CRESST REPORT 794

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USING KEY CONCEPTUAL IDEAS TO IMPROVE TEACHER USE OF FORMATIVE ASSESSMENT DATA

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The National Center for Research on Evaluation, Standards, and Student Testing

Graduate School of Education & Information Sciences
UCLA | University of California, Los Angeles

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Abstract

Making the transition from arithmetic to mathematics, especially algebra, is critical to future academic and economic success, yet many students find this transition difficult and success in algebra elusive. This paper reports on the outcomes achieved from a three year course of professional development, POWERSOURCE©, that helped teachers apply key foundational concepts and formative assessment to math content studied in the 6th, 7th, and 8th grade. The POWERSOURCE© project associated with the professional development reported here found significant gains from students' pretests to students' posttests. Our findings suggest that these gains were associated with changes in teacher thinking and that such change may be easier for more experienced math teachers in earlier rather than later middle school grades.

Introduction

Making the transition from arithmetic to mathematics, especially algebra, is critical to future academic and economic success (Rech & Harrington, 2000); yet, many students find this transition difficult and success in algebra elusive (Ball, 2003; Helfand, 2006; Kollars, 2008; Meehan & Huntsman, 2004; Rubin, 2007; Silver, Saunders, & Zarate, 2008). In part, this difficulty seems to stem from the fact that many teachers of mathematics seldom build on students' prior pre-conceptions, understanding, intuition, or innate problem solving strategies (Donovan & Bransford, 2005). Instead teachers often present math as a set of rules, procedures, and facts to be memorized (Bransford, Brown, & Cocking, 1999); present mathematics in a seemingly random or disorganized manner; and lastly, divorce procedural knowledge from what the processes or results *mean* (Fuson, Kalchman, & Bransford, 2005). Unfortunately, many teachers do not seem prepared to organize and teach from such a conceptual vantage point and do not know where to find help to do so (Kieran, 2003). This paper reports on the outcomes achieved from a three year course of professional development that tried to help teachers apply key foundational concepts and formative assessment to content studied in the 6th, 7th and 8th grades. We first discuss the basis for the intervention as found in the expert-novice and cognitive science literature. Then we discuss how the professional development literature suggests to best structure professional development and how teacher change that is associated with professional development might be directly, rather than obliquely, measured.

Improving the Way Mathematics Is Taught

Research in expert versus novice learning as well as learning research in cognitive science suggests how we might improve the way mathematics is taught. Research that spans as far back as deGroot (1965),continuing through Chi and her colleagues (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Chi & Slotta, 1993),and into the present (Fuson et al., 2005; Kaput, 2004; VanLehn, Siler, Murray, Yamauchi, & Baggett, 2003), hints at the importance of helping teachers present a more “expert-like” conceptualization of domains such as mathematics. In contrast to the piecemeal, context-bound knowledge that novice learners possess, expert knowledge has a relational structure—organized around important, abstract ideas within a given domain (Chi, Feltovich, & Glaser, 1981).

Cognitive studies also suggest that the understanding of a parsimonious set of key organizing principles leads to more flexible and generalizable knowledge use, improves problem solving, makes it easier to understand and master new facts and procedures, and enables transfer (e.g., Rakes, Valentine, McGatha, & Ronau, 2010; Greer, 2003; Carpenter, Fennema, Levi, Franke, & Empson, 2000). Research in the field suggests that, in mathematics especially, subsequent knowledge builds on and helps students abstract the more concrete procedures and concepts developed in arithmetic (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2008; Donovan & Pellegrino, 2004).

This suggests to us that helping teachers organize the mathematics domain around key foundational algebraic concepts, especially concepts already understood by students, might encourage more expert-like conceptions in both teachers and students and thereby improve both teacher pedagogy and student success in algebra and in mathematics as a whole.

Creating Professional Development to Improve Teaching

The expert-novice and cognitive science literature, along with our own experience, suggests that one way to improve mathematics instruction and success with algebra in the United States may be to provide professional development (PD) aimed at increasing the ability of teachers to “help students build and consolidate prerequisite competencies, understand new concepts in depth, and organize both concepts and competencies in a network of knowledge” (Donovan & Bransford, 2005, pp.232).

Overwhelmingly, the body of knowledge on effective PD suggests that quality professional development should focus on: 1) developing the content knowledge of teachers, especially in the content they will actually be teaching; 2) developing a community of teacher-learners that can, among other things, actively share teaching strategies and plan for classroom integration of these strategies; and 3) allowing teachers to examine student work and to explore how student thinking (both accurate and inaccurate student conceptions) develops (Desimone & Ueno, 2006; Fishman, Besta, & Talb, 2003; Garet, Desimone, Birman, & Yoon, 2001; Ingvarson & Beavis, 2005; Seago, 2004; Sparks, 2000; Stigler & Hiebert, 1999). That said, however, as Guskey (2002) points out, "change is a gradual and difficult process for teachers" (p. 386).

Measuring the Short-Term Effects of Professional Development

Since teacher expertise is an important predictor of student achievement (Darling-Hammond, 2007), some have argued that student achievement would be an appropriate measure of change in teacher quality and in teacher beliefs (Hanushek, 2002), and ultimately would be an appropriate measure of the effectiveness of professional development. The fact that such improvements may occur gradually, may be hard to detect on large-scale summative tests, and my be mediated by a number of intervening events suggests a more proximal measure of teacher change is required to determine the short-term effects of professional development efforts.

Our conceptualization of what makes an effective teacher has important implications for how we improve and assess teacher quality. Assessments of teacher knowledge have proven difficult (Tittle, 2006), although not impossible to develop (Hill & Ball, 2004). As Tittle (2006) points out, "assessment methods [in studies of teacher learning and development] are evolving from work within the expert / novice paradigm … and using analyses of content…" (p. 956). With this in mind, researchers at the National Center for Research on Evaluation, Standards, and Student Testing (CRESST) have built on the work of Leinhardt and Ball and their colleagues (see Ball, Lubienshi, & Mewborn, 2001; Leinhardt & Greeno, 1984; Leinhardt & Smith, 1985) to develop a means to measure teacher ability to conceptualize foundational ideas and to use information from formative assessment to plan or modify instruction (Chung et al., 2006; Heritage, Kim, Vendlinski, & Herman, 2009).

