Final Report on an Evaluation of the California Mathematics Diagnostic Testing Project

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Introduction

The California Mathematics Diagnostic Testing Project (MDTP) is a joint venture of the University of California and the California State University that was created to fill a perceived need to support the preparation of high school students for success in college mathematics courses. At the time of the inception of the MDTP it was felt that an inordinately large proportion of incoming college freshman were deficient in the skills required for success in college mathematics and as a result were required to take remedial mathematics courses. This situation places a drain on the resources of the mathematics departments, which must provide the remediation, and of the students, who get no credits toward graduation requirements. The MDTP board, a consortium of teachers from both the university and high school levels, has over the years developed a series of tests intended to support the mathematical preparation of college-bound high school students. These tests are supplied free of charge to any requesting teachers in California high schools. Once the tests have been administered to the students, they are returned to the MDTP, scored automatically, and detailed results are returned to the teachers in a matter of days.

Although the perceived need still exists, in recent years the role of the MDTP as a means of addressing that need has drawn criticism from some advocates of mathematical reform. The most serious charges have gone well beyond questioning the effectiveness of the program; it has even been suggested that the MDTP is antithetical to the reform movement and that it may actually serve as a barrier to reform efforts. Some of these criticisms result from the usage of the MDTP, while others may be linked to its form and function. To gain insight into these issues, a three-pronged evaluation plan was formulated. The first facet utilized results from a survey of MDTP users to examine how the results of the MDTP are used by consumers of the tests; the second facet utilized classroom artifacts collected from a select group of reform-oriented teachers in an attempt to gain deeper insights into how the results of the MDTP might impact instructional practices; finally, the third facet involved the creation of an electronic forum of nationally recognized mathematics educators to discuss issues relevant to the form and function of the MDTP.

Survey of Users

Introduction

This facet of the evaluation was designed to examine how MDTP test results are used by the current user base. Both the MDTP and critics share several concerns. First, there is a concern that teachers may be using the MDTP not as a diagnostic instrument but rather as a proficiency test, and in some cases may be using the MDTP results as components of students' grades. The official position of the MDTP is that such uses are not appropriate, and the MDTP board, both through literature associated with the tests and through workshops convened by site directors, has worked to combat this practice. Second, it is not uncommon for the MDTP readiness tests to be used as placement tests. Since earlier studies of the MDTP have shown a correlation between results on readiness tests and students' success in the targeted classes, there is some validity to such usage. Critics have maintained that this function may be misused, however, and that students may be denied access to classes in which they could have succeeded based only on information derived from the MDTP. This concern is shared by the MDTP, and their position on this issue is that although readiness test results may be useful in making placement decisions, they should not be used as the sole criterion for such decisions. In addition to these major questions, the MDTP is interested in receiving formative information on the utility of MDTP services to users, as well as information on the degree to which MDTP users are informed about and involved in mathematics education reform efforts.

Method

The MDTP board has over the past several years developed a survey that is included annually in one of their periodic mailings to their user base. This survey was modified to meet current concerns and was mailed to users in the fall of 1994. This questionnaire may be found in Appendix A. Results from the survey were tabulated by the Center for the Study of Evaluation (CSE) and were returned to the MDTP in the spring of 1995.

Results

The complete survey report may be found in Appendix B.

Discussion

This report was intended to be descriptive in nature and for the use of the MDTP board. Nevertheless, we can make some comment on the two major issues identified above. First, with respect to the concern that users may utilize MDTP results as a component of a student's grade, it appears that this is not a matter of grave concern. Only 15% of the users surveyed indicated that such uses were made of MDTP results, and only 5% indicated that such uses were either first, second, or third in importance. This is not to say that the MDTP can relax on this issue, as there are still individuals that are misusing the test results in this fashion, but it does appear to be a relatively small problem. There is more room for concern on the second issue, that of the role of the MDTP test results in placement decisions. Ninety percent of the respondents indicated that one of the uses made of test results was that of informing placement decisions, and 56% said that was the most important use of the test results at their site. Not enough information was obtained from the survey to determine whether these uses were inappropriate (i.e., the MDTP results used as sole criterion), but the magnitude of this usage certainly indicates that further examination of the ways in which the test results are used is warranted.

Classroom Artifact Study

Introduction

It has been suggested by some critics that the MDTP may actually be detrimental to reform-oriented instructional practices. The potential negative scenario is that a teacher who administers an MDTP test to her students may respond to the results by providing remediation of the "drill and kill" variety to address deficiencies identified by the class reports. Some MDTP user conferences have provided opportunities for teachers to share materials that they have developed to address deficiencies identified by the MDTP tests, and the examination of these materials indicates that such fears are not without grounding. There have indeed been examples of worksheets targeted at specific, atomic topic areas, but at the same time teachers have also come up with very innovative, high-level tasks that embed the targeted skills and concepts within interesting problem contexts. The type of response a teacher makes to diagnostic information provided may be less a function of the MDTP itself, and rather an expression of the individual teaching styles and competencies of the teacher. The sad truth is that there are teachers in the high schools whose main instructional mode is "drill and kill," and the fact that they respond to diagnostic feedback with more of the same can hardly be causally attributed to the MDTP.

The classroom artifact study was designed to control for teacher entry characteristics. While the MDTP can hardly be held accountable for the instructional practices of the entire teaching population, the question of what effects the MDTP might have on the instructional decisions of reform-oriented teachers does bear examination. The classroom artifact study sought to examine these effects by looking at the instructional artifacts (tests, quizzes, homework assignments, worksheets, etc.) generated by teachers. The original formulation of the evaluation plan called for a quasi-experimental design using paired comparisons between teachers using and not using the MDTP, but this plan proved unfeasible and had to be amended.

Method

Delays in obtaining clearances for funding required some modifications of the original timeline. Under that timeline, teachers were to have been selected for participation over the summer, data collection would have taken place in the fall semester, and the results were to have been analyzed in the spring of 1995. Since the funding was not available until October of 1995, this timeline was moved forward one semester. Hence the teacher selection process was implemented during the fall 1994 semester, and the actual data collection occurred in the spring semester of 1995.

The first task was to identify a group of teachers to participate in this study. As a first step, a mailing list was purchased from the National Council of Teachers of Mathematics (NCTM). This mailing list consisted of all NCTM members in the state of California that are subscribers to the NCTM publication *Mathematics Teacher*. This magazine is targeted at the working high school teacher, and it was felt that this would provide a likely pool from which to select participants. There were 3,353 names on this list, and 500 were randomly selected to be the targets of invitations to participate. Invitations to participate (Appendix C) were sent to these individuals along with business reply cards (Appendix D) that could be sent back to indicate teachers' willingness to participate. Sixty-one individuals responded to the initial mailing. These individuals were contacted by telephone, and the parameters of the study were further explained to them. Follow-up questionnaires (Appendix E) designed to obtain more detailed information on educational backgrounds and training, teaching experience, professional development activities, and personal philosophies on teaching mathematics were mailed to 29 teachers selected in this initial screening. Twenty-five individuals responded to the questionnaire, and from that pool the 20 teachers that were deemed to be most in alignment with reform-oriented mathematics instruction were selected.

Each teacher used one of the courses he or she was teaching as the target course. There were four Algebra 1 courses, six Geometry, five Algebra 2, four Trigonometry/Math Analysis courses, and one Calculus course represented. Teachers selected for the study had teaching tenures that ranged from 3 to 30 years, with the median teaching experience equal to 15 years. They taught in a variety of different school environments in eight different California counties. Eight of the teachers taught in the Los Angeles metropolitan area, five in the San Diego area, two in Orange County, two in the San Francisco Bay area, and one each from Ventura County, San Bernardino County, and Fresno County. Sixteen of the teachers taught in public schools, and the remaining four were teaching in private schools.

During this process it became apparent to the researchers that it would be impossible to pair up teachers in any meaningful fashion. It was decided at that time to modify the design, and instead to have all the teachers use the MDTP tests, looking for effects of MDTP feedback on the stream of classroom artifacts in all teachers. The fundamental research question remained the same: What effect would feedback from the MDTP have on the instructional practices of reform-oriented teachers? The evaluation plan called for teachers to collect and submit to the Center for the Study of Evaluation (CSE) all instructional artifacts generated during the course of the study. Teachers administered the readiness test for their particular course early in the study and then followed up near the end of the term with the mastery test for that course. The instructions to the subjects may be found in Appendix F.

Classroom artifacts. Classroom artifacts were defined as “anything that is generated by you [the teacher] in the course of your instruction which could be collected and examined by us [the evaluators]. Some examples of classroom artifacts include, but are not limited to: homework assignments, worksheets, quizzes, handouts, tests, and any other collectible items” (Appendix F). Each teacher was provided with several pads of Post-it notes that were preprinted to facilitate the collection of information about the artifact. The information asked for on the Post-it notes was (from Appendix F):

1. Type of artifact—Homework, classwork, project, quiz, test, or other.

2. Purpose of the task—To introduce new concepts or skills, to review or practice, to apply concepts or skills in different context, or other.

3. The source of the artifact—Teacher-generated, district-, school-, or department-generated, or a textbook or supplementary source.

4. How and where the work was done—Individually or in groups, in class or outside class.

5. Grading of the task—Graded or checked for completion (no grade).

6. Any comments you feel necessary to explain the artifacts that you collect.

Upon generating an artifact, the teacher was instructed to fill out a Post-it note for that artifact, attach the note to the artifact, and to put the documented artifact into a large business reply envelope provided for that purpose. Teachers were asked to submit their artifact collections to CSE at midsemester and again at the end of the study.

MDTP tests. Teachers were asked to administer an MDTP readiness test appropriate for their particular course at the beginning of the semester, and then to administer the mastery test for that course near the end of the semester. These tests were returned directly to the MDTP where they were scored. Results from the tests were returned to the teachers and copies of those results were also provided to the evaluators.

Artifact coding. As collections of classroom artifacts were received, the individual artifacts were cataloged and coded. In addition to coding for the descriptive information provided on the Post-it notes, a coding scheme based on the NCTM Standards was developed and applied to each artifact. This coding scheme was designed to indicate the degree to which the particular artifact was in alignment with the Standards for Mathematics as Problem Solving, Mathematics as Communication, Mathematics as Reasoning, and Mathematical Connections. For each of these standards a 6-point scale was developed, where a score of 1 indicates the lowest degree of adherence to the Standard and a score of 6 indicates an assignment that could be considered an exemplar of the spirit and letter of that Standard. Thus, in addition to the descriptive information, each artifact was given a score on each of these four facets. A full description of the coding scheme and the anchor points for the NCTM scales can be found in Appendix G.

Results

Data collected. There was a considerable degree of attrition of the subjects. Of the initial 20 subjects, only 9 returned full sets of data. Two subjects returned no data, 4 subjects administered the readiness test but submitted no artifacts, 1 subject submitted artifacts but failed to administer either of the MDTP tests, 1 subject submitted all data except for the readiness test, and 3 subjects submitted all data except for the mastery test. Fifteen subjects submitted artifacts, and a total of 709 artifacts were coded and cataloged. There was considerable variation in the number of artifacts that were turned in by teachers, ranging from a low of 5 artifacts (Teacher 15) to a high of 170 artifacts by Teacher 8. The median number of artifacts submitted was 26.

Alignment with NCTM Standards. Teachers were selected for this study based on their own perceptions of their commitments to reform. The degree to which their classroom artifacts capture the spirit of reform as stated in the Standards is, however, an empirical question. We were able to examine this question through our analysis of the scores received by their artifacts on the four Standards-derived scales. Those scales were Problem Solving, Communication, Reasoning, and Connections. Mean scores for each teacher are presented in Table 1, along with the number of artifacts for each teacher that were assigned valid scores. From looking at this table, we can see that there is considerable variability among the subjects, with mean scores on Problem Solving ranging from a low of 1.86 (Teacher 1) to a high of 5.75 (Teacher 13). Mean scores were quite stable across the four scales, and closer inspection of frequency breakdowns of scale score indicated that for most subjects these breakdowns were also very similar across the four scales. Table 2 provides the breakdown for the Problem-

Table 1
Mean Scores on NCTM-Derived Scales Broken Down by Teacher

Teacher	Problem solving	Communication	Reasoning	Connections	*N*
1	1.86	1.73	1.86	1.45	22
3	3.57	3.80	3.55	3.34	47
4	2.29	1.87	1.99	2.50	68
5	2.62	1.52	2.38	2.43	21
7	3.60	3.36	4.08	2.92	25
8	5.33	5.04	5.35	5.28	164
9	3.38	2.47	3.38	2.98	45
10	4.37	4.14	4.66	4.19	59
11	5.28	4.60	5.40	5.16	57
12	5.75	5.85	5.85	5.85	20
14	3.33	2.17	3.38	3.04	24
15	3.80	3.40	4.60	3.80	5
16	4.45	3.73	5.00	4.82	11
18	2.85	1.6	2.95	2.85	20
19	3.69	2.57	3.37	3.14	35

Solving scale by teachers, and based on the above comments these results may be taken as representative of those for the other scales. Again, we see great variability. The extreme cases are Teacher 1, with 86% of submitted artifacts scored at scale points 1 or 2 , and Teacher 8, with 77% of artifacts receiving scores of 5 or 6.

Qualitative analysis. Susane Moran spent considerable time examining the classroom artifacts; based on this work, as well as responses to teacher questionnaires and telephone interviews that she conducted, she was able to compile descriptive profiles for most of the teachers. These profiles can be found in Appendix H. Once again, these profiles support the notion that there is considerable variability in implementation even among teachers who are committed to reform. Teacher 1, for example, utilized some activities that prompted students to discover mathematical concepts, but the bulk of the work in this teacher's class required mostly routine calculations and memorization of facts. Other teachers, such as Teachers 4, 9, and 14, utilized a blend of rather traditional approaches heavy on computations and memorization with more challenging tasks involving discovery, reasoning, problem solving and

Table 2
Frequencies of Scores on Problem-Solving Scale Broken Down by Teacher

Teacher	Problem-solving scale score 1	2	3	4	5	6
1	9	10	1	1	1	0
3	12	8	6	2	0	19
4	22	21	16	4	2	3
5	4	7	6	2	1	1
7	2	7	5	1	5	5
8	4	8	11	9	7	125
9	2	10	18	7	0	8
10	0	4	15	14	7	19
11	0	1	9	2	6	39
12	0	0	1	1	0	18
14	3	2	9	6	2	2
15	0	1	1	1	2	0
16	0	1	2	2	3	3
18	1	7	10	0	0	2
19	5	1	8	13	2	6
Total	64	88	118	65	38	250
Proportion	.10	.14	.19	.10	.06	.40
Total without Teacher 8	60	80	107	56	31	125
Proportion without Teacher 8	.13	.17	.23	.12	.07	.27

communication. Finally, teachers such as Teachers 7, 8, and 11 almost exclusively used assignments and techniques that are very much in alignment with the NCTM Standards.

Discussion

In our examination of the artifact streams submitted by the teachers, we were able to detect no effects that could be attributable as reactions to the MDTP results. One possible hypothesis is that there is no such effect for teachers such as those used in this study. Under this hypothesis, teachers who are committed to reform are unlikely to be sidetracked from their efforts by feedback as provided by the MDTP. There are alternative hypotheses, of course. One possible alternative is that these teachers had no investment in the MDTP results, and that the lack of effect was due to indifference on their parts to those results. Another alternative is that teachers may have reacted in ways that were undetected by our examination. It is possible (and perhaps even likely) that different teachers reacted in different manners, and that all these hypotheses, and yet other alternatives, may have been operational within our pool of subjects.

An appropriate methodology for examining these questions would be to conduct in-depth, probing interviews of the teachers used in the study to determine what effect the MDTP results had on their instruction, and why those effects occurred. The evaluation plan did call for follow-up interviews of the subjects, but unfortunately both the money and the time allocated to this evaluation expired before that goal could be reached. Our analysis has identified several likely candidates for such follow-up in the persons of Teachers 8, 10, 11, and 12. These teachers appear to be both dedicated to the spirit of reform and highly proficient in the implementation of reform-oriented instruction. Their insights into how the MDTP may or may not support their own efforts could prove enlightening.

Electronic Conference

Introduction

This facet of the evaluation was designed to address issues of concern to the MDTP board regarding the form and function of the MDTP. This report describes an electronic conference conducted in the summer of 1995 by the Center for the Study of Evaluation (CSE) within the context of an evaluation of the California Mathematics Diagnostic Testing Project (MDTP). MDTP board members Alfred Manaster and Philip Curtis had expressed the desire of the MDTP to open a forum on several issues that they felt to be particularly relevant to the testing project, and the role of the MDTP within the mathematics reform movement.

The MDTP has received serious criticism on several major points. First, the MDTP is linked very strongly to the high school curriculum as it existed at the inception of the project. The MDTP board has created a series of multiple-choice tests targeted at each of the traditional mathematics courses in the college preparatory sequence. Each test is intended to assess the readiness of students to succeed at that level, or to assess the students' mastery of skills and knowledge that are targeted in that course. Critics of the MDTP maintain that this structure serves to maintain an antiquated sequence of instruction that most advocates of reform agree needs at least substantial revision.

Second, as the name implies, the MDTP is intended to serve a diagnostic function. It attempts to serve this function by identifying individual skill and knowledge components that are either required as prerequisites for success (readiness) or targets of instruction (mastery). For example, success in algebra may require mastery of operations with integers, while solving systems of linear equations would be a target of instruction for an advanced algebra class. Each test consists of a number of these components, and each component consists of several questions targeted at that component. The detailed score report provides subscores for each of these components, with the objective of identifying areas of weakness both for the benefit of the student and for the teacher. This structure is subject to criticism that it tends to support a traditional notion of mathematics as a collection of topics which are visited in a relatively fixed and long-established sequence.

Finally, one of the most salient features of mathematical reform is a movement towards authentic, performance-based assessments. While the MDTP has made attempts to adapt to changing times (i.e., versions of tests that require the use of calculators), the basic unit of analysis is still the multiple-choice question, with all its inherent limitations. The MDTP has also drawn the criticism that it attempts to serve its diagnostic purpose within an artificial arena, whereas some reform advocates maintain that the only proper context for assessment of all types is within the context of complex problem-solving applications. They maintain that the format of the MDTP precludes valid judgments of students' abilities in the valued outcomes laid out in the NCTM Curriculum and Evaluation Standards (i.e., mathematics as communication, or mathematics as problem solving), instead focusing in on atomic, isolated topical units, which lack relevance to desired post-NCTM Standards curricula.

The MDTP board is sensitive to these criticisms, and while more radical reformers may cast them as a reactionary force, such an over-simplification does them a great disservice. They are a collection of diverse individuals from a variety of educational institutions. They have in common considerable experience in the teaching and learning of mathematics and a devotion to the improvement of mathematical education at all levels, but they certainly are not all of the same mind with regards to the means by which that improvement can be achieved.

Within that body the observer can find a broad spectrum of opinion on most issues related to reform. The range of opinions within the MDTP board is a microcosm of the variety of opinions in the mathematical community as a whole. The NCTM Standards set bold and inspiring targets for the future of mathematics education in this country but provided little in the way of details for achieving that future. It would be safe to say that, as a body, the MDTP is interested in serving the cause of mathematical reform, but, perhaps tempered by their long-term experiences, they differ from more radical reform elements in an unwillingness to completely scrap old practices in favor of new and untested approaches. They are instead motivated by more pragmatic concerns and are interested in discovering what needs to be changed and what is worthy of preserving. For example, most MDTP board members welcome the trend towards learning mathematics through applications, but at the same time, some are concerned that students with procedural and content deficiencies may not be successful at these applications even if they possess the higher order skills detailed in the NCTM Standards; from their perspective, the MDTP tests may provide an efficient means of identifying areas where individual students may have problems.

In the interest of future improvement of the MDTP, and in light of the diversity of opinion within the mathematics education community, the MDTP board requested that CSE convene a forum on several questions relevant to the issues listed above. They requested that this forum be national in scope, and that it should solicit the viewpoints of nationally recognized experts in mathematics education on these questions. Given the short time frame available for the evaluation, the limited funds, and delays in obtaining the funding, it was decided that the most feasible approach would be to convene an electronic conference in which panelists would participate via electronic mail. Thus was born the MathDiag E-Conference.

Method

Participants. In order to obtain a pool of potential panelists, members of the MDTP board and a representative of the California Department of Education (representing the reform point of view) were asked to submit lists of individuals whose input they would value. Invitations to participate in the conference were tendered to these individuals. These invitations described the format of the conference and the topics that would be covered by the conference. One of the benefits of this type of conference is that it greatly expands the pool of potential participants because issues of scheduling conflicts, travel plans, and expenses become relatively insignificant. In this respect, the E-Conference was a resounding success, attracting individuals from across the nation, many with reputations that loom large on the mathematics education horizon. The following individuals accepted invitations to participate:

Geoff Akst	Professor of Mathematics, Borough of Manhattan Community College, City University of New York.
Linda Boyd	Professor, DeKalb College, Atlanta. Former member of the Mathematical Association of America (MAA) Committee on Testing.
Gail Burrill	University of Wisconsin, Madison, Wisconsin Center for Education Research, and president-elect, National Council of Teachers of Mathematics.
Margaret DeArmond	Teacher, Kern High School, Bakersfield, California, and current president-elect of the California Mathematics Council.
Walter Denham	California Department of Education, responsible for mathematics education strategies and staff development efforts.
Marjorie Enneking	Professor of Mathematics and Associate Vice Provost for Research and Sponsored Projects, Portland State University, Portland, Oregon.
John Harvey	Professor of Mathematics and of Curriculum and Instruction, University of Wisconsin at Madison.
Alfred Manaster	Professor of Mathematics, University of California, San Diego, and member of the MDTP board.
Jack Price	Current president of the National Council of Teachers of Mathematics (NCTM); also teaching in the Center for Education and Equity in Mathematics, Science and Technology (CEEMaST), in the College of Science, California State Polytechnic University, Pomona.

