Segment included probing sequence of specific questions	14	5	6	1	2
Segment included general question	10	3	1	6	0
Segment included specific question	18	9	1	6	2
Other					
Bundle of questions	2	0	0	0	2
Leading questions	6	1	0	1	4
Teacher asked no question about student explanation	16	9	0	3	4
Total	66	27	8	17	14

How Teacher Questioning Played Out During Teacher-Student Segments

The previous sections showed that some teacher questioning practices were more likely than others to yield students' elaboration of their explanations and to lead to students giving correct and complete explanations. However, no category of teacher questioning practices uniformly produced a certain kind of student participation. For example, two thirds of segments with specific questions yielded students' elaboration of their explanations, but one third of segments with specific questions did not. In this section, we examine more closely how and why the same kind of teacher questioning produced such different patterns of student participation across different segments.

Probing sequences of specific questions. In all instances the teacher's use of a sequence of specific questions, a probing sequence, was productive in eliciting further elaboration from the students and in six of the nine instances in which the initial student response was either incorrect, incomplete or ambiguous, the probing sequence resulted in the student articulating a correct and complete explanation

The probing sequence was used in one of three ways across the classrooms. It was used when a teacher was unclear about a student's explanation and was trying to understand the student's thinking. In the following example, the teacher did not understand the student's use of the term "partners" so the teacher asks the student a series of questions to figure out what he or she means.

306. T: Okay, who wants to share out their answers? Who wants to share out? K___?

307. K___: It doesn't matter which way you put it…

308. T: Oops, okay hold on. K___?

309. K___: It doesn't matter the way you put it because it still has a partner.

310. T: Oh! What has a partner?

311. K___: The numbers.

312. T: What are you talking about? Could you explain what numbers you are talking about?

313. K___: Two hundred and one.

314. T: One more time.

315. K___: Two hundred and one and the one. Two hundred and the one.

316. T: Two hundred and the one like this are partners?

317. K___: The one and the one are partners and the two are partners.

318. T: Oh, okay, I see what you are saying. So the two hundred and the two hundred are partners and the one and this one are partners. Is that what you are saying? So it doesn't matter which way. These ones are still partners. They are the same. These two-hundreds are partners. They are the still same. So either way we do it, it's still the same on both sides. True?

Teachers also used a probing sequence when attempting to highlight, clarify or make explicit a particular part of a student's strategy. The goal seems to have been to highlight or make explicit some interesting aspect of the mathematics, seemingly for the benefit of the other students. In the following example, the teacher uses a probing sequence of questions to make explicit the steps used in this student's unique solution and to highlight the mathematics.

768. (44:37) Okay, N__, come on up.. Okay, let's all pay attention because N__ found a couple ways to solve this. You can take your paper up. That's completely fine. Okay. Explain what you did.

769. N__: I put sixteen take away one equals fifteen, and fifteen plus…

770. T: Okay, write down what you're saying. M__, I want you to watch what he's doing cause this will give you another strategy of how to figure out this problem. Okay, I have a question for you; where did you get that sixteen from?

771. N__: I got it from the eight plus eight.

772. (45:37) T: Okay. Okay, eyes up here please. He had eight plus eight equals fifteen plus one, and he says this sixteen is what?

773. N__: Eight.

774. T: Eight. Eight plus eight … so he put that here, and then he moved the unknown to this side, so he did sixteen minus blank equals fifteen. Do you guys see how he did that? Are you allowed to do that? (she rewrites on board: 16 – ___ = 15)

777. Students: Yes.

778. T: Are you allowed to move it around like that?

779. Students: Yes.

780. T: That's okay?

781. Students: Yes.

782. T: A__ is saying yes.

783. Students: Yes.

784. T: Okay, where did this one come from? What's this?

785. N__: Fifteen plus one equals sixteen.

786. T: So you just…

787. N__: So I just flipped that around.

788. T: Can he do that? Can he check his work like that?

789. Students: Yes.

Finally, and to a lesser extent, teachers used probing sequences of questions to assist a student in understanding a problem. In the following example, the student had an incorrect solution and gave an incorrect or incomplete explanation. The teacher used a sequence of questions that led the student through the solutions of several related problems. The problems were designed to build on each other and illustrate the relevant concepts needed to solve the target problem.