Our hypothesis is that if teachers could be helped to understand and teach concepts from the position of expertise and analyze student work from this perspective, teacher quality would improve. We believe that professional development that focuses on the key ideas of mathematics will deepen both teacher and student understanding of these ideas as well as the concepts that build upon them, and will allow learners to build the ideas necessary to form solid foundations for the application of mathematics both in and out of school. We also realize, however, that instruction organized in this manner will require that teachers learn to reorganize and teach fundamental concepts in more expert-like ways. Because many teachers are taught in a system that stresses procedure rather than conceptual understanding (e.g., they tend to organize instruction around textbooks, which oftentimes present material in a procedural way and may provide little guidance in regards to how to change their teaching) (Kieran, 2003), such change may be difficult (Ai, 2002).

Based on this hypothesis, we developed a program of professional development designed to help teachers conceptualize mathematics instruction around key foundational ideas, and improve their ability to effectively use formative assessment data in 6th, 7th and 8th grade math classrooms. Both the formative assessments and professional development were designed around a collection of big ideas developed by content experts and math educators (Niemi, Vallone, & Vendlinski, 2006); the formative assessments were designed to elicit student thinking and misconceptions about those big ideas. The program of professional development described in this study supports a larger formative assessment effort as part of a US Department of Education sponsored study called POWERSOURCE©.

Methods

We used the knowledge map of Leinhardt, as refined by Chung and colleagues (Chung et al., 2006; Leinhardt & Smith, 1985), to measure teacher content organization. We used these maps as pre-post measures at the start of professional development and again at end of each year of professional development in order to test our hypotheses and to measure the effects of the teachers' efforts. These combined measures were designed to assess how each teacher organized a limited part of the domain of algebra and how the teachers connected problems and the concepts necessary to solve those problems.

Approximately 230 public school teachers, from seven school districts in Southern California and Arizona participated in various phases of this three year study. The teachers taught 6th, 7th or 8th grade mathematics and were randomly divided into treatment or control conditions. Randomization was done either between school (all teachers from a school were either treatment or control) or within school (some of the teachers in a school were randomly selected to receive the treatment while the remainder were placed in the control group). The within school design was favored unless districts mandated the use of a between school design. Subject teachers were assigned to one of the two conditions in approximately equal numbers.

Each treatment teacher in this study received approximately 9 hours of professional development in small groups (usually between 5 and 20 teachers) each year. These sessions were conducted largely outside of school hours at the district office or at one of the school sites within each district. For the most part, the initial four hours of professional development each year was almost always done prior to the beginning of the academic year. Three 90 minute follow-up sessions with the teachers were conducted in after school settings with the teachers at approximately two-and-a-half month intervals during the remainder of the school year.

In the first year of professional development, the first session focused on conceptual organization of key mathematical concepts and how these concepts appeared in various forms in the appropriate grade level content of the teachers attending the professional development. During the first 45 minutes of each of the follow-up sessions, teachers and researchers discussed student work (from the teachers' students) on the formative assessments associated with a particular foundational concept, possible misconceptions identified by those assessments, and possible instructional interventions to correct those misconceptions given the key concepts. The last 45 minutes of each session focused on another single key concept and its application— how that concept would be developed from its nascent form into abstract concepts in algebra as well as how the concept could be appropriately taught and applied to the specific subject matter of each grade. To aid teachers with their upcoming instruction on each foundational concept, teachers were given an instructional handbook during the latter part of each session. The professional development integrated this instructional handbook (pedagogical content) with the conceptual development of each of the big ideas (content knowledge).

During the second and third year of professional development, teachers participated in four 90 minute professional development sessions. In each of these sessions, the teachers were grouped into dyads; they worked to develop a two day plan to teach each of the four POWERSOURCE© lessons. The teachers were presented with examples of their students' work and the work of other teachers' students from the previous year. Based on this work and their prior experience teaching each lesson, the teachers discussed student misconceptions and the effectiveness of various teaching strategies intended to eliminate those misconceptions; they also went over sample problems that could be used to elicit misconceptions. The teachers then worked to develop their upcoming lesson and shared this with the larger group. Sixth grade teachers in the third year followed the same course of professional development as they had in Year 2, but the professional development meetings were moderated by district personnel rather than researchers.

We began our professional development efforts with 6th grade teachers in Year 1; we conducted professional development for 6th and 7th grade teachers in Year 2; and finally we offered 6th, 7th, and 8th grade teachers professional development in Year 3. Consequently, most 6th grade teachers received three years of professional development, most 7th grade teachers received two years of professional development, and 8th grade teachers received one year of professional development. This phased-in approach allowed us to explore the effects of dosage (one to three years of professional development) and leveraged the matriculation of the student population from 6th to 7th and then from 7th to 8th grade. Furthermore, teachers were able to teach students who had been exposed to the POWERSOURCE© intervention in previous grades. District policy and resource limitations prevented us from varying the treatment dosage by grade for teachers (i.e. one-, two-, and three-year course of professional development for each of the grade levels) or for students.

After each professional development session, the teachers returned to their classrooms and instructed their students on the applicable key concepts for two class periods of approximately 40 minutes each. Following the initial presentation of a concept to their students, teachers were encouraged to continue to use each concept in other instructional units they developed during the year.

In addition to these treatment groups, we created various control groups which were intended to receive equal amounts of other types of professional development. The control teachers received one of three professional development interventions. The first control (in one district) received the usual district professional development; the second control group (in one district) received instruction in determining the technical quality of district benchmark assessments using a data reporting and analysis tool; and the final control (in four districts) received instruction in student self-efficacy and motivation. Aside from the first control, researchers on the project were able to control the amount of time each teacher participated in these professional development programs and were able to ensure the content was rigorous but did not overlap with POWERSOURCE©.

All teachers in the treatment and control groups created a knowledge map prior to any professional development. At the end of each school year, after the completion of treatment and control professional development activities for that school year, teachers again completed the same type of knowledge map.

Teacher Knowledge Maps

To evaluate the maps created by the teachers, each knowledge map was compared to an expert knowledge map created by the researchers conducting professional development. This "expert map" was created by combining the individual maps that those researchers created in isolation from one another and from the teachers. The individual expert maps were identical on more than 98 percent of the relationships and concepts; the researchers met to resolve remaining differences in the map prior to its use as the "expert" standard.