Anita Solow	Professor of Mathematics and chair of the Computer Science/Mathematics Department, Grinnell College. Also chair of the AP Calculus Committee for the Educational Testing Service (ETS).
Elizabeth Teles	Program director in the Division of Undergraduate Education (DUE), National Science Foundation (NSF).
Alba Thompson	Professor of Mathematics, San Diego State University (SDSU), and also a researcher at the Center for Research in Mathematics and Science Education, also at SDSU.
Zalman Usiskin	Professor of Education, University of Chicago, and director of the University of Chicago School Mathematics Project (UCSMP).
Norman Webb	Professor of Education, University of Wisconsin at Madison. Contractor to the State of Wisconsin for developing performance assessments instruments.

More detailed biographical information can be found in the transcripts of the opening statements by the participants (Appendix I).

Topics for discussion. Based on the early input of representatives from the MDTP, an initial list of questions for the forum was generated. This list was submitted for approval and revision to the MDTP board. A cyclic process of revision, resubmission, and approval resulted in the following four questions:

a. What kinds and forms of assessment should be included in the reform efforts and in the intended curricula? In particular, what roles should diagnostic assessment play in these efforts?

b. What are appropriate roles for mathematical content knowledge and procedural techniques in the design of environments that facilitate learning mathematics?

c. How important is the sequencing of instruction for learning mathematics in an effective curriculum? In particular, is student familiarity and comfort with some topics and procedures a necessary prerequisite for developing rich understandings and mastery of other topics? If so, please give examples.

d. What are appropriate roles for applications of mathematics in an effective mathematical learning environment?

Format of the conference. Macjordomo, a standard listserver software package, was used to set up a listserver system. In such a system the participants in the conference, together with the moderator and the primary researcher, are set up as subscribers to the system. All communications are directed to the listserver; then copies of each communication are sent to all subscribers of the listserver. Standardized protocols for message headers were created to facilitate our ability to keep track of the topics of communications, the intended targets of directed communications, and the originators of the communications. Complete details of these protocols can be found in the appendix (Appendix I).

Participants were first invited to submit biographical statements describing their background and interests. They were then asked to submit initial position statements on as many of the four questions they were interested in addressing. This initial phase of the conference lasted from July 6 to July 17. On July 17 the second phase of the conference began, and during this phase participants engaged in open discussion of issues raised in the initial statements. They could ask other panelists for clarification of points, they could further elaborate their arguments, they could respond to points raised by other participants; the intent was to emulate the type of discussion that might occur in a real-time panel discussion, albeit in an asynchronous fashion. This phase lasted until the beginning of August, at which time all participants were invited to make closing comments or statements. The conference concluded officially on August 7. Compiled transcripts of the conference were then sent to the panelists with the understanding that they could edit or amend any of their contributions before the generation of the final report.

Results

The primary product of this conference is the transcript of the actual conference, which may be found in Appendix I. In addition, we have summarized the discussions on the particular threads below. While the primary purpose is to describe what transpired, some general statements can be made, with respect to both the substance of the discussions and the strong and weak points of the E-conference itself.

Assessment. *What kinds and forms of assessment should be included in the reform efforts and in the intended curricula? In particular, what roles should diagnostic assessment play in these efforts?*

The activity related to this question highlights some of the limitations of the electronic format of the conference. There are two major limitations evident here. First, in the closing remarks one of the panelists (Burrill) commented on the difficulty of following a multi-stranded discussion like the one initiated here, and indicated that it might have been more fruitful to have visited each topic in sequence. One consequence of this structure is that some topics may have received less attention than others, and the assessment topic may have been a victim of this phenomenon. Despite a promising set of introductory statements and opening remarks on the topic of assessment, there was very little in the way of discussion of those topics. Panelists stated initial positions, but there was no follow-up discussion. In fact, the discussion consisted solely of Alfred Manaster's responses to the initial statements and his attempts to engage the rest of the panel in continued interaction. These attempts proved fruitless, however, illustrating a second severe limitation of the format; in a linear, synchronous format (face-to-face, or perhaps a conference call) the panelists would have been much more likely to respond to Manaster's efforts to stimulate further discussion.

The most cogent and complete initial statements were made by the two panelists representing the viewpoints that were the stimulus for this evaluation. Walter Denham expressed the views that (a) assessment of students' work should be done primarily by the teacher, (b) while the MDTP may be capable of sending a generalized warning to students, it does not meet his criteria for serving a diagnostic function in that it is incapable of revealing the nature of misunderstandings, and (c) tests in general should not be broken down into topic areas. In his opening statement Alfred Manaster agreed (independently) with Denham that teachers should be the primary assessors of students' higher order skills (creativity and problem solving), but expressed the belief that tests such as the MDTP could be useful to classroom teachers in assessing students' computational and procedural skills, and to a lesser extent their conceptual understanding.

We see from these opening statements that these two stakeholders are largely in agreement on Denham's first point, that the primary responsibility for most valued assessments belongs to the teacher. Their differences of opinion on the utility of the MDTP as a useful instructional tool may hinge largely on different conceptions of what it means for a test to be "diagnostic"; is it enough for a test to simply identify areas of deficiencies, or is it necessary that the test also identify the source and nature of the misconception leading to the deficiency? Although none of the panelists reacted to these statements, Thompson and Price both indicated that the most effective form of diagnosis is that done through the teacher's examination of students' written work. This, we assume, refers to the stronger type of diagnosis mandated by Denham. It does not preclude, however, the utility of a test such as the MDTP to identify areas of possible deficiency, the nature of which could then be explored in more detail by the teacher. Again, in a live discussion with a skillful moderator, panelists could have been probed for their thoughts on this issue, but in the current format it was left unaddressed.

A more fundamental source of disagreement relates to Denham's third point regarding the undesirability of tests that atomize mathematics into a collection of discrete skill and topic areas. Although Manaster does not directly address that issue in his opening statement, it is true that the MDTP tests are indeed structured in this way. Although none of the panelists followed up on this point in this thread, further insights were provided in the thread addressing the role of sequencing of topics in mathematics education.

Content. *What are appropriate roles for mathematical content knowledge and procedural techniques in the design of environments that facilitate learning mathematics?*

This question generated much more in the way of what can be called discussion, with panelists responding both to initial statements and to each other's responses. The primary difficulty in this thread was the inability expressed by several of the panelists to treat this question separately from the fourth question, that involving the role of applications in effective mathematical learning environments. This difficulty arose, perhaps, because of the ambiguity of the phrase "mathematical content knowledge." While there was considerable agreement on what was meant by "procedural techniques," interpretations of what is meant by mathematical content knowledge could conceivably range from simple declarative knowledge (i.e., what is the Pythagorean property) through questions regarding the appropriate "content" for mathematics courses in a much broader sense relating desired outcomes to the activities and practices designed to achieve those outcomes. For example, Usiskin conceptualizes mathematical understanding using a model he dubs the SPUR model, for Skills, Properties, Uses, and Representations. In this model the Skills component corresponds readily to what the panelists understood as procedural knowledge, but mathematical content knowledge could be narrowly interpreted as Properties ("ranging from names for general principles to the doing of proofs") or broadly, encompassing the entire model. Panelists addressing this topic tended towards the broader interpretation, and thus much of the discussion in this thread revolved around the role of applications (Uses, in Usiskin's model) as appropriate content for mathematics courses. In alignment with this broader perspective, only the narrower procedural aspects will be discussed here, while the questions of content will be linked to the synopsis of the discussion regarding the role of applications.

One of the major points raised in this thread, and one that achieved some consensus, was that while procedural knowledge remains an important part of mathematical knowledge, both the nature of the procedures that students need to know and the emphasis that needs to be placed on those procedures have changed drastically. Usiskin highlights the importance of procedures when he states, "Procedures are a means by which we solve problems, by which we explore and represent relationships, and through which we can explain to each other how we have arrived at conclusions. They are not the ends of mathematics, but mathematics cannot be done without them." However, in the age of calculators, the previous emphasis on routine computational skills is no longer justified. Solow indicates that even our definition of what constitutes "routine computational skills" is changing. With the advent of the calculator, tasks such as approximating square roots were relegated to that category; now, with the continued development of software for performing symbolic mathematics, tasks such as computing derivatives and integrals are rapidly moving in that direction.

This does not mean, however, that procedural skills will disappear; instead, new procedures will be developed to solve problems using the available technologies. Usiskin characterizes algorithms as falling into one of three categories: mental, pencil-and-paper, and calculator/computer algorithms. The latter category is a relatively new category, and such algorithms will (and should) predominate in the future. He states, however, that just as many mental algorithms survived the shift towards pencil-and-paper algorithms, we can expect some pencil-and-paper algorithms of particular utility and efficiency to survive the current shift. Harvey, from a strong constructivist position, takes this a bit further, implying that if students are allowed to construct their own technology-assisted algorithms in the process of learning mathematics, they are likely to come up with useful algorithms that we have not anticipated.

While there was general agreement on the role of procedural knowledge in mathematics, there was some dialectical tension regarding the ways in which students should attain that procedural knowledge. In his opening statement on this topic, Price said, "Algorithms, to me, are simple, efficient ways of doing mathematics. The major concern that I have with algorithms is with the way in which they are developed. Children ought to have the opportunity to develop many algorithms on their own. Using a constructivist approach (if you will). Other algorithms may need to be taught." Harvey, in a response to Usiskin's treatment of algorithms, expressed general agreement with Usiskin on most points, but indicated that "being a constructivist I would like, in many places, to replace the word 'teach' with the word 'learn' or 'construct'."

Manaster, in response to these sentiments, posed several questions. "Shouldn't we help students learn how to take advantage of knowledge that our predecessors developed? Indeed, isn't one of the benefits of being human the ability to benefit from the knowledge of others and then build upon it to create better understanding and new knowledge? Isn't it too hard for each individual to reconstruct all the (even relevant) discoveries of the past?" Usiskin in turn expressed strong reservations about some constructivist positions, indicating that ". . . some (not all) of constructivism is rooted in a dangerous anti-intellectualism, a nihilism that denies the knowledge that has been developed by previous generations and our present one, a nihilism that denies that an adult might be able to transmit knowledge to a child, a nihilism that considers books as evil." Making an analogy between constructivism and earlier conceptions of discovery learning, he expresses particular reservations about the utility of more radical constructivist approaches for skill attainment (as opposed to learning concepts) and of the efficiency of such approaches. He states that "some algorithms in school mathematics have developed over centuries; we cannot expect students to construct them."

Denham's response to Usiskin granted that while some forms of mathematical knowledge (history, definitions, useful representational forms) cannot be constructed, the most desirable approach to attaining concepts and procedures is through ". . . giving students more or less proven problem-solving assignments in which they will encounter [those concepts and procedures] . . ." and that ". . . the great bulk of discussion, if it is discussion, has to be among students." While Denham seems to favor minimizing the guiding role of the teacher to setting the stage and providing the initial impetus for students, Harvey expresses a more moderate position. He is in agreement with Denham that the primary motivation for students to construct procedures should come through the provision of ". . . questions/problems/situations/applications/ . . . that will make an algorithm worthwhile we want students to develop," but he continues to state that this does not mean that ". . . after students have invented or tried to invent an algorithms we can't work with them to develop efficient, accurate algorithms that may resemble those we have." Harvey seems to allow for more active direction by the teacher when he states that appropriate roles for teachers in a constructivist learning environment include ". . . giving students sets of 'test data' that will help them discover the errors and shortcomings in their procedures . . . guiding them to discover correct algorithms and procedures . . . sharing with them the theorems that let them organize and consolidate their knowledge so as to produce efficient, accurate schema and maps of their mathematical knowledge."

While this thread of the discussion was successful in identifying some degree of consensus on (a) the continued importance of procedural skills in mathematics, and (b) the changing nature of that procedural knowledge in a technological age, it also indicated that there are fundamental differences of opinion regarding the most appropriate and effective ways of obtaining that procedural knowledge. Unfortunately, the conference ended before those issues could be explored in greater detail.

Sequence. *How important is the sequencing of instruction for learning mathematics in an effective curriculum? In particular, is student familiarity and comfort with some topics and procedures a necessary prerequisite for developing rich understandings and mastery of other topics?* If so, please give examples.

The overwhelming consensus among the panel was that there are many possible ways to sequence a curriculum. Usiskin provided a taxonomy of organizing principles for curricula that proved quite useful during the course of the discussion. Those variations were: (1) logical, (2) historical, (3) utilitarian, (4) problem-oriented, (5) algorithmic, and (6) psychological. He expresses no preferences for any one of these principles, instead arguing that ". . . the more structural frameworks operating in a given course or on a given day, the more likely one is to increase the appeal of the subject matter and the more likely one is to obtain substantive learning."

Other panelists borrowed from Usiskin's taxonomy of organizing principles, but tended to focus in on one or another of the components. This tendency was expressed by Manaster when he stated, "The principal question raised for me is how to select the dominant sequence to use in structuring a curriculum." Harvey agreed with Usiskin's ideas about sequencing, but expressed a preference for the psychological when he when he said ". . . I pay a great deal of attention to sequencing based on our knowledge of learning." DeArmond, in turn, wrote, "I would argue that Zal's [Usiskin's] 'problem-oriented' method of sequencing the curriculum should take top priority." These sentiments are perhaps counter to Usiskin's original intent. In response to DeArmond, Usiskin wrote, "I do not know of a curriculum of more than a year's length in any other country that is based on problems. . . . The problem is efficiency and interest. . . . Problems have not (yet) been analyzed in enough detail to give us a sequence that suggests which ones come first and how we build from one to the next." For Usiskin, all of these various structural frameworks should be used to organize instruction at different "sizes of curriculum," and hence different organizing principles can and should overlay each other at any point in time. To Usiskin, curricula come in at least five sizes: (a) the episode, (b) the lesson, (c) the unit, (d) the year, and (e) the entire school experience, with each step involving an increase in order of magnitude. Due to the great differences in magnitude, ". . . what is a good organizing principle for one size of curriculum may not necessarily be a good organizing principle for another."

There was considerable agreement that traditional curricula based on a sequencing of discrete topics organized into courses such as algebra, geometry, etc. are not appropriate for K-12 education. Denham, in alignment with the California Framework, proposed that curricula should be organized as a web in which "concepts and skills, rather than being 'covered,' are imbedded, to varying degrees in any grade level and to varying degrees according to the particular curriculum chosen." He maintains that "there are only a few truly essential ideas in school mathematics, and they are not ever truly 'mastered.' They should be encountered every year, in gradually increasing depth and, perhaps, complexity." Unfortunately, Denham did not provide a list of those essential ideas, and he seemed content to leave the details of achieving this ideal to the perhaps haphazard efforts of curriculum designers. While agreeing with Denham's conception of curriculum as an evolving web, Enneking addressed some of the practical questions associated with such a conception: "Do we know—agree on—the big ideas? Do we need to? Can each teacher or each school or each district create their own sequence or web which fits their needs?"

Harvey took the web concept considerably further, introducing an (admittedly incomplete) list of big ideas and also illustrating some of the practical difficulties of designing and implementing such a curriculum. While he is in full agreement with the web conceptualization, stating that "one of the problems with the curriculum that has evolved since the New Math era is that it is too linear and too bounded. . . . My (incomplete) list of big ideas needs to be subdivided into units and those units arranged so that the seven ideas are intertwined and connected to each other," his practical experiences in designing and implementing such curricula are illuminating. He maintains, for example, that ". . . giving teachers a web and telling them to choose their own path through it will not be very successful." Instead, he indicates that ". . . within each of these big ideas you [the curriculum developer] can identify some of the things (i.e., units) that need to come first, second, . . . and that you can identify the dependencies of these units on units from the other big ideas." Once the designer has identified these dependencies, ". . . we will have to give teachers a (very) small number of paths through the web that will work and tell them to choose the one they like."

Even Manaster, whose original position that ". . . a well-thought-out sequence of topics and courses is particularly important in any mathematics curriculum" seemed quite traditional, later states that ". . . [the] discussion about sequences has given me some helpful insights. Sequencing does not have to be the rigid categorizing of content in seemingly discrete topics (e.g., algebra, geometry, discrete mathematics)." Sequencing is still important, but its nature has changed, moving towards less rigidity and perhaps breaking some of the old topical molds. Manaster continues, "Instead sequencing might mean understanding which—fairly detailed and specific—understandings, skills, and approaches need to come before others. Thus, the web of connections has some one-way edges. John Harvey suggested this perspective when he mentioned that he had used directed graphs to outline a curriculum." While Denham objects to the idea of one-way relationships and dependencies when applied to the "big ideas" of mathematics, he does ". . . readily agree that if one has a set of five or ten or twenty units, that for several there would be a one-way relationship." The implication here, which is very much in alignment with Harvey's statements in the previous paragraph, is that sequencing becomes increasingly important as we move from the macroscopic level (the big ideas) to the microscopic level (the actual episodes of instruction, to borrow from Usiskin's "sizes of curriculum"). At the level of big ideas sequencing may be unnecessary and even undesirable, while at the smaller "sizes of curriculum" the linear nature of instruction as delivered makes careful attention to sequencing a necessity.

Enneking's question about where the responsibility for developing web structured curriculum lies touched on another cause for concern for the panelists, best voiced by Manaster's statement that "it seems to me that another important issue comes from the mobility of our society. Each year many children move not only from one school to another, but from one district to another and often from one state to another. While a national curriculum seems unobtainable for a number of good reasons, including respect for the tremendous variety of ways that students learn, how do we resolve the competing needs of children who move often and the desire of some for effective national standards with the recognition that many approaches will often be effective, but different ones for different students and different teachers?" From Enneking came, "What about the kids that transfer from one school to another? Will they be repeating things already learned, and not knowing topics the rest of the class knows?"

To DeArmond, this is not a new problem, and indeed is one that may be more troublesome in traditional curricula. "Students do move from school to school. I have often wondered why it seems to be only the mathematics teachers that worry so about this issue. . . . Our problem is the view that mathematics is only a subject of sequential steps and a hierarchy of topics." Enneking touches on this point, and partially answers her own question when she comments, ". . . that question assumes that they actually know the topics covered in the old school. But we all know how far the learned curriculum is from the taught curriculum." Denham adds that ". . . mathematical ideas are not learned/mastered/nailed down at points in time, whereas each class is (we hope) a learning event." It may be inferred from this discussion that in a web structured curriculum, the position of transient students may actually be improved since they will have many opportunities to revisit concepts and topics that they may have missed or failed to learn in their other schools. One final optimistic note on this topic was provided by Usiskin: "The problem is not movement from one school to another . . . students are quite malleable and adjustment would take place."

Applications. *What are appropriate roles for applications of mathematics in an effective mathematical learning environment?*

As mentioned above, for many of the panelists this question proved to be inextricably linked to the earlier question regarding the role of mathematical content knowledge. This was due largely to a perhaps unintended (by the MDTP) tendency by the panelists to focus in on the word "content," reinterpreting these two questions together as relating to a broader question of what content is appropriate for mathematics classes. Harvey makes this reinterpretation explicit when he states that these two questions together can be restated as "1. What is school (collegiate) mathematics?" and "2. What is the role of applications in mathematics instruction?"

For Solow, these questions involve shifting conceptions of what constitutes "mathematical literacy." For her, "mathematical literacy no longer means the ability to do mathematical algorithms by hand. . . ." In response to the recommendations of the NCTM Standards, ". . . mathematics is about ideas, not just procedures; one of the key ideas of mathematics is problem solving." With problem solving an important focus of mathematical study, ". . . the student needs to have problems to solve, and this is where applications come into the curriculum. The particular application is often not very important. Rather it is the experience of applying mathematics in a meaningful environment that is important."

Usiskin provided a very useful historical perspective on these questions when he discusses the ways in which mathematics education has had to adjust to the changing expectations of society. In response to a query from Manaster regarding the definition of the term "average students," Usiskin says, "mathematics courses today are descended from a time in which not all students were expected to need mathematics. For instance, traditional algebra is descended from a time in which the raison d'être for algebra was calculus. They also used to proceed from a notion that the student is self-motivated and thus does not need to be reminded, convinced, or taught of the uses of the subject." Societal expectations have changed, however, and the current climate mandates mathematics (beyond arithmetic) for all students; this change requires adjustments by the mathematics education community. Usiskin goes on to say, "When one decides that 'average students' or 'virtually all students' need mathematics, none of the other assumptions holds. The raison d'être for high school mathematics does not become college mathematics, and a student cannot be assumed to be self-motivated. Applications of mathematics to external situations, which are a primary reason one decides that all students need mathematics, now become a necessity for another reason: They provide a major motivation for taking the subject."

Harvey expressed similar sentiments, saying that "the audience for mathematics grows on a daily basis." He went on to discuss the needs of this audience at the precollege and introductory undergraduate level, as well as the possible consequences of ignoring those needs: "They need a sound knowledge of mathematics so that they can use it to solve problems in their chosen profession (for lack of a better word). If we don't satisfy this audience of 'mathematics consumers,' others will. I feel sure that those of us who teach at the collegiate level have departments within our institutions that teach their own brand of statistics. If we don't respond, they'll teach their own brand of calculus, linear algebra . . ." To Harvey, this scenario is unacceptable: "Mathematics is too vital and necessary a discipline to leave its instruction to 'amateurs'!"