518. T: Who still does not know? K___? Okay, what's your question?

519. K___: I don't know.

520. T: You don't know if it's true? Well then how about this one? Let's look at another one. Let's go back to our third … I think it was the third one … this one. Would you say that that's true, K___? Or false?

521. K___: True.

522. T: Why do you think that that's true?

523. K___: Cause A has a partner and one has a partner.

524. T: A has a partner?

525. K___: And one has a partner.

526. T: With what?

527. K___: With another one.

528. T: Okay.

529. K___: And A has a partner with the other A.

530. T: Okay, so what if we said A was one thousand. We can call A, any number. So then this A has to be … what does that A have to be?

531. K___: One thousand.

532. T: It still has to be one thousand, right? So can't we say from before, K___ … that one thousand plus A is the same as one thousand plus fifty? What does A have to be?

533. K___: Fifty.

534. T: A has to be fifty, right? Why?

535. K___: Cause fifty is left alone.

536. T: Fifty is left alone on this side, isn't it? Okay, so now do you understand? Now would you say that this is true? Okay. A has to be fifty and has to have a partner like you said before. You guys are doing a great job. And, last one … so … oh, okay, we did that. You know what? We are going to take it to a harder one. You guys already said … A plus B … Now we are putting in more missing numbers. Who thinks that this one is true? (some students raise hands)

As can be seen by these examples, the probing sequences provided students with multiple opportunities to express their thinking and provided teachers with multiple opportunities to hear student thinking. Students had an opportunity (through the feedback given by the teacher) to see how their explanations were interpreted and used by the teacher. The students then had the opportunity to adjust their explanations, by changing their language, highlighting a key idea, or clarifying a previously confusing statement. Often times, the teacher's response gave the students cues as to what adjustments were necessary. The interchange between student and teacher also provided the teacher to make adjustments, such as rephrasing questions, asking students for examples, and developing supplemental problems.

General question in response to student explanation. As noted earlier (Table 1), teachers responded to students' explanations with general questions in 14 segments. In nine of these segments, the general question was the predominant form of teacher questioning. In some cases, the teacher asked the student to repeat the explanation either explicitly ("Can you say it one more time?", "What did you say?") or implicitly ("I'm sorry, what did you do right now?"). In other cases, the teacher asked the student to demonstrate further ("Could you come up and show us what you mean?"). In the majority of these segments (7/9, 78%, Table 4), students elaborated on their explanations by adding more detail or explaining more clearly. In many of these segments, the teacher had already repeated or rephrased part or all of a student's explanation just prior to asking the student to explain again, so the teacher's question likely signaled that he or she did not completely follow or understand the student's explanation, or that the explanation was incomplete, and that further explanation or clarification was needed.

For example, below is an excerpt from a segment in which the teacher asked the class whether the number sentence 1000 + 200 = 200 + 1000 was true and to explain why. The student in this segment initially gave a confusing explanation in which the student tried to point out the relationship between the numbers on the left side and the right side of the number sentence (line 1). Her explanation was unclear in part because the student confused thousands and hundreds. The teacher started to restate the student's explanation (line 2), but stopped and asked the student to repeat his or her original explanation. Not only did the student add to her initial explanation by inserting the language "the same answer" (possibly suggesting that the student may have thought that the sum of the numbers on the left side of the number sentence was the same "answer" as the sum of the numbers on the right side, line 3), but the student went on to link the numbers to variables A and B (lines 5 and 7), in implicit reference to a previous problem A + B = B + A.

1. Student: 1000 plus 2000 equals 200 plus 100 because if 100 has 2000 then 2000 needs to have a 1000.

2. Teacher: Okay, so you are also doing the partners. I think you mean, if the 1000, this is 1000 … Can you say it one more time?

3. Student: 1000 plus 200 are the same answer from 200 plus 1000 because 100 ...

4. Teacher: 1000.

5. Student: 1000 needs to be A and 1000 is A.

6. Teacher: 1000 is A.

7. Student: And the other 1000 is A. And 200 is B.

This example shows a common pattern in which a teacher's general request for the student to explain again was not interpreted literally as a directive to repeat the initial explanation verbatim, but was apparently interpreted by the student as a request for further elaboration.