The agreement between each teacher map and the expert map was analyzed for similarity by counting the number of exact matches between teacher and expert propositions. In this case, a match between teacher and expert required that two identical concepts be connected using an identical link. For example, additive inverse (a concept) is a property of (link) arithmetic (a concept). In the exact match comparison, the "direction" of the link was also specified. For the example just given, "Additive inverse is a property of addition" would be scored as a match, but "A property of addition is an additive inverse" would not be scored as a match. In addition to this strictest scoring—we also analyzed the maps without considering the direction of the connection, without the link label, or without either the link label or the direction. In this latter case, the analysis was focused on whether two concepts that were connected by the experts were connected together in any way by the teachers. As noted elsewhere, more relaxed scoring schemes allowed too much noise into our analysis and were not useful for our purposes in the present study (Vendlinski, Hemberg, Mundy, Baker, Herman, Phelan, J., et. al., 2009). The directions for completing the knowledge map, as well as the complete list of concepts and links is provided in the Appendix.

To complete the task, the teachers were also asked to connect various problems to the concept map they had created. Specifically, the teachers were asked to link a problem with a concept if that concept was necessary to solve the particular problem. Finally, the teachers were asked to label each problem-concept link with a "2" if knowing the concept alone was sufficient to solve the problem and with a "1" if knowing the concept was necessary, but insufficient on its own to solve the problem. One might, for example, be able to solve the problem 12 ÷ 4 by understanding only the concept of division; yet, understanding only division, while necessary, would alone be insufficient to find the mean of three numbers.

For the problem-concept part of the mapping task, we analyzed the data in two ways. First, the teacher maps were compared with the experts and rated for exact matches (link and label). Second, the teacher maps were compared with the expert map for any connection (regardless of label) between a problem and concept. As was the case with the concept maps, only first scoring method proved sufficiently concise to be useful (Vendlinski, Hemberg, Mundy, Baker, Herman, Phelan, J., et. al., 2009).

Generally, the 6th grade teachers joined the study in the first year, the 7th grade teachers in the second year, and the 8th grade teachers in third and final year of professional development. Teachers who taught multiple grades and teachers who were assigned to teach different grades between years were allowed to move to different grade level cohorts to accommodate requests from teachers and districts. Given, however, that teachers only generated one knowledge map before professional development, these teachers were only included in the cohort that reflected their first year in the study. Moreover, for the analysis reported here, we only considered teachers who remained with their cohort throughout the course of the study.

The comparisons were evaluated using an independent samples t-test to make point-in-time comparisons. We used paired samples t-tests to determine the significance of changes within groups (pre- to post-) and ANCOVA analysis to determine the significance of overall change between groups when controlling for teacher conceptual organization and problem–concept linking ability prior to professional development.

Results

The longitudinal nature of this study allowed us to investigate the changes in the teachers' organization of data as well as their ability to connect problems and the concepts necessary to solve those problems over time. The number of teachers' concept-based knowledge maps, available for use in each year of the study, is provided in Table 1:

Table 1

Number of Conceptual Knowledge Maps Used for Analysis by Year and Grade Taught

Grade level	Pretest fall 2007	Posttest spring 2008	Pretest fall 2008	Posttest spring 2009	Pretest fall 2009	Posttest spring 2010
6th						
Treatment	47	40		40		40
Control	30	26		26		26
7th						
Treatment			30	22		11
Control			18	18		16
8th						
Treatment					10	9
Control					9	10

Table 2 displays the number of teachers' problem–concept maps used in each year of the study.

Table 2

Number of Problem–Concept Link Maps Used for Analysis by Year and Grade Taught

Grade level	Pretest fall 2007	Posttest spring 2008	Pretest fall 2008	Posttest spring 2009	Pretest fall 2009	Posttest spring 2010
6th						
Treatment	47	35		40		40
Control	30	24		26		26
7th						
Treatment			30	22		11
Control			19	18		16
8th						
Treatment					10	9
Control					9	10

In order to tease out the ability of teachers at the start of the study and at various time points during the study, we performed a series of analyses. First, we compared the treatment and control groups using an independent samples t-test. Next, we wanted to see how teachers in the treatment group changed as they received increasing amounts of professional development. These changes were analyzed using paired sample t-tests. Finally, to determine the significance of differences attributable to our professional development, we analyzed changes between the teachers starting abilities and their abilities after various amounts of professional development between treatment and control groups by using an Analysis of Covariance (ANCOVA) test. We first performed these analyses on the teachers in a group regardless of grade, and then on each of the grade level cohorts of teachers.

Overall Results

As revealed in Table 3, an independent samples t-test, suggests that the treatment teachers as a whole (that is not grouped by grade) begin significantly lower than the control teachers on organizing concepts prior to professional development; yet, after one year there were no significant differences. Similar analysis for the group of teachers who have completed maps before and after two years of professional development suggests that the treatment and control groups are, again, not significantly different. It should be noted that at the end of the first year several teachers, especially those in the control sample, did not complete conceptual knowledge maps; hence, the results after one year of professional development must be interpreted cautiously due to sample size variation.

Table 3

Independent Samples T-Test of Conceptual Knowledge Maps Pre Test and Post After Years 1 and 2

Type of map	*N*	*M*	*SD*	*df*	*t*	*p*
PreMap						
Treatment	27	0.41	1.047	52.37	1.891	0.064*
Control	33	1.12	1.833	52.37	1.891	0.064*
PostMap 1						
Treatment	25	1.08	1.656	49.00	-0.385	0.702
Control	26	0.92	1.230	49.00	-0.385	0.702
PostMap 2						
Treatment	27	0.96	1.506	58.00	1.066	0.291
Control	33	1.45	1.970	58.00	1.066	0.291

$^*p < .1$

Based on these results, we analyzed the significance of the pre- to posttest change that the treatment teachers made after both one and two years of professional development. While the changes after just one year were not significant ($t = 1.455$, $df = 67$, $p = .150$), the growth was marginally significant after two years. Table 4 shows the results of these analyses. The treatment teachers who had at least two years of professional development (i.e. the 6th and 7th grade treatment teachers) were better able to organize the mathematical concepts of interest after this professional development than they were previously. There were no significant pre- to post- changes in the control group.

Table 4

Paired Samples T-Test of Conceptual Knowledge Maps Pre to Post After Year 2 (Treatment Group)

Type of map	*N*	*M*	*SD*	*df*	*t*	*p*
PreMap	27	0.41	1.047	26	2.018	0.053*
PostMap 2	27	0.96	1.506	26	2.018	0.053*

$^*p < .1$

We used the same approach to analyze the data about teachers' growth in the ability to link problems to concepts required to solve those problems. Unlike the conceptual organization tasks, the treatment teachers were not significantly different from the teachers in the control group in their abilities to accomplish this problem linking task at the outset. The two groups did, however, differ significantly after two years (see Table 5). Yet, in this case, the difference seems attributable to the fact that the treatment teachers actually *lost* ground in their ability to connect problems and concepts in expert-like ways.