While the panelists were overwhelmingly in favor of the spirit of applications, they also expressed some reservations. One problem, stated quite succinctly by Linda Boyd, is that "good applications that are accessible to students are very hard to come by." Teles also addressed this issue when she said, "What are exciting, interesting applications for some are totally unknown by others. It is very difficult to find applications which are truly meaningful to the whole class. . . ." Teles also noted that "many examples of 'mathematics' created by those without mathematics backgrounds can just be wrong." Boyd followed up on this remark: "What Liz [Teles] said about the dangers of having people without mathematical backgrounds construct applications is true, but we still need to get ideas from them." She went on to describe how at her institution mathematics teachers are paired with faculty from business and sciences to create applied problems. Relating her experience working with a physics teacher she said, "At first his problems were too simple for the level of calculus I needed for my students. We kept working until he found a problem that was at the correct level, and I was able to help him state it properly."

Akst acknowledged the motivational value of applications, but took issue with the conception that the primary goal of mathematics is to solve applications. He provided a list of possible disadvantages: "It may take too much time to set the stage for an application. Students may not be interested in the particular area of application. The application may assume general knowledge which the student may not possess. The application may place excessive demands on the students' language abilities. . . . The application context may dilute the students' interest in the mathematics of the problem." While many of the panelists agreed with the first few of Akst's points above, his last point tied into an issue that was of considerable concern to this panel of mathematicians, that of the role of pure mathematics in mathematics education.

Manaster, assuming the role of devil's advocate, posed the question ". . . why include material if it cannot be developed through applications?" Price, on this issue, said "Students learn better in context. If they understand when and how something is used they are better able to learn it. Not all mathematics needs to be or should be taught this way." Burrill expressed a similar sentiment when she said, "It seems critical to provide some context in which to learn mathematics but that context can be the mathematics itself." These view were echoed by other panelists as well, but Usiskin provided the most complete and cogent treatment of this subject in his response to Manaster's question, beginning with ". . . even though I am a zealot for applications, I believe strongly that in a curriculum for all students one should include mathematics that is not necessarily tied to real-world applications. Pure mathematics is an essential part of mathematics!" He goes on to provide several reasons for studying pure mathematics. The first reason is that mathematics provides a symbolic language that allows us to efficiently handle problems of all types. He says, "a major part of the power of mathematics—even when doing real-world problems—is that one operates within it without recourse to the situation that gave rise to it. For instance, one does not need to translate every line of the solution of an equation to the real world situation that gave rise to the solution." Another reason for studying pure mathematics is that students need to be introduced to the power and utility of deductive reasoning. To Usiskin, "deduction is a fierce and relatively unyielding game in which one needs to deal in symbols and the logical relationships between propositions. Although reasoning from assumptions in real situations is exceedingly common and very valuable for teaching reasoning in mathematics classrooms, to limit oneself to such reasoning is to ignore over two millennia of mathematical history." And finally, "Pure mathematics is exquisitely beautiful."

These comments illuminate a real source of concern for mathematicians, a concern that is related to the changing audience for mathematics and the resultant changes in societal expectations for mathematics education discussed earlier in this section. That concern may best be summed up by this question: Where is the next generation of mathematicians going to come from? Based on Usiskin's historical perspective (discussed above), it may be inferred that the traditional precollege curricula from algebra through calculus were designed largely to serve the needs of mathematicians. In the current climate, however, mathematicians find themselves more and more in the position of being competing clients for the benefits of precollege mathematics instruction. The implied fear is that while an emphasis on problem solving in real world applications may adequately develop students' abilities to apply mathematical reasoning in those contexts, overreaction in the form of reduced emphasis on mathematics for the sake of mathematics may not adequately prepare students for careers in mathematics. If pure mathematics were sacrificed on the altar of reform, mathematicians could find themselves in the awkward position of being responsible for the stewardship of an educational system that serves everyone's needs but their own.

Solow pointed a way out of this dilemma when she said, "One of my favorite areas of 'applications' of mathematical ideas is in pure mathematics. Not all applications need to come from outside of mathematics." This subtle shift of perspective moves "pure mathematics" out of a category of perhaps (to some) undesirable activities, and into the realm of applications. Mathematics is a part of the real world, and applications of mathematical ideas to mathematics certainly deserve continued emphasis. The key, of course, is to find the right balance. Traditional curricula may lack applications to a deplorable degree, but as Usiskin stated, "Let us not play the pendulum game (with horrible logic) by assuming that the only alternative to a curriculum with no applications is one in which everything is applied." In reality, of the millions of students that have gone through the traditional high school curricula, only a very small percentage have ever gone on to careers in mathematics. One of the objectives to consider in developing new curricula is to do so in such a fashion that students are still provided with enough opportunities to experience the beauty of pure mathematics so that this percentage is at least maintained, or even increased.

Summary

The major flaw in this discussion format proved to be that there was too much to consider in too little time. Of all the participants, only Walter Denham provided anything that approached closure in his final comments, and so his eloquent and all-encompassing submission (Appendix I, I.66) can stand alone as a summary of the closing statements. The other participants generally expressed the view that this conference was an interesting experience, but that it would have been well served by focusing on fewer topics and by a longer time span. In the end, however, the final judgment on the value of this conference must be made by the stakeholders in the MDTP and by the participants in the conference.

Appendix A

1995 Survey of MDTP User Base

California Mathematics Diagnostic Testing Project

School: ______________________________ City: ___________________________

School Use of MDTP Tests and Test Results

(Circle one.)
1. Y N Did *you* use MDTP tests during the 1994-95 school year?
2. Y N Did *you* use MDTP tests during the 1993-94 school year?
3. Y N Did your department administer MDTP tests during the 1994-95 school year?

4. At your school, how many teachers of mathematics courses use MDTP tests? (Check one.)

   Less than 25% About Half More than 75%

5. When are MDTP tests usually given at your school? (Check all that apply.)

   Near the beginning of a course In the middle of a course Near the end of a course

6. How are MDTP test results used at your school? (Check all that apply.)

   a. To focus study efforts of individual students
   b. To modify some aspects of curriculum or instruction for this term's class
   c. To modify some aspects of curriculum or instruction for next term's class
   d. To modify some aspects of curriculum or instruction for courses when taught again
   e. To assist in implementing changes in instructional practice
   f. To help structure summer school program(s)
   g. To assist in student placement decisions
   h. To assess student preparation for this term's class
   i. To show students that they have learned
   j. As a component in students' grades
   k. To facilitate communication with parents
   l. To facilitate communication with counselors
   m. To facilitate communication among faculty
   n. To form the basis for certain teacher inservice activities (If so, please explain.)

   o. OTHERS: (We would especially appreciate descriptions of any faculty uses of MDTP materials that have been particularly helpful to you.)

   Of the uses you checked, list the 3 most important:
   _____ First _____ Second _____ Third

7. Does your school or department have any policies about MDTP tests? If so, please describe them briefly.

Usefulness of MDTP Services

For each of the following services offered by MDTP, circle NOT USED if neither you nor other mathematics teachers at your school have used it in the past three years. If it has been used, please indicate how valuable it was on a scale of 1 for "No value" to 5 for "High value."

8. *Individual Student Test Reports*

NOT USED

1	2	3	4	5
No value		Some value		High value

9. *Class Summary Test Reports*

NOT USED

1	2	3	4	5
No value		Some value		High value

10. *MDTP Users' Conferences*

NOT USED

1	2	3	4	5
No value		Some value		High value

11. *MDTP Presentations at CMC Conferences*

NOT USED

1	2	3	4	5
No value		Some value		High value

12. *School Visits by MDTP Site Director*

NOT USED

1	2	3	4	5
No value		Some value		High value

13. *Telephone Consultation with MDTP Site Director*

NOT USED

1	2	3	4	5
No value		Some value		High value

Suggestions for MDTP

One of MDTP's goals continues to be to help improve the effectiveness of mathematics instruction from prealgebra courses through precalculus courses. We would appreciate any suggestions from you. (Please use additional sheets, if needed.)

14. What can MDTP provide to help you strengthen your mathematics program?

15. How can MDTP help students who have not succeeded in learning mathematics in the past?

16. If you are willing to participate in an interview or discussion about MDTP, please provide the following information.

    Name

    Phone number Contact time

Other Activities of Mathematics Faculty

17. Please check any of the following programs in which your school or department has participated in the last three or four years. (*Please indicate which ones.)

    a. Eisenhower Projects*
    b. California Academic Partnership Program (CAPP) Projects*
    c. Implementation of Math A or Math B
    d. Implementation of UC Davis' College Prep Math Curriculum
    e. Implementation of SFSU/UCB's Interactive Math Project curriculum
    f. Implementation of the University of Chicago's School Mathematics Project curriculum
    g. Implementation of the Southern California Regional Algebra Project (SCRAP)
    h. Implementation of other innovative curricula*

Other Activities of Mathematics Faculty (continued)

18. Please check any of the following programs in which you know some of your school's mathematics teachers have participated in the last three or four years. (*Please indicate which ones.)

    a. California Mathematics Project*

    b. Other teacher institutes*

    c. Regional CMC conferences

    d. NCTM conferences

    e. AP Calculus scoring

    f. CLAS scoring

    g. Continuing education classes*

    h. Other professional mathematics education activities*

About Yourself **(Please answer the following questions to assist in our analysis of this questionnaire.)**

19. Indicate the extent of your involvement with the following organizations by checking all that apply among **N for None, M for Member, J for use Journals regularly, C for attend Conferences, O for held Office.**

None	Member	Journal	Conference	Office	
N	M	J	C	O	NCTM: National Council of Teachers of Mathematics
N	M	J	C	O	CMC: California Mathematics Council
N	M	J	C	O	Regional Section of CMC (Which one(s)?)
N	M	J	C	O	Other professional organizations in which you have recently participated (Please list):

20. For each of the following documents please indicate the extent of your familiarity with it by circling one of the following choices: **U for Unfamiliar, S for Somewhat Familiar** with its main thrust, **F for Familiar, and V for Very Familiar** with the document based upon reading it or attending workshops about it.

Unfamiliar	Somewhat Familiar	Familiar	Very Familiar	
U	S	F	V	NCTM Curriculum and Evaluation Standards
U	S	F	V	NCTM Professional Standards for Teaching Mathematics
U	S	F	V	1985 California Mathematics Framework
U	S	F	V	1992 California Mathematics Framework
U	S	F	V	Statement on Competencies in Mathematics Expected of Entering Freshmen (UC/CSU/CCC, 1989)

21. Please list any recent summer mathematics teacher institutes you have attended.

    California Mathematics project (Where? When?)

    AP Calculus

    OTHER:

MDTP thanks you for completing this questionnaire.

Your time and thoughts are greatly appreciated.

Appendix B

MDTP Survey Report

I. School Use of MDTP Tests and Test Results

Q1 DID YOU USE MDTP TESTS DURING THE 92-93 SCHOOL YEAR?
Q2 DID YOU USE MDTP TESTS DURING THE 91-92 SCHOOL YEAR?
Q3 DID YOUR DEPARTMENT ADMINISTER MDTP TESTS DURING THE 92-93 SCHOOL YEAR?

Coding for questions 1-3:
Variable - Question number.
Mean - Proportion of teachers who answered "yes" to the particular question.
Std Dev - Standard deviation.
Minimum - "0" is used for a "no" response.
Maximum - "1" is used for a "yes" response.
Valid N - Number of responses.

Variable	Mean	Std Dev	Minimum	Maximum	Valid N
Q1	.88	.32	0	1	138
Q2	.86	.34	0	1	139
Q3	.80	.40	0	1	137

Q4 AT YOUR SCHOOL, HOW MANY TEACHERS OF MATHEMATICS COURSES USE MDTP TESTS?

Coding:
Value Label - Possible responses to question.
Value - "1" is assigned to "less than 25%" responses.
"2" is assigned to "about half" responses.
"3" is assigned to "more than 75%" responses.
"9" is assigned to missing responses.
Frequency - Number of teachers that chose each response.
Percent - Percent of teachers that chose each response.
Valid Percent - Percent of teachers that chose each response accounting for the missing responses.
Cum Percent - Cumulative percent of teachers who chose each response.

Value Label	Value	Frequency	Valid Percent	Cum Percent	Percent
LESS THAN 25%	1	29	20.6	21.3	21.3
ABOUT HALF	2	37	26.2	27.2	48.5
MORE THAN 75%	3	70	49.6	51.5	100.0
	9	5	3.5	Missing	
		-------	-------	-------	
	Total	141	100.0	100.0	

Q5B ARE MDTP TESTS GIVEN AT THE BEGINNING OF A COURSE?

<u>Coding for questions 5B, 5M, 5E</u>:
Value Label - Possible responses to question.
Value - "0" is assigned to all "no" responses.
"1" is assigned to all "yes" responses.
"9" is assigned to all missing responses.
Frequency - Number of teachers that chose each response.
Percent - Percent of teachers that chose each response.
Valid Percent - Percent of teachers that chose each response accounting for the missing responses.
Cum Percent - Cumulative percent of teachers who chose each response.

Value Label	Value	Frequency	Percent	Valid Percent	Cum Percent
NO	0	79	56.0	58.1	58.1
YES	1	57	40.4	41.9	100.0
	9	5	3.5	Missing	
	Total	141	100.0	100.0	

Q5M ARE MDTP TESTS USUALLY GIVEN IN THE MIDDLE OF A COURSE?

Value Label	Value	Frequency	Percent	Valid Percent	Cum Percent
NO	0	129	91.5	94.9	94.9
YES	1	7	5.0	5.1	100.0
	9	5	3.5	Missing	
	Total	141	100.0	100.0	

Q5E ARE MDTP TESTS USUALLY GIVEN NEAR THE END OF A COURSE?

Value Label	Value	Frequency	Percent	Valid Percent	Cum Percent
NO	0	13	9.2	9.6	9.6
YES	1	123	87.2	90.4	100.0
	9	5	3.5	Missing	
	Total	141	100.0	100.0	

Q6A-Q6O HOW ARE MDTP TEST RESULTS USED AT YOUR SCHOOL?

Coding for questions 6A - 6O:
Variable - Question number.
Mean - Proportion of teachers who used the MDTP test results for the listed reason.
Std Dev - Standard Deviation.
Minimum - "0" is assigned if the option was not chosen.
Maximum - "1" is assigned if the option was chosen.
Valid N - Number of valid responses.
Label - Describes the options that the teachers are able to choose from.

The options are as follows:

- *A. To focus study efforts of individual students.*
- *B. To modify some aspects of curriculum or instruction for this term's class.*
- *C. To modify some aspects of curriculum or instruction for next term's classes.*
- *D. To modify some aspects of curriculum or instruction for courses when taught again.*
- *E. To assist in implementing changes in instructional practice.*
- *F. To help structure summer school programs.*
- *G. To assist in student placement decisions.*
- *H. To assess student preparation for this term's class.*
- *I. To show students that they have learned.*
- *J. As a component to students' grades.*
- *K. To facilitate communication with parents.*
- *L. To facilitate communication with counselors.*
- *M. To facilitate communication among faculty.*
- *N. To form the basis for certain teacher inservice activities.*
- *O. Others.*

Variable	Mean	Std Dev	Minimum	Maximum	Valid N	Label
Q6A	.56	.50	0	1	141	FOCUS STUDY EFFORTS
Q6B	.23	.42	0	1	141	MODIFY C/I THIS TERM
Q6C	.46	.50	0	1	141	MODIFY C/I NEXT TERM
Q6D	.32	.47	0	1	141	MODIFY C/I WHEN TAUGH
Q6E	.34	.48	0	1	140	CHANGE INSTRUCTIONAL
Q6F	.08	.27	0	1	141	STRUCTURE SUMMER PROG
Q6G	.90	.30	0	1	141	STUDENT PLACEMENT DEC
Q6H	.45	.50	0	1	141	ASSESS STUDENT PREPAR
Q6I	.60	.49	0	1	141	SHOW STUDENTS LEARNIN
Q6J	.15	.36	0	1	141	COMPONENT OF GRADE
Q6K	.41	.49	0	1	141	COMMUNICATE WITH PARE
Q6L	.38	.49	0	1	141	COMMUNICATE WITH COUN
Q6M	.35	.48	0	1	141	COMMUNICATE WITH FACU
Q6N	.02	.14	0	1	141	BASIS FOR INSERVICE
Q6O	.07	.26	0	1	141	Other

Q6M1 OF THE USES YOU CHECKED WHICH IS THE FIRST IN IMPORTANCE?

<u>Coding for questions 6M1, 6M2, 6M3</u>:
Value Label - Option chosen.
Value - Letter corresponding to the option.
Frequency - Number of teachers who chose the option.
Percent - Percent of teachers who chose the option.
Cum Percent - Cumulative percent of teachers who chose the option.

Value Label	Value	Frequency	Percent	Valid Percent	Cum Percent
FOCUS STUDY EFFORTS	A	16	11.3	12.1	12.1
MODIFY C/I THIS TERM	B	1	.7	.8	12.9
MODIFY C/I NEXT TERM	C	5	3.5	3.8	16.7
MODIFY C/I WHEN TAUG	D	8	5.7	6.1	22.7
CHANGE INSTRUCTIONAL	E	1	.7	.8	23.5
STUDENT PLACEMENT DE	G	77	54.6	58.3	81.8
ASSESS STUDENT PREPA	H	14	9.9	10.6	92.4
SHOW STUDENTS LEARNI	I	3	2.1	2.3	94.7
COMPONENT OF GRADE	J	1	.7	.8	95.5
COMMUNICATE WITH PAR	K	3	2.1	2.3	97.7
OTHERS	O	3	2.1	2.3	100.0
Omitted		9	6.4	Missing	
	Total	141	100.0	100.0	

Q6M2 OF THE USES YOU CHECKED WHICH IS SECOND IN IMPORTANCE?

Value Label	Value	Frequency	Percent	Valid Percent	Cum Percent
FOCUS STUDY EFFORTS	A	15	10.6	11.5	11.5
MODIFY C/I THIS TERM	B	5	3.5	3.8	15.4
MODIFY C/I NEXT TERM	C	9	6.4	6.9	22.3
MODIFY C/I WHEN TAUG	D	8	5.7	6.2	28.5
CHANGE INSTRUCTIONAL	E	9	6.4	6.9	35.4
STRUCTURE SUMMER PRO	F	1	.7	.8	36.2
STUDENT PLACEMENT DE	G	18	12.8	13.8	50.0
ASSESS STUDENT PREPA	H	13	9.2	10.0	60.0
SHOW STUDENTS LEARNI	I	28	19.9	21.5	81.5
COMPONENT OF GRADE	J	1	.7	.8	82.3
COMMUNICATE WITH PAR	K	12	8.5	9.2	91.5
COMMUNICATE WITH COU	L	5	3.5	3.8	95.4
COMMUNICATE WITH FAC	M	4	2.8	3.1	98.5
OTHERS	O	2	1.4	1.5	100.0
Omitted		11	7.8	Missing	
	Total	141	100.0	100.0	

Q6M3 OF THE USES YOU CHECKED WHICH IS THIRD IN IMPORTANCE?

Value Label	*Value*	*Frequency*	*Percent*	*Valid Percent*	*Cum Percent*
FOCUS STUDY EFFORTS	*A*	*18*	*12.8*	*14.4*	*14.4*
MODIFY C/I THIS TERM	*B*	*5*	*3.5*	*4.0*	*18.4*
MODIFY C/I NEXT TERM	*C*	*14*	*9.9*	*11.2*	*29.6*
MODIFY C/I WHEN TAUG	*D*	*6*	*4.3*	*4.8*	*34.4*
CHANGE INSTRUCTIONAL	*E*	*10*	*7.1*	*8.0*	*42.4*
STUDENT PLACEMENT DE	*G*	*9*	*6.4*	*7.2*	*49.6*
ASSESS STUDENT PREPA	*H*	*8*	*5.7*	*6.4*	*56.0*
SHOW STUDENTS LEARNI	*I*	*18*	*12.8*	*14.4*	*70.4*
COMPONENT OF GRADE	*J*	*5*	*3.5*	*4.0*	*74.4*
COMMUNICATE WITH PAR	*K*	*16*	*11.3*	*12.8*	*87.2*
COMMUNICATE WITH COU	*L*	*11*	*7.8*	*8.8*	*96.0*
COMMUNICATE WITH FAC	*M*	*4*	*2.8*	*3.2*	*99.2*
BASIS FOR INSERVICE	*N*	*1*	*.7*	*.8*	*100.0*
Omitted		*16*	*11.3*	*Missing*	
		-------	-------	-------	
	Total	*141*	*100.0*	*100.0*	

Q7 DOES YOUR DEPARTMENT HAVE ANY POLICIES ABOUT MDTP TESTS?

Coding:
Value Label - Possible responses to the question.
Value - "0" is assigned to all "no" responses.
"1" is assigned to all "yes" responses.
"9" is assigned to all missing responses.
Frequency - Number of teachers that chose each response.
Percent - Percent of teachers that chose each response.
Valid Percent - Percent of teachers that chose each response accounting for the missing responses.
Cum Percent - Cumulative percent of teachers who whose each response.

Value Label	*Value*	*Frequency*	*Percent*	*Valid Percent*	*Cum Percent*
NO	*0*	*35*	*24.8*	*43.2*	*43.2*
YES	*1*	*46*	*32.6*	*56.8*	*100.0*
	9	*60*	*42.6*	*Missing*	
		-------	-------	-------	
	Total	*141*	*100.0*	*100.0*	

II. Usefulness of MDTP Services

Q8-Q13 For each of the following services offered by MDTP, circle NOT USED if neither you nor other mathematics teachers at your school have used it in the past three years. If it has been used, please indicate how valuable it was on a scale of 1 for "No Value" to 5 for "High Value."