Specific question in response to student explanation. The specific question was the most frequently used question-posing practice to follow up on student explanations. As noted in Table 1, of the 50 segments that had follow-up questions, 26 segments had evidence of teachers using specific questions to prompt the students to elaborate a particular aspect of their initial explanations, clarify ambiguous or incomplete parts of explanations, or consider other important elements of the problems (examples below). The specific question was also the most predominant practice in 18 segments, where no general questions or sequence of questions were asked in conjunction to the specific questions (Table 2). Out of these 18 segments, two thirds of students elaborated on initial explanations, and in the other one third of segments, elaboration was provided either by the teacher or another student.

One way teachers used specific questions was *to highlight a noteworthy aspect about student's correct and complete strategy*. In this example, a student initially gave a complete and correct strategy, explaining that the student solved the problem 1000 – 428 = __ by first subtracting 1 from 1000 and then adding the 1 back in after subtracting 428 (line 1). The teacher asked specifically about why the student subtracted one in order to highlight an "excellent strategy" of using this number relation to simplify the otherwise difficult computation of "subtracting from zeros" (line 2). Here, when the student did not explain further, the teacher posed the same specific question to the class and another student provided further elaboration. Although the student is asked "to clarify" by the teacher, the intention behind the questioning is to have the student make explicit and public a certain concept or strategy that the teacher deems to be important, rather then to simply probe the thinking behind the explanation.

1. Student: One thousand minus one … One thousand minus one equals nine hundred ninety-nine. Nine hundred ninety-nine minus four hundred twenty eight is the same as five hundred seventy-one … plus one equals five hundred seventy-two.

2. Teacher: Okay. Very well done. Now just, you know, to clarify. Why did we subtract one from one thousand? And I know that I explained this to you, and I'm just hoping some of you remember why this is an excellent strategy to use when we are subtracting from zeros.

Teachers also used specific questions *to ask for clarification or elaboration of student's ambiguous or incomplete strategy* in order to unpack student's thinking. Here, the problem posed is 100 + ___ = 100 + 50. The student explained that 50 would go inside the box "because it has to be the same number" (line 1). The teacher asked a specific question to clarify the ambiguity of the student's initial explanation (line 2), leading to a back-and-forth interaction between the teacher and student that yielded further clarification of how the student was considering the relationship of the numbers across the equal sign.

1. Student: One hundred … no, the fifty will go right there because it has to be the same number.

2. Teacher: What has to be the same number?

3. Student: One hundred fifty and the other side has to be one hundred fifty too.

4. Teacher: Okay, so you are adding these together. You said this side has to be one hundred fifty?

5. Student: No.

6. Teacher: Oh, what were you saying?

7. Student: That they have to be those because … ’cause it has to have the same numbers.

8. Teacher: It has to have the same numbers. Okay.

9. Student: Fifty, fifty and one hundred (*student gestures)*.

10. Teacher: Okay, this side has a fifty and this side has a fifty. Okay. I see that relation going across.

Teachers also used specific questions to prompt students who had an incorrect answer to recognize an aspect of the problem that was not part of the students’ initial explanation. In this example, the problem is 4 + 9 = 5 x 3 – 2 (true or false). The student explained: “I thought it was false because four plus nine is thirteen, and five times three is fifteen …. Those two do not match.” The teacher then asks a specific question (line 1), bringing attention to the part of the problem that the student had not included in his or her initial explanation. They discuss that this was the first time they had encountered number sentences with more than one operation (lines 6–7), then the teacher engages with the student and leads him toward a correct and complete explanation.

1. Teacher: Okay, so [Student] said four plus nine is thirteen. Five times three is fifteen. That’s not the same number, so it’s false. Okay, I can agree with that. [Student], what about the minus two? What did you do with that?

2. Student: Oh!

3. Teacher: Did you see the minus two?

4. Student: I thought because five times three that’ll make it fifteen because five times three take away two…

5. Teacher: So it doesn’t make sense? Okay, so what could [Student] do? What could be the next step for [Student]?

6. Student: Because I’ve never done three … five times three take away two.

7. Teacher: This is the first time you've done this. Okay, so while we continue on to operate the different symbols, we can continue on with the math. So this is our first time doing it, right? So now let's take another look at it. What could you do now, now that you know that you can continue?

Conclusions

The classrooms from which we draw this data are clearly places where students are making their mathematical thinking explicit and public. Students are sharing their answers and strategies, and teachers are regularly following up on what a student said or did to get at the student's mathematical thinking. We found that the different approaches teachers used played out differently in terms of how they supported students to elaborate their thinking and how they moved toward getting a complete and correct explanation on the table. We narrowed the analytic focus of this report to examine closely the questions teachers asked to support students to make their mathematical thinking explicit and public.