Table 5

Independent Samples T-Test of Problem–Conceptual Links Pre Test and Post After Year 2

Type of map	*N*	*M*	*SD*	*df*	*t*	*p*
PreMap						
Treatment	27	4.70	4.121	58.000	0.372	0.711
Control	33	5.06	3.307	58.000	0.372	0.711
PostMap 2						
Treatment	27	3.74	3.108	54.812	1.874	0.066*
Control	33	5.70	4.915	54.812	1.874	0.066*

*$p < .1$

As before, we analyzed these changes in the ability of the treatment teachers to connect problems with the concepts necessary to solve those problems after two years of professional development by using a paired samples t-test. This analysis suggests that the changes in the treatment teachers between a time prior to professional development and after two years of professional development were not significantly changed ($t = -1.101$, $df = 26$, $p = .281$).

Finally, in order to account for group differences in the knowledge maps prior to professional development, we conducted an ANCOVA analysis to determine if the type of professional development the teachers received produced significant between group differences. There were no significant between group changes in the teachers' abilities to conceptually organize concepts after one or after two years of professional development. Similarly, there were no significant between group differences when considering the teachers' abilities to link concepts and problems after one year of professional development. As suggested previously, however, the treatment teachers did seem to *regress* in their ability to link problems and concepts, compared to the control group, after two years of professional development. Our ANCOVA analysis (see Table 6) suggests this is a marginally significant change between groups.

Table 6

ANCOVA Analysis of Problem – Conceptual Links After Two Years of Professional Development

Group	*N*	*df*	*t*	*p*
Treatment	27	1	-1.738	0.088*
Control	33	1	-1.738	0.088*

*$p < .10$

Results by Grade Level

We also performed similar analyses on the data for each of the grade levels. While the content of the treatment varied by grade, the structure of the professional development was constant for all the grades. Specifically, in the first year of professional development, all teachers received instruction in content, student misconceptions and formative assessment, but this instruction was tailored to grade. The format of the second and third years (as appropriate) is described in the Methods section of this report. Consequently, analyzing teacher change by grade level seemed appropriate for our purposes.

Since the 6th grade teachers had three years of professional development, the 7th grade teachers received two years of professional development, and the 8th grade teachers had only a single year of professional development—we were able to analyze the effects of one year of professional development at all three grade levels, the effects of two years of professional development for 6th and 7th grade, and a three-year professional development course of treatment only for 6th grade instructors.

Sixth Grade Teachers. An independent samples t-test (see Table 7) suggest that the conceptual knowledge maps of the control and treatment teachers in the 6th grade are not statistically different at the pretest.

Table 7

Independent Samples T-Test of Conceptual Knowledge Maps Before Professional Development for 6th Grade Teachers

Group	*N*	*M*	*SD*	*df*	*T*	*P*
Treatment	47	0.66	1.128	75	0.653	0.516
Control	30	0.50	0.900	75	0.653	0.516

As revealed in Table 8, after the first year of professional development (approximately nine hours), the 6th grade teachers in the treatment group were significantly different from their peers in the 6th grade control group (based on a second independent samples t-test).

Table 8

Independent Samples T-Test of Conceptual Knowledge Maps Pre Test and Post After Year 1for 6th Grade Teachers

Group	*N*	*M*	*SD*	*df*	*T*	*p*
Treatment	40	0.88	1.017	64	1.940	0.057*
Control	26	0.42	0.758	64	1.940	0.057

*$p < .1$

At pretest, the control and treatment teachers also showed marginally significant differences in their ability to attach given problems to the concepts they thought were required to solve those problems. This is presented in Table 9:

Table 9

Independent Samples T-Test of Problem–Concept Links at Pre Test for 6th Grade Teachers

Group	*N*	*M*	*SD*	*df*	*t*	*p*
Treatment	47	4.91	3.781	75	1.705	0.092*
Control	30	3.60	2.343	75	1.705	0.092*

*$p < .1$

After a year of professional development, however, the teachers' means were no longer significantly different from one another. Table 10 shows that it seems as though the ability of both groups of teachers, to link specific problems and concepts, may have decreased after their first year of professional development; though, this *decrease* in the treatment group was larger than the decrease noted in the control group.

Table 10

Independent Samples T-Test of Problem–Concept Links After Year 1 for 6th Grade Teachers

Group	*N*	*M*	*SD*	*df*	*t*	*p*
Treatment	35	4.14	2.614	57	0.97	0.368
Control	24	3.46	3.162	57	0.97	0.368

The 6th grade teachers were not statistically different in their ability to organize concepts or to tie problems to concepts after two or after three years of professional development; however, we were interested in the significance of their growth during the period of professional development; therefore, we compared the gains and losses the teachers made relative to their starting point using a paired samples t-test.

The treatment teachers showed no significant gains in their ability to organize concepts after the first year of professional development relative to their starting point but the teachers had made significant gains (see Table 11) after the second year.

Table 11

Paired Samples T-test of Conceptual Knowledge Maps after Year 2 for 6th Grade Teachers (Treatment Group)

Type of map	*N*	*M*	*SD*	*df*	*t*	*p*
PreMap	26	0.50	0.860	25	1.718	0.098*
PostMap 2	26	1.19	1.789	25	1.718	0.098*

*$p < .1$

These same teachers also show marginally significant improvements in their ability to connect problems and the concepts necessary to solve those problems after two years of professional development as seen in Table 12.