Coding for questions 8-13:
Variable - Question number.
Mean - Mean value for each question.
Std Dev - Standard deviation.
Minimum, Maximum - 0 to 5 scale.
0 indicates that the MDTP service was not used.
1 indicates that the MDTP service was of no value.
3 indicates that the MDTP service was of some value.
5 indicates that the MDTP service was of high value.
Valid N - Number of valid responses.
Label - Describes the MDTP services.
8: Individual Student Test Reports
9: Class Summary Test Reports
10: MDTP Users' Conferences
11: MDTP Presentations at CMC Conferences
12: School visits by MDTP Site Director
13: Telephone Consulation with MDTP Site Director

For the first table the variables have been recoded so that 0 indicates that the service was not used, while 1 indicates that the service was not used. In this table the mean represents the proportion of teachers that used that particular service.

Variable	Mean	Std Dev	Minimum	Maximum	Valid N	Label
Q8	.91	.28	.0	1.0	141	INDIVIDUAL STUDENT RE
Q9	.91	.29	.0	1.0	141	CLASS SUMMARY REPORTS
Q10	.30	.46	.0	1.0	141	USERS CONFERENCES
Q11	.26	.44	.0	1.0	141	PRESENTATIONS AT CMC
Q12	.10	.30	.0	1.0	141	VISITS BY SITE DIRECT
Q13	.38	.49	.0	1.0	141	TELEPHONE CONSULTATIO

For the table below the respondents that used the particular services were selected. The means then represent the average level of satisfaction of teachers that used the particular service.

Variable	Mean	Std Dev	Minimum	Maximum	Valid N	Label
Q8	4.28	.90	1.0	5.0	126	INDIVIDUAL STUDENT RE
Q9	4.26	.86	1.0	5.0	125	CLASS SUMMARY REPORTS
Q10	3.00	1.21	1.0	5.0	39	USERS CONFERENCES
Q11	2.75	1.19	1.0	5.0	32	PRESENTATIONS AT CMC
Q12	1.82	1.47	1.0	5.0	11	VISITS BY SITE DIRECT
Q13	3.61	1.19	1.0	5.0	48	TELEPHONE CONSULTATIO

III. Suggestions for MDTP

Q14-Q16 Suggestions for MDTP

Coding for questions 14-16:
Variable - Question number.
Mean - Proportion of teachers who provided a response to each question.
Std Dev - Standard Deviation.
Minimum - "0" is assigned to those who did not respond to the question.
Maximum - "1" is assigned to those who responded to the question.
Valid N - Number of valid responses.
Label - Describes question asked.

- *14: What can MDTP provide to help you strengthen your mathematics program?*
- *15: How can MDTP help students who have not succeeded in learning mathematics in the past?*
- *16: If you are willing to participate in an interview or discussion about MDTP, please provide the following information.*

Variable	Mean	Std Dev	Minimum	Maximum	Valid N	Label
Q14	.53	.50	0	1	141	HELP MATH PROGRAM?
Q15	.29	.46	0	1	141	HELP STUDENTS?
Q16	.40	.49	0	1	141	INTERVIEW?

IV. Other Activities of Mathematics Faculty

Q17A-Q17H Please check any of the following programs in which your school or department has participated in the last three or four years.

Coding for questions 17A-17H:
Variable - Question number.
Mean - Proportion of faculty who have participated in the listed program.
Std Dev - Standard deviation.
Minimum - "0" is assigned if the faculty has not participated.
Maximum - "1" is assigned if the faculty has participated.
Valid N - Number of valid responses.
Label - Description of programs.

- *17A: Eisenhower Projects*
- *17B: California Academic Partnership Program (CAPP) Projects*
- *17C: Implementation of Math A or Math B*
- *17D: Implementation of UC Davis' College Prep Math Project Curriculum*
- *17E: Implementation of SFSU/UCB's Interactive Math Project curriculum*
- *17F: Implementation of the University of Chicago's School Mathematics Project curriculum*
- *17G: Implementation of the Southern California Regional Algebra Project(SCRAP)*
- *17H: Implementation of other innovative curricula*

Variable	Mean	Std Dev	Minimum	Maximum	Valid N	Label
Q17A	.31	.46	0	1	141	EISENHOWER PROJECTS
Q17B	.06	.23	0	1	141	CAPP
Q17C	.40	.49	0	1	141	MATH A OR B
Q17D	.23	.42	0	1	141	COLLEGE PREP MATH
Q17E	.05	.22	0	1	141	INTERACTIVE MATH PROJ
Q17F	.13	.34	0	1	141	SCHOOL MATHEMATICS PR
Q17G	.06	.25	0	1	141	SCRAP
Q17H	.40	.49	0	1	141	OTHER INNOVATIVE

Q18 CHECK ANY OF THE FOLLOWING PROGRAMS IN WHICH YOU KNOW SOME OF YOUR SCHOOL'S MATHEMATICS TEACHERS HAVE PARTICIPATED IN THE LAST THREE OR FOUR YEARS.

Coding for questions 18A - 18F:
Variable - Question number.
Mean - Proportion of teachers who have participated in the specified program.
Std Dev - Standard deviation.
Minimum - "0" is assigned if none of the school's teachers have participated in the specified program.
Maximum - "1" is assigned if at least one of the school's teachers have participated in the specified program.
Valid N - Valid number of responses.
Label - Description of project.

18A. California Mathematics Project
18B. Other teacher institutes
18C. Regional CMC conferences
18D. NCTM Conferences
18E. AP Calculus scoring
18F. CLAS scoring
18G. Continuing education classes
18H. Other professional mathematics education activities

Variable	Mean	Std Dev	Minimum	Maximum	Valid N	Label
Q18A	.44	.50	0	1	141	CAL MATH PROJECT
Q18B	.52	.50	0	1	141	OTHER TEACHER INSTITU
Q18C	.71	.46	0	1	141	REGIONAL CMC CONFEREN
Q18D	.53	.50	0	1	141	NCTM CONFERENCES
Q18E	.18	.38	0	1	141	AP CALCULUS SCORING
Q18F	.28	.45	0	1	141	CLAS SCORING
Q18G	.48	.50	0	1	141	CONTINUING EDUCATION
Q18H	.52	.50	0	1	141	OTHER ACTIVITIES

Q19AN - Q19DO INDICATE THE EXTENT OF YOUR INVOLVEMENT WITH THE FOLLOWING ORGANIZATIONS BY CHECKING ALL THAT APPLY AMONG N FOR NONE, M FOR MEMBER, J FOR USE OF JOURNALS REGULARLY, C FOR ATTEND CONFERENCES, O FOR HELD OFFICE.

Coding for questions 19AN - 19DO:
Variable - Question number.
Mean - Proportion of teachers who indicated involvement at the specified level.
Std Dev - Standard deviation.
Minimum - "0" is assigned if the teacher indicated no involvement at the specified level.
Maximum - "1" is assigned if the teacher indicated involvement at the specified level.
Valid N - Number of valid responses.
Label - Description of organizations.
A. NCTM: National Council of Teachers of Mathematics
B. CMC: California Mathematics Council
C. Regional Section of CMC
D. Other professional orgainizations

Questions:
AN, BN, CN, DN - asked if the teacher had **No** *involvment with the specified organization.*
AM, BM, CM, DM - asked if the teacher was a **Member** *of the specified organization.*
AJ, BJ, CJ, DJ - asked if the teacher used **Journals** *regularly.*
AC, BC, CC, DC - asked it the teacher attended **Conferences.**
AO, B0, C0, D0 - asked if the teacher had held **Office.**

Variable	Mean	Std Dev	Minimum	Maximum	Valid N	Label
Q19AN	.16	.36	0	1	141	NCTM NONE
Q19AM	.68	.47	0	1	141	NCTM MEMBER
Q19AJ	.57	.50	0	1	141	NCTM JOURNAL
Q19AC	.33	.47	0	1	141	NCTM CONFERENCE
Q19AO	.01	.08	0	1	141	NCTM OFFICE
Q19BN	.23	.42	0	1	141	CMC NONE
Q19BM	.55	.50	0	1	141	CMC MEMBER
Q19BJ	.39	.49	0	1	141	CMC JOURNAL
Q19BC	.38	.49	0	1	141	CMC CONFERENCE
Q19BO	.02	.14	0	1	141	CMC OFFICE
Q19CN	.27	.45	0	1	141	RCMC NONE
Q19CM	.30	.46	0	1	141	RCMC MEMBER
Q19CJ	.11	.31	0	1	141	RCMC JOURNAL
Q19CC	.24	.43	0	1	141	RCMC CONFERENCE
Q19CO	.02	.14	0	1	141	RCMC OFFICE
Q19DN	.23	.42	0	1	141	OTHER NONE
Q19DM	.25	.43	0	1	141	OTHER MEMBER
Q19DJ	.11	.31	0	1	141	OTHER JOURNAL
Q19DC	.09	.28	0	1	141	OTHER CONFERENCE
Q19DO	.01	.12	0	1	141	OTHER OFFICE

Q20A - Q20E FOR EACH OF THE FOLLOWING DOCUMENTS INDICATE THE EXTENT OF YOUR FAMILIARITY BY CIRCLING ONE OF THE FOLLOWING CHOICES: <u>U FOR UNFAMILIAR, S FOR SOMEWHAT FAMILIAR, AND V FOR VERY FAMILIAR</u> WITH THE DOCUMENT.

<u>Coding for questions 20A - 20E:</u>
Variable - Question number.
Mean - Mean value of familiarity with the specified document.
Std Dev - Standard deviation.
Minimum, Maximum, Value -
- *1 unfamiliar with the document.*
- *2 somewhat familiar with the document.*
- *3 familiar with the document.*
- *4 very familiar with the document.*
- *9 response the the question is missing.*

Valid N - Valid number of responses.
Label - Description of documents.
- *A. NCTM Curriculum and Evaluation Standards*
- *B. NCTM Professional Standards for Teaching Mathematics*
- *C. 1985 California Mathematics Framework*
- *D. 1992 California Mathematics Framework*
- *E. Statement on Competencies in Mathematics Expected of Entering Freshmen (UC/CSU/CCC, 1989)*

Value Label - Level of familiarity with documents.
Frequency - Number of teachers that chose each response.
Percent - Percent of teachers that chose each response.
Cum Percent - Cumulative percent.

Variable	Mean	Std Dev	Minimum	Maximum	Valid N	Label
Q20A	2.93	.87	1	4	135	NCTM C&E STANDARDS
Q20B	2.83	.89	1	4	136	NCTM TEACHING STANDAR
Q20C	3.08	.87	1	4	135	85 MATH FRAMEWORK
Q20D	3.24	.80	1	4	138	92 MATH FRAMEWORK
Q20E	2.17	1.10	1	4	132	STATEMENT ON COMPETEN

Q20A HOW FAMILIAR ARE YOU WITH NCTM CURRICULUM AND EVALUATION STANDARDS?

Value Label	Value	Frequency	Percent	Valid Percent	Cum Percent
UNFAMILIAR	1	7	5.0	5.2	5.2
SOMEWHAT FAMILIAR	2	34	24.1	25.2	30.4
FAMILIAR	3	55	39.0	40.7	71.1
VERY FAMILIAR	4	39	27.7	28.9	100.0
	9	6	4.3	Missing	
	Total	141	100.0	100.0	

Q20B HOW FAMILIAR ARE YOU WITH NCTM TEACHING STANDARDS?

Value Label	Value	Frequency	Percent	Valid Percent	Cum Percent
UNFAMILIAR	1	11	7.8	8.1	8.1
SOMEWHAT FAMILIAR	2	34	24.1	25.0	33.1
FAMILIAR	3	58	41.1	42.6	75.7
VERY FAMILIAR	4	33	23.4	24.3	100.0
	9	5	3.5	Missing	
	Total	141	100.0	100.0	

Q20C HOW FAMILIAR ARE YOU WITH THE 1985 CALIFORNIA MATH FRAMEWORK?

Value Label	Value	Frequency	Percent	Valid Percent	Cum Percent
UNFAMILIAR	1	6	4.3	4.4	4.4
SOMEWHAT FAMILIAR	2	28	19.9	20.7	25.2
FAMILIAR	3	50	35.5	37.0	62.2
VERY FAMILIAR	4	51	36.2	37.8	100.0
	9	6	4.3	Missing	
	Total	141	100.0	100.0	

Q20D HOW FAMILIAR ARE YOU WITH THE 1992 CALIFORNIA MATH FRAMEWORK?

Value Label	Value	Frequency	Percent	Valid Percent	Cum Percent
UNFAMILIAR	1	5	3.5	3.6	3.6
SOMEWHAT FAMILIAR	2	16	11.3	11.6	15.2
FAMILIAR	3	58	41.1	42.0	57.2
VERY FAMILIAR	4	59	41.8	42.8	100.0
	9	3	2.1	Missing	
	Total	141	100.0	100.0	

Q20E HOW FAMILIAR ARE YOU WITH THE STATEMENT ON COMPETENCIES?

Value Label	Value	Frequency	Percent	Valid Percent	Cum Percent
UNFAMILIAR	1	51	36.2	38.6	38.6
SOMEWHAT FAMILIAR	2	26	18.4	19.7	58.3
FAMILIAR	3	36	25.5	27.3	85.6
VERY FAMILIAR	4	19	13.5	14.4	100.0
	9	9	6.4	Missing	
	Total	141	100.0	100.0	

Q21A - Q21C   PLEASE LIST ANY RECENT SUMMER MATHEMATICS TEACHER INSTITUTES WHICH YOU HAVE ATTENDED?

Coding for questions 21A-21C:
Variable - Question Number.
Mean - Proportion of teachers that attended the listed institute.
Std Dev - Standard Deviation.
Minimum - "0" is assigned if the teacher has not attended the listed institute.
Maximum - "1" is assigned if the teacher has attended the listed institute.
Valid N - Valid number of responses.
Label - Description of institute.
A. California Mathematics project
B. AP Calculus
C. Other

Variable	Mean	Std Dev	Minimum	Maximum	Valid N	Label
Q21A	.19	.39	0	1	141	CAL MATH PROJECT INST
Q21B	.09	.28	0	1	141	AP CALCULUS INSTITUTE
Q21C	.26	.44	0	1	141	OTHER INSTITUTE

Appendix C

Invitation to Subjects in Classroom Artifact Study

«DATA Novak HD:Projects:MDTP:Form LetterDD»[Date]

«name»
«street address»
«city», «state» «zip»

Dear «fname»,

The Center for the Study of Evaluation at UCLA is looking for a few knowledgeable and innovative mathematics teachers to participate in an evaluation study of a mathematics diagnostic testing program. We are looking for teachers who are

- currently teaching mathematics at the high school level in California,
- familiar with the NCTM standards and the reform effort spurred by those standards,
- actively involved in the implementation of that reform in their own classrooms, and
- willing to have their innovative teaching methods documented within the context of an evaluation.

Teachers selected for participation should be willing

- to administer diagnostic tests appropriate to the course that they are teaching at two or three time points over the course of the school year. Each test administration can be accommodated within a normal class period and the results of the tests have the potential to provide valuable information to inform the instructional process.

- to provide information to the evaluators about instructional artifacts such as textbooks, workbooks, worksheets, or other teacher-prepared or commercially available resources used for instructional purposes.

- to provide the evaluators with information about assessment practices used in the classroom, such as copies of quizzes and tests, scoring and grading procedures, and homework assignments and practices.

- to respond to a questionnaire and a follow-up interview about their instructional practices.

This study will involve little or no disruption of normal classroom routines and practices, and the time commitment required of the teachers will be minimized as much as possible. In addition to the satisfaction of having their innovative classroom practices documented in a manner that can contribute to the professional development of the mathematics teaching community as a whole, participating teachers will be paid a stipend of $100 as a partial compensation for their efforts on the behalf of this study.

If you are interested in participating in this study, and/or you know of other teachers that may be interested, please take a moment and fill out the enclosed information card. If you have any questions, please feel free to call me MWF at (310) 206-1532, fax me at (310) 794-8636, or contact me by electronic mail at john@cse.ucla.edu. I hope to hear from you soon.

Sincerely,

Dr. John R. Novak
Project Director

Appendix D

UCLA - Center for the Study of Evaluation
10880 Wilshire Blvd. Suite 700
Los Angeles, CA 90024-1522

Attn: John Novak

Front

If you are interested in getting more information about participating in this study, or if you know of someone else who might be qualified and willing to participate, then please fill out the contact information below and return this card. Thank you for your consideration of this matter.

Your name: ______________________________

Street Address: ______________________________

City: ____________________ State: ________ Zip: ________

Contact phone number: (   ) ______________________________

Hours that you can be reached at this number: ____________________

Other name: ______________________________

Street Address: ______________________________

City: ____________________ State: ________ Zip: ________

Contact phone number: (   ) ______________________________

Hours they can be reached at this number: ____________________

Back

Appendix E
Questionnaire to Subjects in Classroom Artifact Study

MDTP Evaluation Teacher Survey

Educational Background and Training

1. What were your undergraduate major(s)? minor(s)?

2. Do you have a master's degree? If so, in which discipline?

3. What type of teaching credential do you hold?

4. How long have you had your teaching credential?

5. How many units of college math have you taken?

Teaching Experience

6. How long have you been teaching math?

7. Please list the different math classes that you have taught in the past.

Activities and Professional Development

8. Please list any continuing math and/or education classes that you have taken.

9. Please describe any professional mathematics education activities (such as workshops or inservice activities) that you participate in. Include any recent summer mathematics institutes you have attended.

10. Please describe any leadership roles that you hold at your school.

11. Please describe any innovative curricula that you have helped your school or department implement.

12. Please circle any of the following programs in which you have helped your school or department participate in the last three or four years. (*Please indicate which ones.)

a. Eisenhower Projects*

b. California Academic Partnership Program (CAPP) Projects*

c. Implementation of Math A or Math B

d. Implementation of UC Davis' College Prep Math curriculum

e. Implementation of SFSU/UCB's Interactive Math Project curriculum

f. Implementation of the University of Chicago's School Mathematics Project curriculum

g. Implementation of the Southern California Regional Algebra Project (SCRAP)

13. Please circle any of the following programs in which you have participated in the last three or four years. (* Please indicate which ones.)

a. California Mathematics Project*

b. Other teacher institutes*

c. Regional CMC conferences

d. NCTM conferences

e. AP Calculus scoring

f. CLAS scoring

14. Indicate the extent of your involvement with the following organizations by checking all that apply among **N for None, M for Member, J for use Journals regularly, C for attend Conferences, O for held Office.**

None	Member	Journal	Conference	Office	Organization
N	M	J	C	O	NCTM: National Council of Teachers of Mathematics
N	M	J	C	O	CMC: California Mathematics Council
N	M	J	C	O	Regional Section of CMC (Which one(s)?
N	M	J	C	O	Other professional organizations in which you have recently participated (Please list):

15. For each of the following documents, please indicate the extent of your familiarity with it by circling one of the following choices: **U for Unfamiliar, S for Somewhat Familiar** with its main thrust, **F for Familiar, and V for Very Familiar** with the document based upon reading it or attending workshops about it.

Unfamiliar	Somewhat Familiar	Familiar	Very Familiar	
U	S	F	V	NCTM Curriculum and Evaluation Standards
U	S	F	V	NCTM Professional Standards for Teaching Mathematics
U	S	F	V	1985 California Mathematics Framework
U	S	F	V	1992 California Mathematics Framework
U	S	F	V	Statement on Competencies in Mathematics Expected of Entering Freshmen (UC/CSU/CCC, 1989)

Personal Statement

16. During the past few years, research on student learning has encouraged educators to adopt new methods of teaching mathematics. Please write a brief description (1 -2 paragraphs) of your teaching philosophies and how you have implemented reform-oriented education in your classroom.

Appendix F

Instructions to Subjects in Classroom Artifact Study

Date here

«name»
«street address»
«city», «state» «zip»

Dear «fname»,

By now you should have received a classroom set of the MDTP examinations in the mail. If you have not already done so, please plan to administer the tests to your students sometime in the near future.

We will be trying to get information about the relationship between the MDTP tests and your instruction. Our primary means for obtaining information about your instructional practices is the collection of what we call **classroom artifacts**. A classroom artifact is anything that is generated by you in the course of your instruction which could be collected and examined by us. Some examples of classroom artifacts include, but are not limited to: homework assignments, worksheets, quizzes, handouts, tests, and any other collectible items. What we would like you to do is to put aside one copy of any such artifact to be sent to us at a later date.

To make this process as easy as possible, we have enclosed two pre-addressed Federal Express envelopes and two pads of post-it notes. We would like to ask you to take a few minutes each day to fill out one of the post-its to record the following information regarding each of the artifacts for that day:

1. Type of artifact - Homework, classwork, project, quiz, test, or other.

2. Purpose of the task - To introduce new concepts or skills, to review or practice, to apply concepts or skills in different context, or other.

3. The source of the artifact - Teacher generated, district, school, or department generated, or a textbook or supplementary source.

4. How and where the work was done - Individually or in groups, in class or outside class.

5. Grading of the task - Graded or checked for completion (no grade).

6. Any comments you feel necessary to explain the artifacts that you collect.

Attach the post-it to the artifact and slip it into one of the envelopes. For example, suppose that on a typical day you give your students a quiz, and then for homework you have them complete a teacher generated performance assessment. For that day you would have at least two artifacts, the quiz and the worksheet; each would get a post-it note and then be added to the envelope. Remember, the purpose is to document as fully as we can, within the limits of our budget and your time, what is going on in your classroom; whatever you can reasonably provide to assist with that task will be appreciated. So, for example, if you have lesson plans that you could easily include, please go ahead and do so. If there is any question in your mind about whether or not to include something, then go ahead and include it. If you have ready access to a copying machine and you have assigned students problems from a book, then a photocopy of the actual problems would be very helpful. If that is not feasible, then just a list of the page and problem numbers will suffice.