Previous research examining teacher change found that as teachers engage in professional development, their practice shifts to include asking initial questions about how students solved problems. Teachers readily ask, "How did you do that?" to elicit student strategies and explanations. What teachers find much more difficult is to do something with the students' responses to these initial questions (Franke et al, 2001). Teachers have difficulty figuring out how to ask the next set of questions, probe student thinking, and compare student ideas. In the classrooms we examined here, however, the teachers were frequently able to follow up on students' initial responses and did so in a range of different ways. At times they probed one student in a focused manner over a series of turns. Other times they asked one specific question related to something the student had said. Sometimes they asked leading questions, and sometimes they did none of these. Our goal in this report was to begin to specify what teachers did to move beyond the initial question and help students make their thinking explicit and public.

Our research shows not only that teachers can and do follow up on students' initial responses, but that they do so in particular ways that relate to what students say and do. Additionally, our findings suggest that asking follow-up questions (of various kinds) is necessary but not sufficient to insure that students articulate complete and correct strategies. We found that it did matter that teachers asked follow-up questions; many times these questions supported students in making their thinking more explicit and even added to it in ways that led to a correct and complete strategy being shared. However, we also saw that there were times when asking the follow-up questions, almost regardless of type, did not lead to students' sharing complete strategies. It became clear as we looked closely at some of these questioning techniques that teachers used them in different ways depending on their goals, the ways in which students responded, and how the interaction was situated within the context of the conversation.

We recognize that the narrow examination of how teachers support students to make their mathematical thinking explicit through questioning is only the beginning of what needs to happen for a comprehensive understanding of teacher discourse practices. We know and have seen a range of other teacher moves that may have also played a role in what happens for students. For instance, when asking a specific question, teachers often began by revoicing the student's strategy. Because we can now begin to categorize and characterize some of the types of questions teachers use, we can look at the patterns of how these questions unfold in relation to other moves teachers make such as revoicing. We see what we have accomplished in this report as not only an existence proof of teachers being able to make student thinking explicit but also as a methodological argument about the detail necessary to understand the relationship between what teachers do and how students take it up.

We see these findings as consistent with the work of Wood, Sherin and others looking at the details of mathematical discourse. Although our findings reflect a more specific look at particular types of questions, they show, as do the others, that details matter, that focus matters, and that teachers need to choose what to pursue in student thinking and know how to do so. Our hope is that as we begin to understand the moves teachers make in relation to what students say and do, we can use what we learn to articulate the principled ideas that drive them. For instance, as we examined the sequences of probing questions teachers asked a given student, we saw that the teacher spent focused time with a single student, followed up on the details of what the student said, and highlighted what the teacher considered important about the mathematics embedded in the student's strategy. As we look across patterns of teacher moves, we plan in the future to tease out these types of principled ideas so we can help teachers understand how particular moves can support or not support students without having to provide a list of practices to follow.

We want to be careful in situating the implications of this work. First, we see there are implications both for teachers and for research, but those implications must be understood within the context of how we have begun to examine the teacher practices. One cannot take these teacher moves, such as asking specific questions, and make any broad claims about their use. It is important to note that all we have done here is to begin to detail these practices in relation to student participation. We have not looked at the ways in which these practices relate to one another, the patterns within which they occur, and the mathematical ideas embedded within them. So, we see the implications for teachers as being that thinking beyond the initial question matters. There are a variety of questions one can ask in response to what students say, and different ways to support students in explaining their thinking. Planning out the kinds of questions teachers might ask and then watching to see how students take them up can help teachers fine tune which types of questions work for them and their students.

We also see that this work has implications for research. Our work shows that close attention to what students say and do in relation to what a teacher does and says allows us to understand the details of practice that matter for student learning. It becomes imperative to not only be able to hear the details of what many students say but also to examine student participation in relation to teacher participation and the context of the classroom. This type of analysis is difficult as one cannot strip what teachers say from the context in which it happens or from how students engage with classroom interaction. Yet, this type of analysis in conjunction with a variety of student outcomes can help us understand the ways in which teachers can support students' mathematical understanding through classroom dialogue that supports students in explaining their thinking. We recognize that our analyses are only the initial foray into this work. We want to look more closely at the relationships among the teacher moves in relation to student participation, link this to student outcomes, and understand teachers and their classrooms in relation to student participation.

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