Table 12

Paired Samples T-Test of Problem– Concept Links After Year 2 for 6th Grade Teachers (Treatment Group)

Type of map	*N*	*M*	*SD*	*df*	*t*	*p*
PreMap	26	4.46	3.075	25	1.716	0.099*
PostMap 2	26	5.58	3.657	25	1.716	0.099*

*$p < .1$

Seventh grade teachers. Similar trends are noted for the 7th grade teachers. After two years of professional development, the treatment teachers seem dramatically improved in their ability to organize concepts in a more expert-like manner. These changes, however, did not rise to the level of statistical significance. Furthermore, the teachers changed in their ability to connect problems and requisite concepts. While the treatment teachers nearly doubled the number of problems and concepts they connected, the control teachers showed a significant *decrease* in their ability to make such connections. The change in the treatment teachers does not meet statistical significance. This is presented in Table 13:

Table 13

Paired Samples T-Test of Conceptual Knowledge Maps After Year 2 for 7th Grade Teachers

Type of map	*N*	*M*	*SD*	*df*	*t*	*p*
Pre Map						
Treatment	6	4.17	2.483	5	1.221	0.277
PostMap 2						
Treatment	6	8.17	7.139	5	1.221	0.277
Pre Map						
Control	12	7.83	5.306	11	-1.993	0.072*
PostMap 2						
Control	12	4.33	4.376	11	-1.993	0.072*

*$p < .1$

Eighth grade teachers. Like teachers in the lower two grades, the 8th grade teachers did not show significant changes in their ability to organize data after the first year. Table 14 reveals that unlike their counterparts in the lower grades, the 8th grade teachers in the treatment group show a significant *decrease* in their ability to connect problems to concepts after one year of professional development.

Table 14

Paired Samples T-Test of Problem–Concept Links After Year 1 for 8th Grade Teachers (Treatment Group)

Type of map	*N*	*M*	*SD*	*df*	*t*	*p*
PreMap	9	6.89	6.194	8	-2.388	0.044**
PostMap 1	9	3.00	3.808	8	-2.388	0.044**

**$p < .05$

As before, we also conducted grade level ANCOVA analyses on each of the groups between each of the years in order to account for group differences in the knowledge maps prior to professional development. As expected, after one year of professional development, 6th grade teachers in the treatment group perform marginally significantly better in their ability to organize concepts compared to 6th grade teachers in the control group, when controlling for their ability prior to professional development. These effects are shown in Table 15:

Table 15

ANCOVA Analysis of Conceptual Knowledge Maps After One Year of Professional Development for 6th Grade Teachers

Group	*N*	*df*	*t*	*p*
Treatment	39	1	1.720	0.091*
Control	25	1	1.720	0.091*

*$p < .10$

An ANCOVA analysis on these same groups of 6th grade teachers shows that while the treatment teachers were not statistically different from the control group in their ability to connect problems and concepts after one year of professional development, they were more able to do so after two years of professional development. As shown in Table 16, the treatment effects are significant when controlling for ability prior to professional development.

Table 16

ANCOVA Analysis of Problem–Conceptual Links After Two Years of Professional Development for 6th Grade Teachers

Group	*N*	*df*	*t*	*p*
Treatment	26	1	2.520	0.016**
Control	16	1	2.520	0.016**

**$p < .05$

A similar set of ANCOVA analyses with 7th grade teachers suggested that the conceptual maps of these teachers only change significantly after their second year of professional development. The result of this analysis is shown in Table 17.

Table 17

ANCOVA Analysis of Conceptual Knowledge Maps After Two Years of Professional Development for 7th Grade Teachers

Group	*N*	*df*	*t*	*P*
Treatment	6	1	3.002	0.009***
Control	12	1	3.002	0.009***

***$p < .01$

Relative to the control group, the 7th grade treatment teachers' ability to connect problems to the concepts necessary to solve those problems did not change significantly after one or after two years of professional development once ability prior to professional development was controlled. Similarly, relative to the control, neither the 8th grade treatment teachers' ability to organize concepts nor their ability to connect problems to the concepts necessary to solve those problems changed significantly over one year of professional development after controlling for initial ability.

Teacher experience and credentials. To place these findings in context, we analyzed the math teaching experience of the teachers in the 6th and 7th grades. In addition, we analyzed the scope of the credential possessed by the teachers in each group. In particular, we wondered whether the 6th grade treatment teachers had less teaching experience and were more likely to hold multi-subject credentials compared to their peers in the 7th grade treatment group.

Contrary to our expectations, comparing all 6th grade teachers in the study to all 7th grade teachers in the study, we found that the 6th grade teachers had significantly more experience teaching math in general ($t = 3.787$, $df = 177.957$, $p < .001$), and in middle school in particular ($t = 2.967$, $df = 176.102$, $p = .003$). The 6th grade teachers were also significantly more likely to have single subject credentials ($t = 2.743$, $df = 180.849$, $p = .007$).

Based on the significant findings reported above, we also compared the 6th and 7th grade treatment teachers who had complete first year data sets in order to identify significant differences in the experience and credentials of this more restrictive sample. The only significant difference between these 6th and 7th grade teachers was that, once again, the 6th grade teachers had significantly *more* experience teaching math than their 7th grade counterparts ($t = 2.152$, $df = 54$, $p = 0.036$). Although 6th grade teachers also had more experience teaching middle school math, and were also more likely to have a single subject credential in math, these differences were no longer significant.

Conclusions

Our results suggest that professional development, which focuses on organizing conceptual understanding around key foundational ideas, does seem to produce more "expert-like" conceptualizations of a domain in the minds of teachers. We saw this change both in the way teachers organize their content knowledge and how they connect problems with the concepts necessary to solve these problems. These changes, however, are not always apparent at first blush and the abilities to accomplish each of the two tasks seem to develop at different rates. We suspect that this is a function of both the grade level of the teachers and the type of professional development the teachers received during a specific year.

Overall, while the treatment teachers in 6th and 7th grades do seem to make significant gains in their ability to organize concepts relative to their starting point prior to professional development, we saw no significant gains in the treatment teachers' ability, as a whole, to organize concepts in more expert-like ways after one or two years of professional development (when we consider their starting point relative to the control group but do not consider grade). While we do see significant changes in the ability of the treatment teachers, relative to the control group, to connect problems and concepts, these changes actually represent *decreases* in this ability relative to the teachers' starting point prior to professional development. In fact, our analysis suggests that both the treatment and control groups are less able to perform this task after two years, if grade is not considered.

The outcomes are very different when we separate the groups by grade level taught; this becomes very important considering the teachers in various grades were exposed to different amounts of professional development by design. The 6th grade teachers in the treatment group seemed to make significant gains the most quickly on organizing mathematical concepts. After one year of professional development, the 6th grade treatment teachers were better able to organize concepts than were their peers in the control group after pre-professional development ability was controlled for. After two years of professional development, these teachers were also more able to connect problems with the concepts necessary to solve those problems, relative to the control and after ability prior to professional development were controlled for. Since professional development first focused on foundational concepts, student misconceptions, and formative assessments, it seems entirely logical that the teachers' conceptual organization would change. Moreover, given that the teachers' second year of professional development focused more on using foundational concepts, student misconceptions, and assessment data to design instruction, increases in the teachers' ability to connect concepts with problems of the type useful for instruction seems well aligned with our findings.