We ask that you return one envelope at mid-semester, and other at the end of the year. Just seal the envelope and drop it into the nearest Federal Express drop box, or leave if for the Federal Express pickup at your school. If it looks like you are running short of post-it notes or envelope capacity, please let us know and we will send you more.

Once again, I would like to thank you for your participation in this study. You will help to provide us with very valuable information regarding the Mathematics Diagnostic Testing Project. If you have any questions, please feel free to call me MWF at (310) 206-1532, fax me at (310) 794-8636, or contact me by electronic mail at john@cse.ucla.edu.

Cordially yours,

John R. Novak
Project Director

Appendix G

Artifact Coding

ID: Each assignment was given a five-digit ID number. The first two numbers correspond to a unique ID given to each teacher and the last three numbers correspond to the particular artifact. For example, the artifacts received from teacher 1 were coded as 01001, 01002, ... , 01022.

I. Alignment with NCTM Standards

Each artifact was examined to determine its adherence to the guidelines set by the NCTM.

Math as Problem Solving

NCTM states that mathematics curriculum should seek to accomplish the following tasks:

- Use problem-solving approaches to investigate and understand mathematical content.
- Apply problem-solving strategies to solve problems from within and outside mathematics.
- Recognize and formulate problems from situations within and outside mathematics.
- Apply the process of mathematical modeling to real-world problem situations.

Descriptions of scale points

1-2 Assignments consist primarily of routine calculations which can be solved using the same or similar techniques. See artifact 8160, 8108.

3-4 Students are asked to solve problems that require different types of solutions but can still use routine procedures to arrive at the solution. Although some questions may require higher level thinking, students can still complete part of the assignment with the knowledge of a few formulas and procedures. See artifact 8107, 8120.

5-6 Requires students to analyze problems using their existing knowledge and develop appropriate techniques to solve the problem. The assignment encourages investigation so that students can recognize and formulate their own problems. Students use the problem-solving process to solve real-world problems. See artifact 8110, 8119.

0 Not applicable.

Mathematics as Communication

The goals of communication within mathematics should be the following:

- Formulate mathematical definitions and express generalizations discovered through investigation.
- Students should reflect upon and clarify their thinking about mathematical ideas and relationships.
- Express mathematical ideas orally and in writing.
- Read written presentations of mathematics with understanding.
- Ask clarifying and extending questions related to mathematics they have read or heard about.
- Appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.

Descriptions of scale points

1-2	Students work individually and are not asked to explain their mathematical reasoning. Assignments consist primarily of routine calculations which focus on answers rather than procedures. See artifact 08043, 19036.
3-4	Students work in cooperative groups and are asked to explain some of their thinking. Although much of the focus is still on answer rather than procedure, students must sometimes explain how an answer was derived. See artifact 19004, 1021.
5-6	Students are required to read about, write about, speak about, reflect on, and demonstrate mathematical ideas. Students should write convincing arguments that validate their own generalizations and work cooperatively in order to share ideas and reasoning. See artifact 7023.
0	Not applicable.

Mathematics as Reasoning

NCTM states that mathematics curriculum should seek to extend logical reasoning skills which will enable students to do the following:

- Students make and test conjectures.
- Formulate counterexamples.
- Follow logical arguments.
- Judge the validity of arguments.
- Construct simple valid arguments.

Descriptions of scale points

1-2 Students are asked to perform routine calculations without explanation for their thinking. See artifact 9014, 18093.

3-4 Although most of the problems require a numerical answer, students are sometimes required to explain the procedures used while problem solving. See artifact 18011, 08076.

5-6 Students are required to provide simple valid arguments as justification for their solutions to specific problems. Students are asked to examine patterns and make generalizations based on their observations. They should verify their findings by using a logical test or finding a counterexample. See artifact 18019, 10054.

0 Not applicable.

Mathematical Connections

NCTM states that the goal of making mathematical connections should be that the student:

- Recognize equivalent representations of the same concept.
- Relate procedures in one representation to procedures in an equivalent representation.
- Use and value the connections among mathematical topics.
- Use and value the connections between mathematics and other disciplines.

Descriptions of scale points

1-2 Students are asked to complete assignments that fail to demonstrate any relationship to other mathematical topics.

3-4 Students can see different representations of the same concept (for example, if a student is to solve a problem and graph the results). Part of the assignment makes connections among math topics or among other disciplines.

5-6 Students are able to apply and translate among different representations of the same problem situation or same mathematical concept. Students are asked to relate procedures (for example, solve a problem, graph the results, and do a presentation). The assignment makes connections with other disciplines. The real-world value of the activity is evident.

0 Not applicable.

II. Description of Assignment

Type

Exam:	1 = Y 0 = N
Quiz:	1 = Y 0= N
Project:	1 = Y 0 = N
Homework:	1 = Y 0 = N
Exercise:	1 = Y 0 = N
Other:	1 = Y 0 = N

Purpose

Introduce new concepts or skills:	1 = Y 0 = N
Review or practice:	1 = Y 0 = N
Apply concepts or skills in different contexts:	1 = Y 0 = N
Other:	1 = Y 0 = N

Source

Teacher-generated:	1 = Y 0 = N
District-, school-, or department-generated:	1 = Y 0 = N
Textbook or supplementary source:	1 = Y 0 = N

Work Done

Individual:	1 = Y 0 = N
In Groups:	1 = Y 0 = N
In Class:	1 = Y 0 = N
Outside Class:	1 = Y 0 = N

Graded

1 = Yes
2 = Checked for completion / no grade
0 = No response

Comment

1 = Comment
0 = No comment

Appendix H

Teacher Profiles

H.1 Teacher 1 Geometry

Teaching Philosophy

"I believe students learn the most when they are actively involved in the learning process. The teacher has the role as more of a facilitator rather than a transmitter of information. Discovery learning is a better approach for the students I have had. I have found that they retain the knowledge better and enjoy learning when I use this approach.

Traditional methods have their place and time, and I do use them on occasion. However, the students and I both enjoy it when we use my additional activities with the textbook. In addition to the activities, I also use manipulatives when appropriate. This gives the students a more concrete way to understand the concepts discussed in class."

Activities and Professional Development

Teacher 1 has continued taking education classes on Mathematics Assessment through the Kings County Office of Education and has attended workshops on State Curriculum for Math B and CLAS Scoring and Preparation. Among the leadership roles that she holds at her school are Academic Decathlon Coach and Class of 1995 Advisor. Upon her arrival at Fowler High School, she helped to implement the state's Math A and B curriculum.

Classroom Activities

Teacher 1 had students do various real-world problems in order to understand ratios, proportions, and right triangles (01001, 01016). Her lessons on similar polygons were done using the traditional two-column proofs. Teacher 1 emphasized parts and properties of geometric figures and considered terminology an important part of learning. Students are not only required to state definitions, however; they are also required to draw a diagram to explain the definitions (01008).

Teacher 1 taught students properties of circles by assigning group discovery projects. Students were asked to observe various properties and write their own geometric theorems based on their observations (01009). Although her tests were usually teacher-generated, they didn't require the students to use the problem-solving skills required to solve the word problems in some of the assignments (01017). Upon investigating perimeter and area among polygons, Teacher 1 again used the discovery approach by providing students with activities that asked them question like "What do you notice about the perimeters of the different polygons?" and "What do you notice about the areas?" But besides a few of these activities that prompted students to explain their answer, most exercises asked for routine calculations and tested memorization.

H.2 Teacher 2
Algebra/ Geometry

Teaching Philosophy

"I am currently working toward completion of the CSUN waiver program in mathematics. I try to participate in the various conferences (e.g., LACTMA, CMC) when close to home. I have been developing a fledgling team of students to participate in the 'Math Olympics' at the invitation of Glendale School District (last year was our first) with my department chair and other interested math department members at Poly High School. I believe efforts in developing the modality of cooperative learning in mathematics teaching. To that end I employ it more and more frequently either on an informal or formal basis. I strongly believe that 50% (at least) of our mathematics students are sadly neglected or discouraged by bias and/or cultural inhibitions (i.e., the women in our society) and do everything to encourage their participation and examination of opportunities available. I am currently attempting to complete a review of the NCTM Standards and the 1992 California Framework for implementation in my classroom and our math program at Poly. As much as possible I try to encourage use of calculators and other manipulatives (some very simple) and am constantly looking for new ideas from other professionals. (CSUN mini manuals and LACTMA conferences have been very productive to this end.) And, last but not least, I have followed up on your invitation."

Activities and Professional Development

Teacher 2 has completed all required courses at CSUN within the Department of Secondary Education in addition to history of mathematics, linear algebra, discrete math, statistics, differential and integral calculus, analytic geometry, modern algebra, as well as methods in teaching mathematics. Teacher 2 has also participated in the continuing education Eisenhower program within the LAUSD where he learned about the use of graphing calculators. He also participates in the LACTMA conference yearly and other district-sponsored Eisenhower workshops. Teacher 2 sponsors several clubs at his school like the Asian club, the 11th grade lunch-time tutoring, and the Math Olympics. In addition, he is developing coordination of WISE (Women in Science and Engineering) as the liaison with the CSUN School of Engineering.

Classroom Activities

N/A

H.3 Teacher 3
Algebra 1

Teaching Philosophy

"What 'we' have done to students in mathematics over the past years is atrocious. Students (particularly?) in Algebra 1 are exposed to extremely abstract ideas and are expected to master such concepts in a limited time frame. It is now known that students must be given the time to discover patterns and relationships and 'construct their own learning.'

Eight years ago, I began using cooperative learning groups in my classes, and had my students maintain a journal of their learning. I have also incorporated the use of manipulatives to teach abstract ideas—integer chips (or tiles) for integer operations, balances for equation-solving (and new 'cups and tiles'), and 'strips and singles' (Lane Co. Algebra) tiles to introduce the distributive property. I also use integer tiles to learn factoring. I am beginning to incorporate portfolios as part of student assessment. Tests have moved toward explanation of thinking and applications, and less on manipulation of symbols."

Activities and Professional Development

Teacher 3 currently attends various professional mathematics education activities including the California Math Council Conference, UC Davis College Preparatory Mathematics Workshops, and Sheltered Math Workshops. In the past she has served as a mentor teacher, department chair, union representative, Math Engineering Science Achievement (MESA) advisor, and Students for Social Responsibility (SSR) advisor. She is currently a member of the Scholastic Circuit, a support network for "middle 50%" students planning on education after high school. Among the activities that she has helped her school implement are the Math A State Framework, UC Davis College Prep Math for Algebra 1, and writing in mathematics.

Classroom Activities

N/A

H.4 Teacher 4
Pre calculus

Teaching Philosophy

"My philosophy on teaching mathematics revolves around two central themes: The first is that I, as the teacher, am not the sole source of knowledge and information in the classroom, and the second is that math is real and is everywhere.

Concerning the first, students have a great deal of knowledge that can be tapped for the benefit of the entire class; they also oftentimes have a certain amount of intuition that can be applied to various problems that reflect real-world phenomena. Which brings me to the second theme. Students need to see the uses, applications and purpose of mathematics in order to appreciate the existence and beauty of it. We work on real-world problems, often introducing a new problem at the beginning of the unit to be solved at the conclusion of it. This seems to fuel student motivation to a degree unknown in the traditional paradigm of math teaching."

Activities and Professional Development

Teacher 4 is currently working to get his teaching credential and has attended the Arizona Advanced Placement Institute and the California Math Council Conference. Among the leadership roles he holds at school are senior class advisor and computer lab supervisor. He has helped his school implement the University of Chicago School Math Project (UCSMP).

Classroom Activities

Teacher 4's classroom activities are an even balance of routine calculations and explorations. Many homework assignments include true/false and multiple-choice questions to test student knowledge of vocabulary and procedures (04001, 04003, 04007, 04051) but almost all assignments also require students to go beyond memorization of formulas and apply concepts in different contexts.

One of the most common applications of knowledge that Teacher 4 uses in his class is the graphing of functions (04001, 04003, 04022, 04027, 04030, 04031, 04038-04040). In order to understand sinusoidal functions students are generally required to identify and find solutions to equations and then explain their answers with a graph (04031). The understanding of the functions was further enhanced with the use of graphing calculators (04035, 04039). Using these calculators, students are required to either check their work or identify characteristics of certain functions.

Teacher 4 was able to extend his assignments by incorporating a variety of real-world problems into his lessons. One of Teacher 4's strategies used to teach sinusoidal functions was to discuss how math and music are related (04013, 04016). In this manner, he explained how tone can be determined by the amplitude and period of a sound wave. Another creative idea involved the use of average monthly temperatures to develop sinusoidal equations which the students could use the predict the temperature on any given day (04020). Teacher 4 also incorporated various word problems related to science to promote higher level thinking and to help students make connections between math and science. These problems included topics such as pendulums, radar, voltage, waves, and springs (04018, 04027, 04039, 04040).

Teacher 4 encouraged cooperative learning within his classroom, especially on assignments where there was more than one correct method for solving a problem so that the students could learn different problem-solving strategies. Whenever students were required to prove theorems or come up with trigonometric relationships they were put into groups (04005, 04017, 04020, 04033, 04039) so that they could collaborate on ideas.

H.5 Teacher 5
Trigonometry/Math Analysis (Pre calculus)

Teaching Philosophy

"It is the teacher's desire and expectation that each student will be successful in this class; that each will find the algebra and/or geometry interesting and understandable; and that each will enjoy creative problem solving, discovering new methods, growing intellectually, making academic progress, and developing new insights and understanding. It is the teacher's hope that each student will come to appreciate Galileo's famous and beautiful observation: 'Mathematics is the alphabet with which God has written the universe.'

The achievement I am most proud of recently is that a group of my students won first place in the American Statistical Association's Poster Contest for 1993-4, the best out of 462 entries! Another group of mine won honorable mention. Three years ago another of my groups won first place; and the story is attached, published in the December 1992 issue of *Mathematics*

Teacher. This project involves groups, cooperative learning, connections with other disciplines, writing across the curriculum, and alternative assessment."

Activities and Professional Development

Teacher 5 has received several teaching awards including the Tandy Technology Scholars Outstanding Teacher Award in Mathematics/Science/Computer Science and the *Mathematics Teacher* Presidential Award for excellence in Mathematics Teaching. She has also been recognized by the *Valley Tribune*, California State University, Los Angeles, and *The Los Angeles Times* for her outstanding teaching. Teacher 5 has been involved in several research projects covering a broad range of subjects including "Factors Influencing Student Productivity," "The Influence of Income on Self-Perceived Success Among California State University Physics Professors," and "Computer-Assisted Instruction, and Algebra 1 Texts." She has published several articles in *Mathematics Teacher*, some of which are "Preparing Posters Promotes Learning," "Open ended Question," "A Pythagorean Party," and "Indirect Proof: The Tomato Story." Her professional activities involve speaking at various conferences and in-services on topics such as "Integrating Statistics Into the Traditional Mathematics Curriculum," "Teacher-Researcher," "Motivators In the Classroom," and "Creativity in The Mathematics Classroom."

Classroom Activities

N/A

H.6 Teacher 7 Geometry

Teaching Philosophy

"I obtained my teaching credential three years prior to the 1985 California Mathematics State Framework was published. During that time, I began to look for new strategies to make mathematics more meaningful for my students. I also became an advocate of creating parent awareness of the changes in mathematics to gain community support. In 1991, I became a trainer for the California School Leadership Academy. This association provided me with information regarding the vision of a quality mathematics program. In the module, we taught administrators, in a two-day comprehensive workshop, all of the changes involved in creating a quality mathematics program. Math Counts, the 1992 California State Framework, and the National Council of Teachers of Mathematics Professional Standards for Teaching Mathematics were all focal points in the presentation.

I believe all students can be mathematically powerful, given the right motivation. I enjoy creating challenging activities that relate the mathematics learned in class with real-world problems. I enjoy watching my students present what they've learned or discovered in class through writing assignments and oral presentations. My reform efforts include the daily use of calculators by all students in my classroom and the use of manipulatives for various math activities. My goal is to create a mathematics program that is challenging and interesting, but also accessible to a variety of students."

Activities and Professional Development

Teacher 7 has continued her education by taking computer courses at UCSD. She has participated in various mathematics education activities, including the California School Leadership Academy and the Golden State Exam, and has been a speaker at the Greater San Diego Mathematics Council Annual Conference, California Mathematics Council Southern Section Annual Conference, Annual Mentor Conference, and the California Association for the Gifted Annual Conference on Math. Teacher 7 demonstrates her leadership in her school participation as Engineering, Technology, and Design Career Path resource teacher, chairperson of the Advisory Action Team, and Governance Team member. Among the innovative curricula which she has helped implement in her school are CLAS/CAP open-ended math problems contest, schoolwide math projects, and development and use of alternative assessment practices for the entire math department.

Classroom Activities

Teacher 7 integrated various projects and problem-solving activities into her classroom and used very few traditional methods as part of her lessons. Her geometry class was very rarely asked to prove theorems using the traditional two-column proofs; instead, the focus of her class was to allow students to prove theorems in an open-ended manner. Students were asked to respond using an essay style which allowed for more flexibility in each answer (07012, 07013, 07019, 07024).

The paragraph responses for proofs are characteristic of Teacher 7's integration of writing into her classroom. Another writing activity used by Teacher 7 is a journal (07021) that students are required to submit after every chapter. In this journal, students give a brief summary of the unit/chapter, explaining what they have learned. The students also give examples of problems they have studied and how they are solved and reflect upon what was easy or difficult for them.

Problem solving was heavily emphasized in Teacher 7's class where she challenged them by giving open-ended questions (07023, 07025, 07026) and requiring them to write a response justifying their answer. One question (07023), for example, asked the student, "Write everything you could tell a friend about the Pythagorean Theorem, why it works and how you would use it." This method of asking students to explain a concept not only tests the knowledge of the formula but also assures that the student knows its significance and when to apply it. Another way in which Teacher 7 presented her students with open-ended problems was to integrate them into interesting anecdotes so as to gain the interest and attention of the students (07025). One problem-solving technique used to develop student understanding of concepts was pattern recognition (07007, 07008). Upon learning about characteristics of different polygons, for example, students were asked to record all of the possible diagonals in different geometric shapes. Based on these findings they were then asked to develop a relationship between the number of vertices and the number of diagonals. This method of having the students develop their own generalizations helped their understanding of formulas and developed their problem-solving skills. The creativity required for developing these generalizations was developed in spatial visualization activities (07011). This type of assignment helped students to think more abstractly and find different ways to solve a problem.

Teacher 7 developed several activities for her students in order to allow them to apply concepts in different contexts and to help them see the mathematical connections to the real world (07001, 07002, 07021, 07023, 07025, 07026). One project that Teacher 7 assigned to each student was to develop a scale model of his/her room (07001). This project involved the use of ratios and required the students to include the exact measurements of the room and the furniture within it, along with the scaled measurements. Another project developed to help the students see the connections between math and the real world was a Career Poster Project (07002). This project required the students to interview someone who uses mathematics in his or her job. The students had to give an example of a problem that this person has solved in his or her work and illustrate the job. This project helped students recognize the importance of learning mathematics and helped to develop enthusiasm for the subject material.

The traditional activities that Teacher 7 used in her class were usually the tests and quizzes. The students were generally given practice tests (07014 - 07018, 07020) which tested vocabulary and memorization usually using true/false or multiple-choice formats but they were allowed to work in groups. The actual tests and quizzes (07022) also followed the same format and were textbook generated for the most part. Although Teacher 7 did incorporate some traditional assignments into her lessons, her teaching style generally reflected her beliefs in using new methods for teaching mathematics.

H.7 Teacher 8
Algebra 1

Teaching Philosophy

"My approach to teaching algebra is different form the traditional. Rarely do students learn or practice an algebra skill without a meaningful context preceding the skill. I want students to be able to move easily between context – chart – graph – function – equation – solution – decision/comparison. Students begin the year translating situations into charts and graphs. We then focus on representing the same thinking process in symbolic (function) form. We use graphing calculators to reinforce the power of symbolic representation. We use the graph to solve, evaluate, and predict. We spend a little time developing linear functions to represent approximately linear data (line-of-best-fit). The whole time we are using functions to evaluate and graphs to solve, compare or predict. Equations keep popping up as specific cases of the more general function. We move easily into learning procedures for solving equations arising from a variety of linear contexts. The second semester of algebra follows a similar process developing understanding and skills for quadratic functions. Students work in groups, use technology (TI-81), write sentences to explain the meaning of their answers, work on short-term projects and give presentations (occasionally)."

Activities and Professional Development

Teacher 8 is a writer for the ARISE (COMAP/NSF) curriculum project during the summertime. He has been a presenter at the California Math Council Annual Math Conference and has also given presentations related to the ARISE curriculum. Among the leadership roles he holds at his school are Math Department Representative on Staff Development Committee, Mentor Teacher, and Member of Prep Tech/Career Advisory Committee. He is writing 95% of the algebra curriculum used in his math class, much of which is used by other department members who also teach algebra.

Classroom Activities

The students in Teacher 8's class are arranged in groups of four. Students often work together with other group members during the class period. During class, students may be asked to: work at the chalkboard, explore a problem using TI-82 graphing calculators, review skills through group competition, tackle an open-ended problem as a group, prepare and give a group presentation, or review skills as a class. Teacher 8 wrote all of the materials he submitted which require students to problem solve and understand the meaning behind each activity. One of the concepts which he emphasizes most is the students' ability to create and interpret charts and graphs. For example, one problem given to the students was the "Riding the Metrolink Problem (08005)" where students are given the fares to ride the train and are then asked to decide upon the best payment plan by using charts, graphs, diagrams or pictures. Students are also asked to submit a written recommendation to a Metrolink rider. Every task assigned requires students to complete a chart that demonstrates the representation of the values. All assignments require higher level thinking that cannot be completed simply by memorizing a formula; the tasks demand conceptual understanding. To help students understand graphs , Teacher 8 assigns students to graph real-world situations. One assignment, "A Graph is Worth a Thousand Words (08012)," for example, asks students to sketch a graph that compares Magic Johnson's yearly salary to the number of years he played professional basketball. Another question asked students to choose their own topic to graph and write at least two complete sentences describing the meaning of their graph. Teacher 8's introduction to operations with negative numbers began as pattern recognition rather than a set of rules. Students were then asked to form the rules for addition, subtraction, multiplication, and division on their own. Teacher 8's introduction to linear equations was also very closely related to real-world problems. In asking students to write and interpret functions, Teacher 8 poses problems such as the following. "Anthony is buying a new car. He makes a down payment of $3,500 and monthly payments of $400. Use words to write an expression explaining how to find his total payments. Use mathematical symbols to represent the same expression." By making his assignments and lessons conceptual, Teacher 8 helps his students interpret real-world data mathematically and enables them to use what they have learned outside of the classroom.

H.8 Teacher 9
Math Analysis (Pre calculus)

Teaching Philosophy

"In the past 25 years of teaching, I have taught all levels of mathematics at both the junior and senior high level to students of many ethnic backgrounds. In planning and teaching my classes, I feel the standards I set for myself are important parts of the role I model to the students. These standards of preparedness, respect and organization are values I consider important and ones I share with all my classes. To provide quality and equitable educational experiences to my students I have continued my own education. I was selected to attend a NEWMAST seminar at Cal Tech and was selected for the past two years to participate in the NSF institute for new math assessment techniques. Over the years I have shared my skills by writing curriculum guides in the area of mathematics, serving on district advisory committees for math proficiencies, higher level competencies and textbook adoptions.

In my classes I have developed units that incorporate the use of more writing into the students' work. The activities are couched in problem solving real world, mathematical, or whimsical. Students carry out investigations in a variety of situations where they sometimes work individually, or in small groups, or as an entire class. One major writing activity I have used for several years is a term paper project in the second semester. The students choose their own groups of three to write a 5-10 page paper on a subject of their choice but showing the relationship to mathematics. Then they prepare a 15-20 minute oral presentation. The feedback is always positive. They like seeing and developing a connection between a subject they enjoy and the mathematics they have learned."

Activities and Professional Development

Teacher 9 has participated in NEWMAST seminars and in the NSF Institute for Development of Materials for Authentic Assessment. She is actively involved in her school and has shown her leadership in her roles as department chair, GATE advisor, and Grade Level Class advisor, and as a member of the Governance Team, Principal Advisory Committee, and Scholarship Advisory Committee. She has developed a writing project for her Honors Geometry class that is incorporated into the second semester coursework.

Classroom Activities

Teacher 9 had an even balance of both routine practice exercises and innovative mathematical explorations. During the past few weeks she has covered the topics of vectors and polar coordinates as well as conic sections. Most of her tests and quizzes on vectors and polar coordinates were designed to test computational skills such as addition and subtraction of vectors. They were also used to test a student's ability to remember definitions such as magnitude and dot product. Students were able to use this knowledge out of context, however, in a problem of the week that required them to know about magnitudes of vectors.

One of the projects that was assigned to the students was related to the solving of Fermat's Last Theorem (09017). Students were taught about the current mathematician Dr. Andrew Wiles who spent seven years solving the problem. Students were then given exercises to convince them that the theorem was true and were asked to do research on the solving of the problem.

The conic sections unit was a set of activities that Teacher 9 had created herself in order for the students to explore the characteristics of different surfaces. Not only did students become familiar with the equations and their relevant graphs, but they learned about the characteristics of each conic section. In groups students were given similar shapes and were asked to take measurements and tally their results in order to recognize patterns (09020). Students were also asked to make constructions.

Teacher 9 incorporated the use of graphing calculators in order for students to understand the components of a quadratic equation (09021). The students were given different exercises so that they could recognize patterns and see how changes in the equation could produce changes in the graph.

H.9 Teacher 10
Calculus AP

Teaching Philosophy

N/A

Activities and Professional Development

N/A

Classroom Activities

Teacher 10 uses traditional problems for teaching calculus as she prepares her students for the AP test. She has provided many assignments that she has her students complete in preparation for the AP Calculus test. The homework is normally taken out of a book and includes drill and practice problems with a few standard word problems (10003, 10004, 10006, 10008, 10011, 10013, 10016, 10017, 10022). Some homework includes only word problems, also taken from a text, one of which Teacher 10 does in class as an example (10019). With some other traditional problems, the class does them as a whole, with a group of remaining problems done as homework (10023).

Teacher 10's exams are all teacher-generated and include traditional problems with some recall of definitions (10002). In a few problems she asks her students to explain their answers, and she includes a bonus question that is a word problem that relates the concepts learned in class to real-world problems (10002 and 10020). Her tests generally resemble the AP test and do include multiple-choice in addition to "essay" type problems (10012 and 10026). Teacher 10 does use group quizzes to encourage discussion among students (10005 and 10007). These quizzes include questions such as "explain why" and "justify your answer" after some of the problems, with all problems being traditional. In addition, Teacher 10 gives individual student quizzes that the class discusses afterwards (10009). Other group work includes preparation for quizzes that utilize drill and practice problems that resemble the homework problems (10014). The particular quiz that this group work was designed to prepare for was a no-calculator quiz that emphasized speed with very traditional problems. The final exam is a most recent version of the full AP exam.

Teacher 10 uses videos to help her students understand certain concepts; she used a video from Cal Tech to explain the use of π in rotations (10007). In addition Teacher 10 provides projects to help students visualize and understand rotation and volumes. For example, she provides flat shapes and asks the students to physically cut them out of paper and rotate them to determine the volume that they make upon rotation (10010). Some projects, from text books, provide students with a way of discovering concepts in a step-by-step fashion (10018). Teacher 10 has students go through this discovery process that includes such procedures as making and comparing graphs, both in groups and on one's own.

As a part of her teaching she uses graphing calculators to solve traditional problems (10001, 10021). Also, Teacher 10 gets her students to write calculator programs that the students use to solve traditional problems at home (10025). Some problems the students first do by hand and then develop calculator programs to solve the same problem (10024). Teacher 10 continues to use graphing calculator exercises during the few weeks prior to the AP exam (10036) a portion of which is done in groups. In addition, Teacher 10 uses progressive, thematic problems; one assignment is about the properties of an electron and includes questions such as "How did you get that?" Also, Teacher 10 uses a "Tic Tac Toe" game to review concepts and quiz students on the topics of the course (10047); this game gets individual students to answer questions with those individual answers determining the outcome of the entire team.

In the final stretch before the actual AP Calculus exam, Teacher 10 has her students work AP-type problems in groups and then discuss them with the entire class (10028, 10029, 10032, 10033, 10040, 10041, 10049 - 10059); these assignments are not graded, but only checked for completion. One of these group exercises was adapted from the C-4 Institute (10043). Also, in class, individual work is done and discussed within the class as a whole (10037). Traditional, individual homework assignments are also given (10030, 10034, 10038, 10042, 10046). In addition she has her students write journal entries that ask questions about various concepts within her course, what techniques the students know to approach certain concepts (10031) and commenting on topics that they are most and least comfortable with (10027). During this final preparatory time Teacher 10 continues to give in-class quizzes with traditional AP type questions (10035 and 10045) that are sometimes discussed afterwards (10039). Teacher 10 also provides students with hand-outs that students can browse through on their own for review after she briefly describes the material in class (10048). At the end of the course Teacher 10 gives a survey to her students asking them about her teaching regarding graphing calculators, group work, and their preparation for the AP exam.

H.10 Teacher 11
Geometry Honors

Teaching Philosophy

"I believe that girls can achieve in mathematics and teaching in an all girls school. I continually try to encourage my students to share my belief.

In my classes there are many and varied opportunities for them to be successful. Working within the framework of the UCSMP curriculum, I am able to include critical thinking and problem-solving activities, investigations and explorations, cooperative experiences and special projects. My students are actively involved in their learning and are expected to share their thinking verbally and in written form. For the past three years portfolios have become a part of my assessment enabling both me and my students to further assess their thinking and progress.

I have become more of a facilitator, leading my students to discover learning, than a traditional teacher. Through UCSMP my students are aware of the real-life uses of the mathematics they are learning. They are active learners and thinkers. 'Math is no longer a spectator sport!' (UCSMP)"

Activities and Professional Development

Teacher 11 has taken continuing classes in the area of computers and attends workshops and conferences, including the California Math Conference, on a regular basis. She has demonstrated her leadership as Math Department Chair and Computer Coordinator for the past twelve years. She has helped her school to adopt a new math program based on the University of Chicago School Mathematics Project thus changing the four-year math curriculum at St. Joseph High School.

Classroom Activities

Teacher 11 has effectively implemented reform-oriented teaching methods in her classroom. One activity that she emphasizes heavily within her class is writing and communicating orally about mathematics. Students in her class keep journals where they are asked to explain a mathematical concept or discuss their feelings about a topic that has been covered. Artifact 11002, for example, asks students "Discuss your feelings about this chapter. Did you find it easy or difficult. Why?" Student journals are used to access students' needs and anxieties and also to help students make their own connections. For example, Artifact 11017 asks students to "Explain how surface area and volume in 3-D are related to perimeter and area in 2-D." Oral communication is also heavily emphasized as students are required to present one problem per chapter to the class. During this presentation, a student must restate the problem in his/her own words, speculate on a solution, describe the strategy to the class, attempt a solution, discuss the solution attempt, and create a problem that can be solved using a similar strategy (11006).

Teacher 11 includes challenging math puzzles to help students develop their own creativity and style for solving problems. She has assigned her students problems of the week and has given her students assignments based on the four-color theorem and Conway's Cube puzzle. She has also asked her students to build a kite using their knowledge of geometric figures and submit a report describing the process of building it and the geometry used.

In order to help her students understand the concepts that are being taught in class, Teacher 11 always includes activities that help with visualization. She gives her class activities on spatial visualization and lets students work with actual shapes and figures. For example, students were given a piece of 8.5 X 11 paper and were told that the paper could be rolled to make two different sized cylinders with the same lateral areas. They were then asked which to find which cylinder had greater volume (11024). Students were also asked to cut out three pyramids and put them together to make a prism.

Teacher 11's teaching methods closely follow the NCTM standards. She requires her students to problem solve and make mathematical connections, and she also shows students the real-world significance of the knowledge that they acquire. She challenges her students and makes the subject material interesting.

H.11 Teacher 12
Geometry/ Algebra I

Teaching Philosophy

"I believe in the constructivists approach and I have always tried to teach that way and to acquire curriculum that also follows that approach."

Activities and Professional Development

Teacher 12 attended the California Math Project at TCMP for two summers. In addition, he has participated in the Project T.I.M.F. He has attended many CMC-SS annual conferences and Math A as well as CPM workshops. Teacher 12 has also led workshops for CSUDH Math Project and was a speaker at a CMC-SS conference. In addition to outside of school activities, Teacher 12 helped to implement Math A, CPM 1 and CPM 2 curricula at his own school.

Classroom Activities

Teacher 12 uses current educational reform innovations such as group work and portfolios within his classroom. He gives daily assignments that are done in groups within the classroom and completed outside of the classroom in the form of homework (12001, 12002, and 12003). These assignments seem to be primarily drill and practice without application of the concepts to real-world situations.

At the end of each unit students compose a summary of things that they learned in that unit (12004 and 12005). In these summaries they write about not just mathematical concepts that they learned or had problems with, but also about learning cooperative skills and how to solve problems generally. The summaries are included with the student's portfolio assignments that are turned in for each unit. These portfolio assignments sometimes include within them problems that use concepts within the class to solve real-world problems like using graphs to solve problems (12006). After and during the solving of each problem, the students write explanations as to why they are solving the problems in a particular fashion (12006 to 12014). In addition, within their unit portfolios, the students have the opportunity to discuss their strengths and weaknesses by illustrating them through solving problems that the students consider they know well and those that they do not know very well (12006 to 12014). Teacher 12 seems to explain to his students why he is teaching particular concepts and tries to give a glimpse of the overall picture to them. For instance, the students write in their portfolios that the class is studying how to solve subproblems as a strategy for solving large problems (12010 to 12014). Students are free to provide their own examples of how subproblems can be of benefit. As the last portion of the portfolio assignment for each unit the students describe their working within their group. Teacher 12 has his students keep all of their unit portfolio assignments in a folder (portfolio) in order for them and him to see their progress. The grading is done on a four-point scale (Well Done, Acceptable, Revision Needed, and Restart).

In a note, Teacher 12 explains his grading scheme: Homework counts for 100 points, a group test counts for 50 points, portfolio is 100 points, individual test is 100 points, weekly problems are 10 points each, and projects are 40 points with about two per semester.

Teacher 12 provided examples of group tests that he administered to his students. Artifact 12018 contains an assortment of group tests while artifact 12019 contains all the group tests for a single student. In addition, Teacher 12 provided answer keys (and thereby examples) of all tests that he administered—both individual and group tests (12020). His assessment style is progressive as he uses group tests during which students are encouraged to do peer teaching (12018, 12019, and 12020). He randomly chooses a single test to grade from each of the groups for every unit covered. Teacher 12 normally tests his students twice at the end of each unit—once using a group test and once using an individual test (12020). During the individual tests the students are encouraged to use their notes, books, and homework. Teacher 12 has a mix of questions that he asks his students to solve: Some are word problems while others vary in complexity from quick answers to graphing to explaining in a short statement why a certain answer is correct. In addition, Teacher 12 uses pictures in his tests to evaluate the extent of his students' knowledge.

H.12 Teacher 14
Geometry

Teaching Philosophy

"My first couple of years of teaching, I was pretty rigid and by-the-book. My grades were based on how well the students could do the proofs, and how well they did on the quizzes and tests that were given. Pretty much the way it was done when I was in Catholic school in 1968. Homework was collected, graded for correctness and completeness and returned, making up a significant part of the grade.

Now my instruction is a cooperative group setting and using a discovery approach to get students to figure it out. There is still some lecture, although much less. Homework is no longer collected, but checked routinely for completeness, not correctness. I have also incorporated project-type activities which are about equal in weight to quizzes and tests. I also allow notes for tests and push the use of calculators. I also try to get students on a computer whenever possible. Although I feel it was unsuccessful, I also tried a 'Build a Book' class."

Activities and Professional Development

Teacher 14 attends 2 to 3 mathematics conferences per year and was a workshop presenter at the October and December 1994 Asilomar Conferences. During the summer he is employed by Key Curriculum Press as an Institute Facilitator for the workshops they sponsor, which have included Discovering Geometry, Geometer's Sketchpad, Calculus, and Graphing Calculators. At his school, Teacher 14 has helped his department use Geometer's Sketchpad to help enhance learning in geometry and has developed two units for use in Math A.

Classroom Activities

The majority of tests and quizzes that Teacher 14 used in his geometry class seemed to test memorization of formulas and vocabulary and were designed to test students on previous concepts. One quiz (14003), for example, tested students on 35 different vocabulary words. Another quiz (14018) asked students to find the volume of 13 different geometric figures. A student who had memorized the 13 formulas would have a perfect score. Yet another quiz (14007) asked students to directly list a formula. A question on this quiz was "What is the formula for finding the sum of the measure of the interior angles of a polygon?" Some of his assessments did challenge students to apply concepts in different contexts. While studying sequences, for example, he extended the exercise by asking students to find the next term not only in number sequences but also in figure sequences (14002). He also had the students use compasses to make constructions. Teacher 14 did, however, allow students to apply their knowledge of geometry to a couple of real-world projects. One project was to write a children's book that incorporates the 5 Platonic solids. This project is done in conjunction with a first grader and is given to the first grader upon completion. Another project planned by Teacher 14 is designed to help students discover the geometry around them and requires students to find 25 different examples of geometric figures, identify them and display them.

H.13 Teacher 15
Algebra 2

Teaching Philosophy

"During the past three years I have implemented two new teaching aids into my classroom. I now use cooperative learning to assist in learning some difficult concepts. I also use the TI-82 as an aid to visualization of complex graphs in my classroom.

During the past year, I have also used the idea of open-ended questions to assist in evaluating the understanding of my trigonometry students."

Activities and Professional Development

N/A

Classroom Activities

N/A

H.14 Teacher 16
Algebra 2 Honors

Teaching Philosophy

"My classroom used to be a very traditional classroom; teacher lecturing followed by student work.

After my involvement in the UCSD Math Institute my classroom has changed.

We do a problem of the week at almost all levels. They are seated in groups instead of rows. They do projects and group presentations, including a game show project as last semester's Pre-Calculus final. I have taught the UC Davis change from within and am currently in my second year teaching the Innovative Math Project. These two new courses have necessitated a drastic change in my classroom."

Activities and Professional Development

Teacher 16 is enrolled in a masters program in educational technology and has attended the UCSD Math Institute and the Discrete Math Institute. Her leadership roles at her school include Math Department chair, chairperson of a focus group for WASC evaluation, and site coordinator for Innovative Math Project (IMP). Through her involvement in IMP she has been instrumental in guiding her department towards the California Mathematics Framework and the NCTM Standards.

Classroom Activities

Teacher 16 submitted only six artifacts for evaluation. Although it is difficult to assess the major concepts that she has covered in class during these past few weeks, it is obvious that she likes to challenge students to solve problems creatively. She has a challenging problem that she gives weekly to her Algebra 2 class which requires students to use geometric skills, past knowledge, and recognition of patterns. All of her tests are teacher-generated and usually involve real-world questions. Artifact 16006, for example, asks “A tumor 4.87 cm below a patient’s skin is to be treated with a beam of gamma rays. Because the tumor is underneath a vital organ, the radiologist moves the gamma machine over 7.5 cm. At what angle to the patient’s skin must the radiologist aim the gamma machine to reach the tumor?”

H.15 Teacher 18
Math Analysis (Pre calculus)

Teaching Philosophies

“As I began teaching mathematics, many of my students did not have the enthusiasm or the ‘spark’ to learn mathematics. My students would always ask, ‘Why should I learn this?’ or ‘When am I ever going to use this in my everyday life?’ This inspired me to have students at all math levels write a math term paper on a subject that they liked and show how mathematics is related to that subject. These term papers changed the attitudes of many students and showed that there is a purpose in learning mathematics. My students also give an oral presentation of their term papers. These projects demonstrated that a student needs to develop writing and speaking skills for a mathematics course or career.

Cooperative learning, problem solving, open-ended questions, calculators, hands-on manipulatives, computers, writing skills, and speaking skills should be developed in all math courses regardless of the paths the students take through high school.”

Activities and Professional Development

Teacher 18 has continued her education by taking computer classes and has participated in several mathematics education activities including the San Diego Math Assessment Institute, San Diego Math Project, San Diego City Schools Mentor Program, and San Diego City Schools Gate Program. She is very active within her school and has assumed positions as Math Mentor Teacher, California Scholarship Federation Advisor, Math Team Advisor, Pep Club Advisor, Future Educator of America Advisor. She is also a member of the Principal Advisory Committee, Accreditation Steering Committee, and Race/Human Relations Committee. Among the new methods of teaching that she is helping to implement into the Honors Geometry Program at her school are term papers, oral presentations, cooperative learning, and partner testing.

Classroom Activities

Most of Teacher 18’s class activities were drills that were designed for review or practice of a particular concept. During the past few weeks, she has covered three major concepts within her classroom: logarithms, permutations and combinations, and conic sections. The methods used for teaching logarithms and logarithmic equations did not deviate much from the traditional method of paper/pencil computation. The teaching materials used for this section were all district-, school-, or department-generated, and all tested a student’s ability to manipulate equations and derive the correct answer. There was no indication of the teacher trying to relate logarithms to real-world problems (such as earthquakes) or any indication that students knew what logarithms were.

Students were asked, however, to graph exponential functions so that they could form a relationship between a function and its graph.

The probability unit incorporated more of the NCTM standards. During this unit, assignments were given to groups and even one of the quizzes was a partner quiz (18006). The questions that students were asked in this section were much more relevant to their everyday life activities. For example, one quiz question posed to Teacher 18's Advanced Math 7-8 class asked, "There are 38 students in an Advanced Math 7-8 class. How many committees of 4 can be chosen." Although every assignment and quiz tested memorization and evaluation of mathematical formulas (for example, "Evaluate 8p4" [18005]), each assignment also tested the ability of the student to apply these definitions in a nonroutine setting.

During the conic sections unit, the teacher emphasized the connection between the function and the graph that the function represents. Again, the exercises seemed to be routine but the teacher at least allowed students to work in groups.

Teacher 18 did incorporate problems of the week in order to elevate her students' enthusiasm for math and encouraged creative solutions to problems.

Appendix I

Transcript of MDTP Electronic Conference

I.1

Addressed Topics: moderator, assessment, content/procedures, sequencing, application
Key Words: diagnostic testing, content knowledge, procedural knowledge, curriculum

Subject: E-Conference/ Genesis/ AJC
Date: 7/6/95

Dear colleagues:

Thank you for agreeing to participate in the UCLA Center for the Study of Evaluation's panel discussion about mathematics education reform. The individuals who will be participating in this electronic conference are Geoff Akst, Linda Boyd, Gail Burrill, Margaret DeArmond, Walter Denham, Marjorie Enneking, John Harvey, Alfred Manaster, Jack Price, Anita Solow, Elizabeth Teles, Alba Thompson, Zalman Usiskin, and Norman Webb. Based on the availability of these panel members, we have scheduled the conference to start today, July 6, and to close toward the end of July.