The 7th grade findings were less aligned with our expectations. As with the 6th grade teachers, after their first year of professional development, we expected 7th grade teachers to make significant gains in their ability to organize concepts in more expert-like ways. We also expected these teachers to be more adept in their ability to connect concepts and problems after the second year of professional development. Like the 6th grade teachers, the 7th grade treatment teachers did make significant gains in their ability to organize concepts relative to their peers in the control group. Unlike the 6th grade teachers, however, these gains were only evident after two years of professional development.

The differences between the 6th and 7th grade outcomes were somewhat surprising in that both groups received professional development designed to address conceptual organization in the first year of the study. Here, the experience of the subjects in this study is enlightening. In the treatment groups, the 6th grade teachers had significantly more experience teaching mathematics than their 7th grade counterparts. While we might have suspected that experience would make teachers reticent to change, it seems that more experience is associated with an ability to conceptually reorganize one's thinking about a topic more quickly. Although the 7th grade teachers eventually did reorganize their conceptual maps, it took these teachers two years of both thinking about key conceptual concepts and designing instruction around these concepts, student misconceptions and analyzing student work to do so. This same trend seems evident in the 8th grade teachers, a group more similar to the 7th grade teachers in terms of credentials and experience teaching math. However, since the 8th grade teachers only had a single year of professional development, these similarities cannot be confirmed.

Unlike recent studies of the effectiveness of mathematics professional development (e.g. Randel, Beesley, Apthorp, Clark, Wang, Cicchinelli, & Williams (2011) which found no significant gains in student achievement, the POWERSOURCE© project associated with the professional development reported here did find significant gains from students' pretests to students' posttests (Phelan, Choi, and Vendlinski, 2011). Our findings suggest that these gains in student achievement are associated with the changes in teacher thinking. This association is important because it seems as teachers change the way they think about organizing a domain of knowledge, they may be better able to effect changes in student learning. Specifically, our findings suggest that student achievement is positively correlated with the way teachers organize their thinking around key conceptual ideas and use materials that are also organized in that same way. We are currently investigating this link between teacher change and student achievement.

While Guskey (2002) notes that "change is a gradual and difficult process for teachers" (p. 386), our research suggests that changing teachers' conceptual organization is possible, and such change may be easier for more experienced math teachers in earlier rather than later middle school grades.

References

Ai, X. (2002). *District mathematics plan evaluation: 2001-2002 Evaluation report*. Los Angeles, CA: Los Angeles Unified School District.

American Educational Research Association. (2006). *Do the math: Cognitive demand makes a difference*. Retrieved from http://www.aera.net/uploadedFiles/Journals_and_Publications/Research_Points/RP_Fall06.pdf

Ball, D. L. (2003). *Mathematical proficiency for all students*. Santa Monica, CA: RAND.

Ball, D. L., Lubienshi, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of Research on Teaching* (*4th ed*., pp. 433-456). Washington, D.C.: The American Educational Research Association.

Berkner, L., & Chavez, L. (1997). *Access to postsecondary education for the 1992 high school graduates (NCES 98-105)*. Retrieved from http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=98105

Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2003). *Assessment for learning: Putting it into practice*. Buckingham, UK: Open University Press.

Black, P., & Wiliam, D. (2004). The formative purpose: assessment must first promote learning. In M. Wilson (Ed.), *Towards coherence between classroom assessment and accountability, 2* (pp. 20-50). Chicago, IL: University of Chicago Press.

Black, P., & William, D. (1998). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan, 80*(2), 139-148.

Bowie, L. (2009, February). State edges past New York. *The Baltimore Sun*. Retrieved from http://www.baltimoresun.com/news/education/k12/bal-te.md.ap05feb05,0,4601402.story

Carpenter, T. P., Fennema, E. E., Levi, L., Franke, M. L., & Empson, S. B. (2000). *Children's mathematics: Cognitively guided instruction: A guide for workshop leaders*. Washington, DC: The National Council of Teachers of Mathematics, Inc.

Chi, M. T. H., Bassok, M., Lewis, M. W., Reimann, P., & Glaser, R. (1989). Self-explanation: How students study and use examples in learning to solve problems. *Cognitive Science, 13*, 145-182.

Chi, M. T. H., & Roscoe, R. D. (2002). The processes and challenges of conceptual change. In M. Limon & L. Mason (Eds.), *Reconsidering conceptual change: Issues in theory and practice* (pp. 3-27). Netherlands: Kluwer Academic Publishers.

Chi, M. T. H., & Slotta, J. D. (1993). The ontological coherence of intuitive physics. *Cognition and Instruction, 10*, 249-260.

Choi, K. C., Phelan, J., Niemi, D. N., & Vendlinski, T. P. (2009). *The effects of POWERSOURCE © on student performance.* Los Angeles: UCLA / CRESST.

Chung, G. K. W. K., Baker, E. L., Brill, D. G., Sinha, R., Saadat, F., & Bewley, W. L. (2006). *Automated assessment of domain knowledge with online knowledge mapping* (CRESST Tech. Rep. No. 692). Los Angeles: University of California, National Center for Research on Evaluation, Standards, and Student Testing (CRESST).

Darling-Hammond, L. (2000). Teacher quality and student achievement: A review of state policy evidence. *Education Policy Analysis Archives, 8*(1), 1-44.

deGroot, A. D. (1965). *Thought and choice in chess*. The Hague, Netherlands: Mouton and Co. Publishers.

Desimone, L. M., & Ueno, K. (2006). Are teachers who need sustained, content-focused professional development getting it? *Educational Administration Quarterly, 42*(2), 179 -215.

Donovan, M. S., & Bransford, J. D. (Eds.). (2005). *How students learn: History, mathematics, and science in the classroom*. Washington, D.C.: The National Academy Press.

Fishman, B. J., Besta, S., & Talb, R. T. (2003). Linking teacher and student learning to improve professional development in systemic reform. *Teaching and Teacher Education*, *19*, 643-658.

Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical Understanding: An introduction. In M. S. Donovan & J. D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom*. Washington, D.C.: The National Academy Press.

Garet, M. S., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? *American Educational Research Journal, 38*(4), 915-945.