As you know from the initial invitation, we will focus on four questions:

a. What kinds and forms of assessment should be included in the reform efforts and in the intended curricula? In particular, what roles should diagnostic assessment play in these efforts?

b. What are appropriate roles for mathematical content knowledge and procedural techniques in the design of environments that facilitate learning mathematics?

c. How important is the sequencing of instruction for learning mathematics in an effective curriculum? In particular, is student familiarity and comfort with some topics and procedures a necessary prerequisite for developing rich understandings and mastery of other topics? If so, please give examples.

d. What are appropriate roles for applications of mathematics in an effective mathematical learning environment?

For the conduction of the electronic conference, in order to discuss the above questions, we have set up an e-mail address (MathDiag@CSE.UCLA.edu) that will forward messages to all members of the panel of which you are a member. This is the way that you received this e-mail about the e-conference. In sending messages to this e-mail address we ask that you follow some basic procedures:

1. Compose the subject heading to include one of the four discussion questions/ short subject topic/ and your initials so that question (a) has the subject heading of "Assess," question (b) "Content," (c) "Sequence," and (d) "Apply," and the short subject topic and your initials are separated by slashes. As an example, if a message was to be sent regarding question (b) and I was interested in Alfred's expanding on the use of calculators within a classroom, I could design the subject to read "Content/Alfred: calculators-classroom/AJC." The "Content" stands for the topic of the discussion question (b), "Alfred: calculators-classroom" is the creation of the composer of the message to let others know about the general topic of the message, and "AJC" stands for my initials—Alexander J. Chizhik. Please try to limit the length of the middle portion of the subject heading.

2. If possible, try to limit each message to address a single topic. In addition, the length of the messages should be kept reasonably short so that everyone will have enough time to read all messages and be fully integrated within the panel discussion.

3. Throughout the e-conference it is important to check your e-mail often as to keep abreast about the evolution of the discussions about each of the four topics. We recommend that you check your e-mail and participate in the e-conference at least once a day by reading the messages and replying to any of the messages regarding which you have information that you would like to contribute. In addition, it is important that any questions directed specifically to you be answered as soon as possible with an understanding that some questions may take some time to answer because they may require checking references.

4. Please save the messages that you receive through the e-conference for reference throughout the discussion (especially this particular message). We do ask, however, that you do not disperse the contents of the e-conference without first asking for permission from the originators of the messages.

As the preliminary step in our e-conference that will begin today, July 6, we would like each of you to send a short description of your background and interests as they relate to any of the four discussion questions. Use the subject heading of "Interests/your initials" in sending this e-mail to "MathDiag@CSE.UCLA.edu." This will let all the participants know about each other and should make our discussion run better since everyone will be somewhat acquainted with each other. In addition, you may want to include the name that you would like to be called in personal communication within this message.

After sending biographical information about yourself, we ask each panel member to provide a position paper addressing any or all of the four questions that this electronic conference is hoping to shed light upon. Please send up to four messages (one each for any of the four discussion topics you choose to address) to "MathDiag@CSE.UCLA.edu" and include a subject heading as described in the procedures listed above. Please try to e-mail your opening statements between today, July 6, and July 14 after which date we will begin the second step of this electronic conference.

Once all the initial statements have been received, we will start the less structured second step in which each of the panel members is free to expand on or respond to any issues raised during the opening statements made by any of the panel members. This phase continues as a normal discussion among the panel members regarding all four subjects that will be brought to light by the panel members' diverse backgrounds. This phase should continue for at least one week until the discussion has been deemed to have brought forth enough issues that surround the four discussion questions of this e-conference.

During the third and final step you will have an opportunity to provide a closing statement regarding any of the four discussion questions. This will be the time to sum up important issues surrounding the topics of the e-conference. For those individuals who want to continue discussing mathematical reform, the "MathDiag@CSE.UCLA.edu" address will continue to be active and disseminate messages to the panel members beyond the time of the conference. Those individuals who no longer want to receive messages via "MathDiag@CSE.UCLA.edu" will need to contact me at that time. In addition, if anyone new would want to join the discussion, they would also need to contact me.

As we indicated before, after the discussion is concluded, a transcript and summary, prepared by the moderators, will be prepared and presented to the participants and to other interested parties. If further dissemination is deemed worthwhile, each panelist will be given the opportunity to review and comment upon any materials that might be submitted for broader publication or dissemination.

Again, any questions about the content or purpose of the e-conference may be sent to John Novak at the UCLA Center for the Study of Evaluation [john@cse.ucla.edu, (310)206-1532], Phil Curtis at the UCLA Mathematics Department [pcc@math.ucla.edu, (310)506-6901], and Alfred Manaster at the UCSD Mathematics Department [amanaster@ucsd.edu, (619)534-2644].

If there are any technical questions throughout the conference please let me know, Alexander Chizhik <Alex@CSE.UCLA.edu>. Thank you again for your participation in this worthwhile discussion. I am looking forward to working with you.

Sincerely,
Alexander Chizhik

I.2

Addressed Topics: interests, assessment, content/procedures
Key Words: technology, NCTM, diagnostic testing

Subject: Interests/ABM
Date: 7/10/95

Alfred Manaster:

Dear Colleagues:

This is in response to Alex's request for a short description of my background and interests as related to the four discussion questions of this conference. In 1965 I received a Ph. D. in Mathematics from Cornell University. My research area was mathematical logic. Since 1967 I have been a member of the Mathematics Department at the University of California, San Diego. In 1980 I became a member of the California Mathematics Diagnostic Testing Project. I have continued working with that project, serving as its administrative coordinator since about 1986. Finally, I am one of the founding faculty members of a doctoral program that is offered jointly by UCSD and San Diego State University in Mathematics and Science Education.

Throughout the thirty years that I have been teaching undergraduate mathematics, there has been a steady increase in my interest about what students are learning, what we want them to learn, and how they can learn more effectively. My affiliation with the Mathematics Diagnostic Testing Project has naturally led to a more focused interest in assessment in general and desired mathematical preparation of entering college students in particular. Working in these areas during the development and first steps of implementation of the 1989 NCTM Curriculum and Evaluation Standards has heightened some questions and raised others. Another factor affecting the ways in which we can help students learn is the pervasive impact of ever-changing technology. It is clear that this is an exciting time to have concerns about student learning of mathematics. It is also a challenging one since efforts to move in new directions always require careful thought and re-examination of old practices to make sure that changes have minimal negative, though unintended, effects.

The situation just described is intended to provide some of the setting for my interest in the topics of this conference. Since it appears to me that all four of the focus questions fit into that context, and since Alex suggested that these remarks be brief, let me stop here. I, too, am looking forward to an interesting and informative discussion. I hope that the questions are provocative and that they will lead to thoughtful deliberations.

Cordially,

Alfred

I.3

Addressed Topics: interests, assessment, content/procedures
Key Words: calculators, curriculum, problem solving

Subject: Interests/WD
Date: 7/11/95

Walter Denham:

I'm Walter Denham. I've been primarily responsible for mathematics education strategies and staff development work at the California Department of Education since 1983. As you might assume, I am an advocate of the California Mathematics Framework. I believe, for example, that the great majority of classroom time should be used for students doing mathematics in context, with calculators always available. The emphasis should be on getting mathematical work done, including problem formulation and interpretation and communication of results.
It's harder to make a short statement about testing. One view I subscribe to about as strongly as anyone is that the form as well as the substance of tests that matters has a very heavy (no longer incredible) influence on instruction. Multiple choice testing seriously inhibits problem solving in the curriculum.

I also despair about the continued dominance of norm-referenced testing, but I don't believe that needs to play a large role in this forum

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I.4

Addressed Topics: assessment
Key Words: diagnostic testing, performance tasks

Subject: Assess/Statement/WFD
Date: 7/11/95

Walter Denham:

Performance tasks should receive the main attention. Some might be ten or twenty minute tasks, some an hour or two, and a few should be over a longer period of time. This priority influences instruction in the right way, and, equally important, indicates to students what it means to do mathematical work, to pursue and achieve a purpose in a situation amenable to mathematical formulation.

Ideally, classroom teachers would be the primary assessors of students' work, so the time needed for "testing" would not be an issue. We do not have the ideal, of course, nor are we about to anytime soon. Teachers' assessments, to understate, are not uniformly valid, and as long as the public takes a competitive view of measuring achievement, teachers' assessments are not trusted.

But this forum exists because of a particular set of tests that use the label "diagnostic." I believe those tests are satisfactory for sending a generalized warning to a student. The questions on the "Algebra readiness" test, for example, are not unduly difficult, and a student scoring well below 80% is, indeed, in poor shape to begin high school (or even seventh grade, for that matter). I disapprove of the adjective "diagnostic," however. To be diagnostic, an assessment would reveal the nature of the misunderstanding, i.e., just how does the student have the mathematics screwed up? Or, in what reasoning patterns does the student use consistently well? How did the context (or lack of context) influence what the student did with a problem?

As a generalization, I'd say that teachers should be responsible for diagnosing. Outside agencies can administer "predictive" tests, or potentially even performance tasks (although political forces are undermining any governmental role in performance assessment, at least in California).
To make a narrower point, I strongly object to any test being broken into "topic" or "topic area" components. In high school especially the teaching of topics continues to impede reform. A broader related point is that I strongly believe that secondary mathematics courses should not have strand names, most especially there should not be a course called Algebra 1. I am particularly interested in the (perhaps unintended) effect that a given test may have on instruction

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I.5

Addressed Topics: sequencing
Key Words: procedural knowledge, curriculum

Subject: Sequence/Statement/WFD
Date: 7/11/95

Walter Denham:

I support the Framework's discussion of the best way to think of what instruction in mathematics is "about." Curriculum should consist of coherent units. The Framework first discusses the possibility of designing curriculum in terms of content "strands," and by implication topics within strands. It then considers focusing on "unifying mathematical ideas." But neither of these is judged a good basis for curriculum design.

To put it very briefly, a curriculum of coherent units is quite unlike a curriculum of topics. Concepts and skills, rather than being "covered," are imbedded, to varying degrees in any grade level and to varying degrees according to the particular curriculum chosen. The big ideas must recur within the curriculum, and it is the responsibility of the curriculum designers to bring it off. But this is altogether different from a sequence of topics.

Skills/procedures is a bit harder. I subscribe to spending very little time specifically on skill development, although I acknowledge the great frustration for teachers at any grade above four from the insufficient skills of some students. Still, the time that is taken out for skill practice should be recognized as just that: time taken away from doing mathematical work. It's the carpenter practicing with his/her electric (not manual) saw, etc.
There are only a few truly essential ideas in school mathematics, and they are not ever "mastered." They should be encountered every year, in gradually increasing depth and, perhaps, complexity.
I see curriculum as an evolving web, not as a sequence of topics.

I.6

Addressed Topics: interests, content/procedures
Key Words: technology, calculators, calculus

Subject: Interests/AES
Date: 7/12/95

Anita Solow:

Dear Colleagues:

Here is my background information. I received my Ph.D. in mathematics from Dartmouth College in 1978. Since 1980, I have been at Grinnell College, a small liberal arts college in Iowa. I am currently Professor of Mathematics and Chair of the Mathematics/Computer Science Department.

Since 1986, I have been involved with the Advanced Placement Calculus Program. I am currently chair of the AP Calculus Committee. This group is responsible for writing the examinations and course descriptions and for making recommendations about technology requirements for the examinations. In 1995, graphing calculators were required for the first time on the AP Calculus examinations. We are currently rewriting the course description from a sterile list of topics to a descriptive document emphasizing the concepts of calculus.

I have also spent several years working with calculus reform efforts at the college level. I am the editor of "Learning by Discovery: A Lab Manual for Calculus," and "Preparing for a New Calculus," both published by the MAA.

Anita Solow

<u>I.7</u>

Addressed Topics: application
Key Words: technology, algorithms, NCTM, procedural knowledge, curriculum, problem solving, arithmetic, algebra, calculus

Subject: Apply/Statement/AES
Date: 7/12/95

Anita Solow:

Role of applications in an effective learning environment:

One question that I have been pondering is "What is mathematical literacy?" This question will probably have different answers for different levels of education, but I think it is a basic question that needs to be dealt with.

For example, the definition of computer literacy has changed rapidly over the past 10 or 15 years. Computer literacy used to mean the ability to program a computer. It now means the ability to use software packages. It is not clear that we would consider it important to program at all for this basic definition of computer literacy. Note that computer literacy and competence in computer science are quite distinct, and have, in fact, moved further apart.

The definition of mathematics literacy is also undergoing a change. Some of the impetus for this change has to do with the NCTM Standards. But a large push is coming from technology. Mathematics literacy no longer means the ability to do mathematical algorithms by hand: arithmetic, algebraic manipulation, etc. What the NCTM Standards articulated was that mathematics is about ideas, not just procedures; one of the key ideas of mathematics is problem solving. Since technology is available that performs the algorithms and procedures of mathematics, one must question what mathematical content is important.

If problem solving really is to become the focus of much of mathematical study, then the student needs to have problems to solve, and this is where applications come into the curriculum. The particular application is often not very important. Rather, it is the experience of applying mathematics in a meaningful environment that is important. This means that applications need to be approached at least some of the time from a modeling perspective, where the student will have to spend some time figuring out what the problem is asking and what mathematics may be appropriate to bring to the situation. This type of applied experience takes time and often does not fit neatly into one class period. Technology enables students to focus on problem solving by performing the calculations for the student.

I will give an example from calculus, since this is where I devote most of my energy. The important outcomes of calculus are for the student to understand the derivative, the integral, and be able to apply the derivative and the integral in a variety of situations. Since available technology can do many of the manipulations traditionally taught in calculus, it is not good enough for students to demonstrate that they can compute derivatives and integrals. They need to demonstrate that they can apply the IDEAS of calculus to situations that they have not seen before. There are many ways to help students attain this ability, but all involve applications. For example, one may teach a course that is entirely problem solving based (Project CALC), one may have students do several applied projects throughout the semester (Resources for Calculus), or one may use a book that has a constant supply of applied situations as the basis for problems (Harvard Calculus).

I.8

Addressed Topics: interests, content/procedures
Key Words: NCTM, curriculum, abstraction

Subject: interests/JP
Date: 7/12/95

Jack Price:

I have been through various reform movements since I began teaching in 1952, beginning with the NDEA and continuing through SSI and USI. My major interests have been in curriculum and instruction particularly for those students who have been classified as “at risk.” Through the years I have come to believe that every student can learn mathematics, not a dumbed-down mathematics, if it is taught in a manner consistent with the child’s learning style. The ideal to me is the mathematics of the SMSG/UICSM projects wedded to the teaching of the Madison Project. Children are capable of doing much more than we give them credit for. They can generalize and participate in abstract thinking but we need to have a classroom environment that fosters this thinking.

At present I am completing a circle, teaching in the Center for Education and Equity in Mathematics, Science and Technology (CEEMaST), in the College of Science at Cal Poly Pomona, after teaching elementary-high school, being a supervisor and other district level administrators. My term as president of NCTM ends April, 1995. I am looking forward to the comments and hope I can keep up with them both in thought and time! Jack Price

Subject: interests 2/JP

It has been brought to my attention that my term ends April, 1996. Oh well, it was a good try! Guess I’m there for another nine months. Jack

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I.9

Addressed Topics: interests, assessment, content/procedures, application
Key Words: technology, calculators, NCTM, curriculum, arithmetic, algebra, geometry

Subject: Interests/ZU
Date: 7/13/95

Zal Usiskin:

Dear Colleagues:

I am Zalman Usiskin (Zal), a professor of education at the University of Chicago and director of the University of Chicago School Mathematics Project.

I am interested in all aspects of K-12 mathematics education, but particularly on the questions "What mathematics (and what about mathematics) should students learn?" "How is that best organized to optimize learning?" I am also intensely interested in society's views towards mathematics, and the (often distorted or unwise, in my opinion) ways in which mathematics is conceptualized, not only in the media (where we can naturally expect such things) but even by professionals.

A quick rundown on background: 1963: bachelor's degrees (one in mathematics, one in education) are from the University of Illinois (I was there in the heyday of UICSM). Same year: first publication in a mathematics (not math ed) journal (on probabilities in the voting paradox). 1964: MAT from Harvard; 1964-66 taught high school mathematics full-time, then for the next nine years taught part-time at least one class in grades 8-11. 1969: Ph.D. from the University of Michigan. My work there, mainly with Art Coxford, dealt with the development of geometry for average students through transformations (reflections, rotations, size changes, etc.).

In 1969 I became a faculty member at Chicago and continued my work with transformations, matrices, and groups in the study of second-year high school algebra. At the same time, I became converted to the notion that applications are not just a nice adjunct to a mathematics classroom but an important aspect of mathematics without which a student's mathematics education is incredibly lacking. From 1973-76, I directed an NSF-supported project entitled "First-Year Algebra Through Applications Development Project" in which the goal was to see if the content of elementary algebra could be developed for average students through applications. (A one-sentence answer: Yes, but not all of first-year algebra content was appropriate for such an approach; e.g., we could find no application for factoring the general trinomial.) In 1978-79 I became interested in testing the van Hiele theory, and Sharon Senk and I wrote a geometry course (never published) developed by going up the van Hiele levels.

This work led to two projects from 1979-82. The first project was with Max Bell dealing with Applications of Arithmetic, an attempt to identify the applications of arithmetic for teachers. Realizing that calculators took away the backbone of school arithmetic, one goal was to see whether applications could be organized and sequenced to provide an alternate backbone. Another goal was to take applications out of the fuzzy realm in which many people think there are so many applications that they cannot possibly all be learned into an environment in which there are basic important applications which can be identified and from which other applications follow quite nicely. The second project was with Sharon Senk and others and dealt with the testing of the van Hiele level theory on a nationwide scale; it included the first nationwide assessment of geometry proof competence that we know of.

In 1983 the University of Chicago School Mathematics Project began. All of this earlier work was then applied to create a curriculum for the vast majority of students in grades K-12 that would increase levels of performance, decrease the stultifying amount of review that occurs in many K-8 classrooms, employ the latest technology, have applications everywhere (carefully sequenced), not have artificial barriers between branches of mathematics, work hard to make students independent learners, and so on. For eight years, the Secondary Component, which I directed, worked hard on the 7-12 curriculum, which involves six student books and a host of ancillary materials (activities, technology, tests, etc.). We know that our early success was one of the factors in convincing NSF to put money into other curriculum projects. We also realize that we had a reasonable amount of influence on the NCTM Standards.

The success of UCSMP has been beyond anyone's expectations; we estimate that over 2 million students have been using the 7-12 materials each year for the past couple of years. Curiously (or perhaps we should expect this), with the success comes the view of some that we didn't go far enough, but in fact virtually all the school districts that move to UCSMP materials are worried that they have gone too far.

Since 1992 we have been working on the four books that are in second edition. They are characterized by advances in technology (more sophisticated technology recommended for all courses, required for some), by the inclusion of projects in all books (they were only in the last two first edition books) and by significant attempts to make the learning more active, as well as a broader array of assessment materials.

Zal Usiskin

<u>I.10</u>

Addressed Topics: sequencing
Key Words: algorithms, content knowledge, curriculum, problem solving, arithmetic, algebra, geometry, calculus

Subject: Sequence/ZU
Date: 7/13/95

Zal Usiskin:

Sequencing is very important, but there are a large number of possible sequences. Following are a number of ways to sequence:

1. the logical. This is traditional in geometry and in higher mathematics. It starts with postulates or tacit mathematical assumptions and is motivated by the question "What can we prove about X from what we have?"

2. the historical. This is sometimes called the genetic approach. It is (was) used in traditional school arithmetic. E.g., decimals and negative numbers come well after fractions. All algebra comes after arithmetic. The historical sequence can be motivated by the question "What happened next?" but it only works if the actual history is discussed.

3. the utilitarian. We begin with concepts, ask how they are used, and organize around the uses. Consumer mathematics courses have often been organized with this in mind. The natural question is: How could mathematical idea X be used?

4. the problem-oriented. One begins with problems considered important and lets the content develop as an outgrowth of attacking the problems. Some Polya-style courses have been organized in this way. The natural questions are: How can we solve this? What else can we solve?

5. the algorithmic. We begin with simple algorithms and progress through more and more complex algorithms. Arithmetic, algebra, and calculus have been traditionally taught this way. The natural question is: What new things can we now do?

6. the psychological. We take a psychological theory (e.g., Piaget or van Hiele) and sequence according to that theory. As far as I can tell, there is no natural question that moves the curriculum forward when one has this sequence.

I would like to argue that the more structural frameworks operating in a given course or on a given day, the more likely one is to increase the appeal of the subject matter and the more likely one is to obtain substantive learning.

I believe very strongly in the advice of William Brownell, who by many is considered the first great researcher in mathematics education. Brownell is known for advocating "meaningful learning" over "drill" and for doing research to back up his advocacy. But he had even more distaste for "incidental learning," in which mathematics was thought to be learned through natural behavior if one merely set up a rich environment. (See his article opening the 1935 Yearbook of NCTM.)

All of this is to reiterate that I am a great believer in both structure and sequence in curriculum, and a believer that there are many possible structures and many possible sequences as well as many combinations of them.

Zal Usiskin

I.11

Addressed Topics: content/procedures, application
Key Words: technology, algorithms, content knowledge, procedural knowledge, representations

Subject: Content/ZU and Apply/ZU
Date: 7/13/95

Zal Usiskin:

As my background may have suggested, I do not see why or how questions (b) and (d) can be separated. Applications of mathematics are as much a part of mathematics as "mathematical content knowledge and procedural techniques." For instance, a student who knows slope as (y1-y2)/(x1-x2) but does not have some real instances of rate of change does not have a full knowledge of the mathematics "content"; these ideas must be connected (at least) for that. Conversely, a student who has some sense of chance but does not connect that with fractions is also sorely lacking.