Guskey, T. R. (2002). Professional development and teacher change. *Teachers and Teaching: Theory and Practice, 8*(3/4), 381-391.

Hanushek, E. A. (2002). Teacher quality. In L. T. Izumi & W. M. Evers (Eds.), *Teacher Quality* (pp. 1-12): Hoover Institution Press.

Helfand, D. (2006, January 30). A formula for failure in LA schools. *Los Angeles Times,* pp. A1, A14-15.

Heritage, M., Kim, J., Vendlinski, T. & Herman, J.L. (2009). From evidence to action: A seamless process in formative assessment? *Educational Measurement: Issues and Practice*, *28*(3), 24-31.

Heritage, M., & Vendlinski, T. (2006). *Measuring teachers' mathematical knowledge*(CRESST Tech. Rep. No. 696). Los Angeles: University of California, National Center for Research on Evaluation, Standards and Student Testing (CRESST).

Herman, R., Dawson, P., Dee, T., Greene, J., Maynard, R., Redding, S., et al. (2008). *Turning around chronically low-performing schools: A practice guide* (No. NCEE#2008-4020). Washington, DC: Institute of Education Sciences, U.S. Department of Education.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*(5), 330-351.

Ingvarson, L., & Beavis, A. (2005). Factors affecting the impact of professional development programs on teachers' knowledge, practice, student outcomes and efficacy. *Education Policy Analysis Archives, 13*(10), 1-28.

Jacobson, L. (2008). *California board mandates Algebra 1 for all 8th graders: Governor lauds change, but state schools chief sees resources lacking.* Retrieved from http://www.edweek.org/login.html?source=http://www.edweek.org/ew/articles/2008/07/16/43algebra.h27.html&destination=http://www.edweek.org/ew/articles/2008/07/16/43algebra.h27.html&levelId=2100

Kester, L., Kirschner, P. A., & vanMerriënboer, J. G. (2005). Timing of information presentation in learning statistics. *Instructional Science, 32*, 233-252.

Kieran, C. (2003). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook on Research on Mathematics Teaching and Learning* (pp. 390 - 419). Reston, VA: National Council of Teachers of Mathematics.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding + it up: Helping children learn mathematics*. Washington, D.C.: National Academy Press.

Kollars, D. (2008, May 12). Despite high school algebra focus, more students need remedial college math*. Sacramento Bee,* pp. A1.

Kushman, J., Hanita, M., and Raphael, J. (2011). *An experimental study of the project CRISS reading program on grade 9 reading achievement in rural high schools.* (NCEE 2010-4007). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

Leinhardt, G., & Greeno, J. G. (1984). The cognitive skill of mathematics teaching. *Journal of Educational Psychology, 78*(2), 75-95.

Leinhardt, G., & Smith, D. A. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology, 77*(3), 247-271.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Earlbaum.

Marzano, R. J. (2006). *Classroom assessment and grading that work.* Alexandria, VA: Association for Supervision and Curriculum Development.

Meehan, K., & Huntsman, H. (2004). *Pathways through algebra project: Improving student success rates in elementary algebra*. Retrieved from http://72.14.205.104/search?q=cache:z55AbRizN_AJ:www.ijournal.us/issue_09/ij_issue09_09_MeehanAndHuntsman_01.html+%22algebra%22+%26+success&hl=en&ct=clnk&cd=11&gl=us

Nathan, M., Koedinger, K., & Martha, W. (2001, April). *The expert blindspot: When content knowledge and pedagogical content knowledge collide.* Paper presented at the annual American Educational Research Association conference, Seattle, WA.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Niemi, D., Vallone, J., & Vendlinski, T. (2006). *The power of big ideas in mathematics education: Development and pilot testing of POWERSOURCE assessments* (CRESST Tech. Rep. No. 697). Los Angeles: University of California, National Center for Research on Evaluation, Standards, and Student Testing (CRESST).

Novak, J. D., Mintzes, J. J., & Wandersee, J. H. (2000). Learning, teaching, and assessment: A human constructivist perspective. In J. J. Mintzes, J. H. Wandersee & J. D. Novak (Eds.), *Assessing science understanding* (pp. 1-13). New York, NY: Academic Press.

Phelan, J. A., Choi, K., & Vendlinski, T. P. (2011, April). *The effects of a formative assessment intervention on student understanding of basic mathematics principles*. Poster presented at the annual American Educational Research Association conference, New Orleans, LA.

Phelan, J. A., Kang, T., Niemi, D. N., Vendlinski, T. P., & Choi, K. (2009). *Some aspects of the technical quality of formative assessments in middle school mathematics.* (CRESST Report 750). Los Angeles: University of California, National Center for Research on Evaluation, Standards, and Student Testing (CRESST).

Randel, B., Beesley, A. D., Apthorp, H., Clark, T.F., Wang, X., Cicchinelli, L. F., & Williams, J. M. (2011). *Classroom assessment for student learning: The impact on elementary school mathematics in the Central Region.* (NCEE 2011-4005). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

Rech, J., & Harrington, J. (2000). Algebra as a gatekeeper: A descriptive study at an urban university. *Journal of African American Studies, 4*(4), 63-71.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other?. *Journal of Educational Psychology, 91*(1), 175-189.

Rockoff, J. E. (2004). The impact of individual teachers on student achievement: Evidence from panel data. *AEA Papers and Proceedings, 94*(2), 247-252.

Rubin, J. (2007, July 11). Locke High's weary teachers face a hard multiple-choice test. *Los Angeles Times.* Retrieved from http://articles.latimes.com/2007/jul/11/local/me-lockehigh11

Saxe, G. B., Gearhart, M., & Seltzer, M. (1999). Relations between classroom practices and student learning in the domain of fractions. *Cognition and Instruction, 17,* 1 - 24.

Seago, N. (2004). Using video as an object of inquiry for mathematics teaching and learning. In *Using video in teacher education: Advances in research on teaching, 10,* (pp. 259 - 286). Orlando, FL: Elsevier.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4-14.

Siegler, R. S. (2003). Implications of cognitive science research for mathematics education. In J. Kilpatrick, W. B. Martin & D. E. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 219 - 233). Reston, VA: National Council of Teachers of Mathematics.

Silver, D., Saunders, M., & Zarate, E. (2008). *What factors predict high school graduation in the Los Angeles Unified School district?* Santa Barbara, CA: UC Santa Barbara.

Sparks, D. H. (2000). *A national plan for improving professional development*. Oxford, OH: National Staff Development Council.

Sternberg, R. J., Forsythe, G. B., Hedlund, J., Horvath, J. A., Wagner, R. K., Williams, W. M., et al. (2000). *Practical intelligence in everyday life*. New York, NY: Cambridge University Press.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: Free Press.

Sturino, G. (2002). *Teaching and textbooks: The case of grade 9 mathematics*. Retrieved from http://www.oise.utoronto.ca/field-centres/TVC/RossReports/vol7no2.htm

Szydlik, J. E., Szydlik, S. D., & Benson, S. R. (2003). Exploring changes in pre-service elementary teachers' mathematical beliefs. *Journal of Mathematics Teacher Education, 6*(3), 253-279.

Tittle, C. K. (2006). Assessment of teacher learning and development. In P. Alexander & P. Winne (Eds.), *Handbook of Educational Psychology* (pp. 953-980). Mahwah, N.J.: Lawrence Erlbaum Associates.

VanLehn, K., Siler, S., Murray, C., Yamauchi, T., & Baggett, W. (2003). Why do only some events cause learning during human tutoring? *Cognition and Instruction, 21*(3), 209-249.

Vendlinski, T. P. (2009, April). *The importance of intention and order: teaching for conceptual understanding using handheld technology.* Paper presented at the annual American Educational Research Association conference, San Diego, CA.

Vendlinski, T.P., Hemberg, B.C., Mundy, C., Baker, E.L., Herman, J.L. Phelan, J., et. al. (2009). *Designing professional development around key principles and formative assessments to improve teachers' knowledge to teach mathematics*. Meeting of the Society for Research on Educational Effectiveness. Crystal City, VA.

Wiliam, D. (2006). *Does assessment hinder learning?* Retrieved from http://www.uk.etseurope.org/home-corpo-uk/news-home/?print=1&news=136&view=detail&no_cache=1

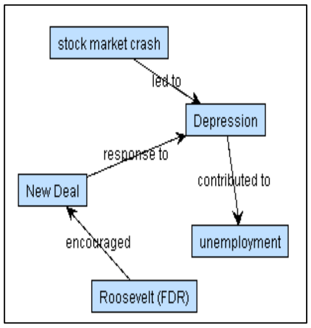
Wiliam, D., & Thompson, M. (2007). Integrating assessment with instruction: What will it take to make it work? In C. A. Dwyer (Ed.), *The future of assessment: Shaping teaching and learning*. Mahwah, N.J.: Lawrence Erlbaum Associates.

Zevenbergen, R. (2005). Primary preservice teachers' understandings of volume: The impact of course and practicum experiences. *Mathematics Education Research Journal, 17*(1), 3-23.

Appendix

Knowledge Mapping Task

sample concept map



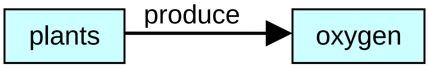
Part 1: Concept Map

Create concept map about mathematics.

Connect the concept labels (labels with concepts in black rectangles) to each other **with the link labels** (labels with black arrows and linking terms).

The **link label** indicates **how** the concepts are related, and the **arrow** indicates the **direction** the thought is read.

For example:



Here, “Plants” and “Oxygen” are the concepts and “produce” is the link. The thought is read as

Plants --> produce--> Oxygen.

If you run out of the preprinted link labels, use a blank arrow label and **write in a link label using only the list provided.**

<u>Please limit yourself to 15 minutes of work on Part 1</u>

Part 2: Problem Map

Connect mathematics problems to the **concepts** in the knowledge map based on what concepts are relevant to solve the problem correctly.

Use the blue arrow labels to connect the problems (labels with blue borders) to the concepts and **rate the relevancy** of the connections by writing **either a 1 or a 2 in the blue box**, using the following scale:

1 = necessary, **but not** sufficient (you need to know this concept, but it is not enough to solve the problem correctly)

2 = necessary **and** sufficient (if you know this concept, you can solve the problem correctly)

Please limit yourself to 15 minutes of work on Part 2

Knowledge Mapping Task

Concepts and Links for Part 1

For your reference, here are the concepts and links to be used when creating your mathematics concept map.

Concepts
Additive Identity
Additive Inverse
Additive Property of Equality
Arithmetic Operations
Comparing Values
Distributive Property
Equation
Equivalence
Evaluation
Expression
Factoring
Fractions
Multiplicative Identity
Multiplicative Inverse
Multiplicative Property of Equality
Number
Order of Operations
Problem Solving
Properties of Arithmetic/Algebra
Proportions
Rational Numbers
Ratios
Simplifying
Transformations

Links
applies to
can represent
is a
Obeys
property of
Shows
type of
used to
Uses

Problems Associated with the Knowledge Mapping Task

Solve for the unknown.

$5x+3(x+4)=28$

40

Solve for the unknown.

$15+17+c=40$

2

Solve for the unknown.

$x^2+5x=16$

901

Solve for the unknown.

$4+b=3$

91

Solve for the unknown.

$5+x^2=19$

512

Solve for the unknown.

$3x+1=7$

4

Find the volume of a cube with a side = 4.

242

Evaluate:

$f(x)=x^2$
$x=-2$

53

The perimeter of a square is equal to 4*s* where *s* is the length of one side. Find the perimeter of the square with a side = 7 ft.

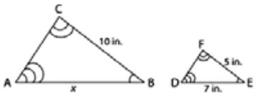
544

Find the length of a side of the triangle if the perimeter is equal to 40, and the other two sides are:

$a = 15$
$b = 17$

402

The two triangles are similar. What is the length of side $\overline{AB}$?



1

The perimeter of the triangle is equal to the sum of the lengths of its sides: $a + b + c$
Find the perimeter of the triangle if the sides are:

$a = 8$
$b = 15$
$c = 17$

22

Write the fraction as a decimal:

$\frac{3}{8}$

20

Determine which fraction is larger:

$\frac{1}{5}$ or $\frac{1}{3}$

300

Add the following fractions:

$\frac{2}{4}+\frac{3}{2}$

521

Evaluate the expression when $y = 2$:

$y+3$

5

Evaluate the expression x^3 when $x = 5$.

140

Evaluate the expression when $x = 4$:

$(4x)^3$

90

Evaluate the expression when $y = 2$:

$8y$

50

Solve for the unknown.

$3-\frac{3}{4}x=-6$

904

Solve for the unknown.

$\frac{1}{5}=\frac{\square}{15}$

411