In UCSMP, we have a conceptualization of understanding we call SPUR (Skills, Properties, Uses, and Representations). Skills ranges from the rote application of algorithms (mentally, by paper-and-pencil or using technology) to the invention of new algorithms. Properties ranges from names for general principles to the doing of proofs. Uses ranges from simple, one-step applications to the development of new models. Representations includes concrete materials, graphs, visual mnemonics, etc. One of the major reasons for this conceptualization was to ensure that when we examined a bit of mathematics subject matter, we would involve at least these four dimensions of understanding. At times we add a fifth dimension: Culture.

Zal Usiskin

I.12

Addressed Topics: moderator, assessment, content/procedures, sequencing, application
Key Words: diagnostic testing, content knowledge, procedural knowledge

Subject: E-Conf/ Phase 2/ AJC

Date: 7/14/95

Dear colleagues:

Thank you all for wonderful opening statements that have illuminated a lot of issues in mathematics reform. It seems like some panel members have not sent their opening statements as expected by today. These statements are necessary for providing the foundation for a provocative discussion. For this reason we are elongating the duration of the first phase to include this week-end in addition to overlapping with the second phase of the electronic conference on Monday, July 17th. That day will end the first phase of the electronic conference that was in the form of position statements and begin the second phase. The job of the panel will then reside in digesting the information that has been put forth by the means of discussing those issues.

As we indicated at the start of the electronic conference, this will be a less structured phase in which each of the panel members is free to expand on or respond to any issues raised during the opening statements made by any of the panel members. This phase will continue as a normal discussion regarding all four subjects that have been brought to light during phase one. Again, the discussion phase will start Monday, July 17th, and continue through next Sunday, July 23rd, by which time the discussion should bring forth enough issues that surround the four discussion questions of this e-conference. The final phase of this electronic conference is slated for the week of July 24. During this step you will have an opportunity to provide a closing statement regarding any of the four discussion questions. This will be the time to sum up important issues surrounding the topics of the e-conference.

For those who have been held up somewhat in sending opening statements—you can continue to send them to MathDiag@CSE.UCLA.edu throughout the next few days. It is imperative that the other panel members receive these statements as soon as possible in order to have a more meaningful discussion about mathematics reform that spans the diverse backgrounds of all the knowledgeable individuals who are participating in this electronic conference.

Here again are the procedures for the conduction of the electronic conference:

1) Compose the subject heading to include one of the four discussion questions/ short subject topic/ and your initials so that question (a) has the subject heading of "Assess," (b) "Content," (c) "Sequence," and (d) "Apply," and the short subject topic and your initials are separated by slashes. As an example, if a message was to be sent regarding question (b) and I was interested in Alfred's expanding on the use of calculators within a classroom, I could design the subject to read "Content/Alfred: calculators-classroom/AJC." The "Content" stands for the topic of the discussion question (b), "Alfred: calculators-classroom" is the creation of the composer of the message to let others know about the general topic of the message, and "AJC"

stands for my initials—Alexander J. Chizhik. Please try to limit the length of the middle portion of the subject heading.

2) If possible, try to limit each message to address a single topic. In addition, the length of the messages should be kept reasonably short so that everyone will have enough time to read all messages and be fully integrated within the panel discussion.

3) Throughout the e-conference it is important to check your e-mail often as to keep abreast about the evolution of the discussions about each of the four topics. We recommend that you check your e-mail and participate in the e-conference at least once a day by reading the messages and replying to any of the messages regarding which you have information that you would like to contribute. In addition, it is important that any questions directed specifically to you be answered as soon as possible with an understanding that some questions may take some time to answer because they may require checking references.

4) Please save all the messages that you receive through the e-conference for reference throughout the discussion. We do continue ask, however, that you do not disperse the contents of the e-conference without first asking for permission from the originators of the messages.

If there are any questions, they can be e-mailed directly to me at Alex@CSE.UCLA.edu.

Thank you for your continued participation,
Alex

I.13

Addressed Topics: interests, assessment, content/procedures, sequencing, application
Key Words: procedural knowledge, performance tasks, geometry

Subject: opening remarks nlw
Date: 7/15/95

Norman Webb:

I have appreciated glancing at the remarks of others. You have picked as the opening discussion as the one week in July I am away from the e-mail. Here are some of my initial thoughts on the topics:
Background—I have been working in the area of assessment for 12 years and was directing a project developing performance assessment instruments for the state of Wisconsin until July 1 when the Wisconsin legislature deleted from the state budget

any funding for the development or administration of performance assessment over the next two years. Sound familiar?

Assessment: Assessment is critical to reform. Assessment has to match (be aligned with) reform. The performance assessment we were developing was in the right direction. Our knowledge of assessment is very little other than traditional norm-referenced test. We need more information about assessment.

Procedural Knowledge and Content Knowledge: Procedures are essential to mathematics. I cannot envision mathematics being taught or a learning environment without giving some consideration to developing procedural knowledge. I believe that instructional demands are very much related to content. Instruction needs to be content specific.

Sequence: Sequence for some things are critical, for others not so critical. I just recently interviewed a high school geometry teacher who said that her students learned the material on congruence in a weeks time what took previous students four weeks because she moved work on similarity from the middle of second semester to first semester before work on congruence.

Applications: Applications should be included if they have utility for the students, have or pedagogical value but not at the expense of good mathematical thinking, some of which is done without any application other than relation to mathematics.

Norman Webb

<u>I.14</u>

Addressed Topics: interests, assessment
Key Words: technology, diagnostic testing, calculus

Subject: Interests/GA
Date: 7/16/95

Geoff Akst:

Hi to old friends and new friends.

My name is Geoff Akst. I teach mathematics at Borough of Manhattan Community College—one of the two-year colleges in the City University of New York.

My work background sounds extremely BORING, but it's not: I've been at BMCC more than 25 years, teaching the usual array of lower division math courses: remedial, statistics, logic, probability, calculus, nursing math, etc. However some years ago when everyone else took one step backward, I took over the reins of my college's developmental math program, and that has been a responsibility and an interest ever since.

Here's where I studied: Columbia AB (math); NYU/Courant Institute (math); Columbia Teachers College (teaching of college math).

So there's a contrast between my academic background (very pure) and my work life (should I say applied or sullied?). No wonder I go around muttering to myself.

Assessment (dare I say testing?) is important to me for many reasons:

* For each Monday morning, I need to make up tests in my own classes.
* I realize that probably more than anything else, testing tells our students (and us if we keep our eyes open) what in our courses we think is important.
* I took the lead in making up my departmental quizzes in our developmental program.
* I chair the math task force of my university's Freshman Skills Assessment Program, which produces math (and language) tests that students must pass to get their baccalaureates; this task force is generally conservative on the technology issue.
* I serve on the Committee on Testing, an MAA group which is mainly concerned with developing college placement tests and which is progressive on the technology issue.
* I've designed and programmed on-line math tests, and am especially interested in the feasibility of using computers to diagnose students' strengths and weaknesses. (Will we agree to use the word diagnose in our discussions in this restricted sense, I hope?)

Well, what else to say. This is my first e-mail conference, and I'm suspicious that some dean dreamed it up as a plot to withdraw travel funds from us underpaid instructors!

I.15

Addressed Topics: assessment
Key Words: content knowledge, procedural knowledge, curriculum, problem solving

Subject: Assess/Statement/ABM
Date: 7/16/95

Alfred Manaster:

It seems to me that any effective mathematics curriculum requires a variety of assessments. In the first place, different forms of assessment are needed to answer different questions. There are also different needs for results of assessments for individual students, for their parents, for teachers, for schools, for districts, and for the nation. In this statement, I want to focus on students and teachers. Even with this restricted focus, it seems clear that a variety of assessments are essential, ranging from ongoing assessment that occurs during every class—whether documented or not—to classroom tests and final examinations largely developed by teachers.

Since we want students to have a variety of learning experiences in mathematics, that variety needs to be represented in the assessments given to students. Among others, students need to develop procedural skills, conceptual understandings, ways of approaching new ideas and problems, effective tools for communicating their solutions to problems, and abilities to recognize and apply mathematics to situations beyond the mathematics classroom. Some of these traits are most effectively and efficiently assessed using tests developed outside the classroom while others require the creative talent of the classroom teacher for both development and scoring.

In particular, the best, if not the only, way to test student creativity and problem solving in new contexts is through student responses to tasks set by the classroom teacher. I think this is a consequence of the teacher's unique knowledge of exactly what the student has been taught. While other mathematics educators can create interesting problems, which can be valuable for giving a sense of what is expected to teachers and others, in the final analysis only the teacher can understand how large a leap is required for students to respond to a problem. The point here is that even the most creative problem can be reduced to an almost robotic task with adequate (but often inappropriate) training. If it is known that students will have to answer a particular type of question, then it is almost certainly possible to give them enough practice to be able to solve the problem in a fairly mechanical way.

On the other hand, ensuring that students have mastered required computational and procedural skills can often be done very effectively with tests that are developed outside the classroom. Such tests can also measure, to some extent, students' conceptual understanding. It is possible to provide teachers with indications of both individual students' strengths and weaknesses and summaries of the class's performance on specific topics, skills, and concepts. Such externally developed tests can save the teacher's time that otherwise might be needed to prepare, score, and summarize the results of such tests. In addition, it seems to me that it is possible and helpful to use this as one form of testing to see how completely students have incorporated certain procedural techniques and conceptual understandings into their mathematical habits of mind.

I hope it is clear that this relatively brief statement is not intended to be comprehensive. Since both old and new curricula call both for mathematical thinking and problem solving and for development of the techniques and understandings that form the basis of those activities, it seems to me that both forms of assessment described here are critical components of any mathematics education program. Partly because of lack of space and time, and partly because of lack of experience, I have not mentioned other equally important forms of assessment that are needed to measure student proficiencies in communicating mathematics, in working on long-term problems, and in other desired aspects of learning and doing mathematics.

Alfred Manaster

I.16

Addressed Topics: content/procedures
Key Words: calculators, algorithms, NCTM, content knowledge, procedural knowledge, curriculum, motivation, problem solving, abstraction

Subject: Content/Statement/ABM
Date: 7/16/95

Alfred Manaster:

It seems to me that a fairly robust understanding of mathematical concepts and facility with a variety of mathematical procedures are prerequisites to any substantial mathematical problem solving. This only reinforces my position that student learning of mathematical concepts and of procedures have to play central roles in any curriculum intended to help students learn mathematics. If the goal is to teach students mathematics, then we first need to agree about what mathematics is to be learned. Of course, as the 1989 NCTM Curriculum Standards emphasized, mathematics is much more than concepts and procedures. Even so, it still seems to me that these are the defining aspects of the subject, that is, what makes the intellectual activity mathematics and distinguishes it from other subjects that also require thinking, communicating, solving problems, applying existing knowledge to new situations, etc.

A particularly challenging question to me is the extent to which students must master computational skills that computers and calculators can now execute faster and more accurately. How significant is the process of learning specific algorithms that always produce correct answers in the development of a person's sense of some aspects of the power of mathematics? It seems to me that the experiences of getting answers that both the student and society know to be correct helps build the student's confidence and under-standing that precision and formal processes can lead to solutions in a strong sense. Indeed, for some of us one of the appeals of mathematics is that its abstract nature allows more complete analyses and more thorough answers than other fields of study or situations. Since there are less factors at work, it is easier to find the "causes" of results. This, in turn, enables mathematics to serve as one paradigm for science and other forms of understanding.

Returning to the "Content" question, while I am arguing that content knowledge and procedural techniques should be central to any mathematics curriculum, they certainly do not provide the entire basis of an effective curriculum. Applications and modeling are mathematical activities. Often they can provide motivation for the mathematics being learned, sometimes reflecting the historical development of the subject. In addition they can provide good contexts to help students strengthen both their understanding of concepts (and their applicability) and their procedural skills (by seeing how to use them, comfortably, to gain insights into other problems without being side-tracked by computational or organizational difficulties that have been solved by the mathematics and its notation). An effective mathematics curriculum must include these approaches, as well as placing a strong emphasis on communicating the reasoning that is done in mathematical work. All that said, it still seems to me that the center of the curriculum needs to be the concepts and processes that classically are regarded as "mathematics."

Alfred Manaster

I.17

Addressed Topics: sequencing
Key Words: algorithms, content knowledge, procedural knowledge, curriculum, problem solving, algebra, geometry, calculus

Subject: Sequencing/Statement/ABM
Date: 7/16/95

Alfred Manaster:

It seems to me that a well-thought-out sequence of topics and courses is particularly important in any mathematics curriculum. Students need to be able to build upon what they already know, simultaneously using that knowledge to solve new problems and develop new understandings while also strengthening their existing knowledge by seeing its applications in new settings. Even more important perhaps is the sense that most mathematics has a strong logical character and is most fully understood through a combination of experiences that include both explanations or justifications of why things work the way they do and examples of the general principles that can be independently verified to give evidence of the truth and usefulness of those principles or rules.

One example that is well known but may be worth discussing is addition of fractions. It is my impression, based upon California data and recollections of other data, that students who are unable to add fractions often have serious difficulties with a first course in algebra, to say nothing of any later courses. Why is this? One fairly obvious reason is that such students are doomed to confront major difficulties when trying to add rational forms. They need to be able to test possible rules with numbers and readily see that some of those "obvious" rules fail. A few experiences like this should help them learn that caution is needed and that the simplest rules are not always the right ones—indeed all of this should also help them see that mathematics is not an arbitrary collection of rules to be learned from "authority" and followed blindly but rather includes powerful techniques that can be seen from examples and verified through reasoning. A more general, but also more debatable, example is the need for algebraic proficiency in order to learn calculus. While this seems obvious in the more traditional courses, where derivations of differentiation rules are given and based upon algebraic manipulations, it continues to be important for at least some of the newer approaches to calculus. One reason is that algebraic manipulations are often helpful in applications of the ideas of calculus to less mathematical settings and in simply carrying out the rules for integrating and differentiating functions as one step in solving many problems. My experience following the text of the Harvard Calculus Consortium fairly closely the past two years was that some students were seriously impeded by the weakness of their algebraic skills. They could not translate word problems into mathematical terms or, if they could do that, they were then stopped from applying the ideas of calculus because they could not use the mathematical terms anyway.

Recent mathematics education research has shown that developing a good conceptual understanding of the function notion is quite difficult for many students. This raises a question of what level of understanding students should have of that concept before taking a calculus course. Since the concept is so difficult, how should the need for that kind of conceptual understanding be balanced against the need for students to develop a comfortable understanding of some fundamental synthetic and analytic geometry and their need for certain algebraic proficiency?

Let me close with a broader question about sequencing. While many students' lack of ability to perform what elementary computations hinders their progress in college, another weakness in student preparation is also important. Many students come to college convinced, as evidenced by their behavior, that mathematics is a subject to be learned as algorithmically as possible. Attempts at understanding are considered a diversion if not a waste of time. I am not sure this is a sequencing issue, but it does seem that a good mathematics curriculum must simultaneously strengthen students' knowledge of mathematical concepts, their skill in executing mathematical procedures, and their awareness of the need for understanding why assertions in mathematics are correct.

Alfred Manaster

I.18

Addressed Topics: application
Key Words: motivation

Subject: Apply/GA
Date: 7/16/95

Geoff Akst:

Math applications are important in a math class and on math tests for many reasons:

They can be highly motivating, and so often serve as a good introduction to a topic. Students may be in career or professional programs, where their goal is clearly applications-oriented. Applications can not only show why a particular math topic is worth learning, but also what the topic means in a concrete way. Some people (not me) take the point of view that the main goal of mathematics is to solve applications.

On the other hand, applications have disadvantages too:

It may take too much time to set the stage for an application. Students may not be interested in the particular area of application. The application may assume general knowledge which the student does not possess. The application may place excessive demands on the students' language abilities—a particular problem with ESL students. The application context may dilute the students' interest in the mathematics of the problem.

As in all things before and since Aristotle, the question of pure vs. applications in a math environment is one of balance.

I.19

Addressed Topics: assessment
Key Words: diagnostic testing

Subject: Assess/GA
Date: 7/16/95

Geoff Akst:

Let's decide what we mean by diagnostic assessment before we discuss it. The stricter interpretation is to assess a student's strengths and weaknesses, but the looser usage includes placement too. What do we mean?

Geoff

I.20

Addressed Topics: content/procedures
Key Words: procedural knowledge, geometry, precalculus, calculus

Subject: Content/GA

Date: 7/16/95

Geoff Akst:

What's the intent of this question? Is it to contrast content/knowledge with procedure in the classroom and in an assessment? If so, the question reminds me of the issue what we should do with geometry in the college. So many students go on to college knowing little or no geometry. What do we need to teach them—the facts of geometry (areas, perimeters, terms, simple theorems) or the process of geometry (deductive, analytic, etc.). From the point of view of pre-calculus instruction, the content/knowledge may be sufficient. But is that the heart of geometry? If that's all that's worth learning, why do we bother with the rest?

Geoff

I.21

Addressed Topics: interests, assessment, content/procedures
Key Words: technology, diagnostic testing, problem solving

Subject: Interests/ ET
Date: 7/17/95

Liz Teles:

I'm Elizabeth Teles, one of the mathematics Program Directors in the Division of Undergraduate Education (DUE) at the National Science Foundation (NSF). In DUE, as a mathematics Program Director I work in all programs including the Undergraduate Faculty Enhancement (UFE), the Course and Curriculum Development (CCD), the Instrumentation and Laboratory Improvement (ILI), the Mathematical Sciences and Their Applications Throughout the Disciplines, and the Collaboratives for Excellence in Teacher Preparation. In addition, I am the Lead Program Director for the Advanced Technological Education (ATE) program. I received my Ph.D. from the University of Maryland in mathematics education in 1989 some 20+ years after receiving my MAT from Johns Hopkins University. I taught mathematics at Montgomery College, a two-year college right outside Washington, DC, from 1969 to 1991 when I joined the Foundation. In addition, I have spent some time this past year (not as much as I wished) on developing the new Maryland assessment tests.

At NSF we are very interested in both assessment of student learning as well as evaluation of programs. I am particularly interested in kinds and forms of assessment which should be included in the reform efforts. We are becoming much more accountable for the projects which we support. I'm not sure if we are restricting these communications to K-12 efforts or if we are also considering undergraduate issues. I am assuming that the question about "kinds and forms of assessment" refers to assessment of student learning, not overall evaluation of programs, although I understand that the two issues are intertwined. I am a strong believer in applications being used throughout although by the time most students are in college they dread almost all applications, having been taught for the most part almost totally contrived applications. To me the best applications are those that students have enough context to appreciate, understand, and ultimately use. I think we should use more technology as a tool more with students and utilize longer problem sets when possible. By diagnostic testing in the context of this discussion, does that means assessing what skills (broadly defined, I hope) students bring to a reform mathematics classroom rather than those with which they leave and the implications of that on the mathematical experiences that students and teachers together can have during the course?

I.22

Addressed Topics: interests, assessment, content/procedures
Key Words: NCTM, content knowledge, curriculum, calculus

Subject: Interests/AT
Date: 7/17/95

Alba Thompson:

Hi—

I'm Alba Thompson, a professor of mathematics in the Department of Mathematical Sciences and a math ed researcher at the Center for Research in Mathematics and Science Education, both at San Diego State University. I started my undergraduate work in mathematics and physics (astronomy) at the University of Havana, but finished it at the University of Miami in 1973. My masters and doctorate are both in mathematics education, the former from UM (1975), the latter from the University of Georgia (1980). I've taught at the university level since 1980. Before that, I taught school mathematics and science. I've actually taught every grade level from 2nd through 12th except for 4th and 10th grades . . . and survived it all.

My research interests are in: (a) teachers' mathematical knowledge and its role in teaching; (b) the development of children's quantitative reasoning; and (c) the knowledge-base required for teaching mathematics conceptually.

Through the years, I've been involved in a number of professional activities having to do with assessment/evaluation. The most worthy of mention here are: I was one of the six authors of the evaluation component of the NCTM Curriculum and Evaluation Standards; I currently chair the College Board's SAT II committee which is in charge of writing the Mathematics Achievement Tests; for many years I've worked with NAEP (National Assessment of Educational Progress)—the most recent work was to write the 8th-grade instrument for the upcoming 96 assessment.

I've also tried my hand at curriculum development. At present I am directing an NSF-sponsored project that develops instructional materials for the mathematical preparation of elementary and middle school teachers. From 85-90 I co-directed a project that created a new program for the education of middle school mathematics teachers in Illinois. As part of that project we developed several new undergraduate mathematics courses, including a conceptual calculus course and a course in elementary applications of mathematics. I've also co-authored Addison-Wesley's new secondary series (Focus on Algebra).

The most informative (in that I learn the most from) of all my professional activities are teaching and advising undergraduate math majors in our department. From interacting with students I've gained insights into how it is that they know the mathematics that they have studied. This is of great interest to me, but also a source of great concern.

Alba Thompson

I.23

Addressed Topics: assessment, content/procedures, sequencing, application
Key Words: algorithms, constructivism, diagnostic testing, content knowledge, procedural knowledge, curriculum, performance tasks, problem solving, interdiscipline, trigonometry

Subject: brief answers to questions
Date: 7/17/95

Jack Price:

As late as I am, I would like to give some brief answers to some of the questions.
Assessment/JP
I believe there is a role for many different types of assessment and assessment instruments in the intended curricula. One of the assessment standards calls for coherence, using an instrument or assessment practice for the purposes for which it or they were designed. For example, we wouldn't use the SAT if we were evaluating the effectiveness of a program. What we know very little about, I suspect because there has been little written about it, at least in NCTM journals, is performance assessment. With respect to diagnostic assessment, an experienced teacher can learn a great deal about a student's knowledge and ability to use mathematics simply by observation or by interview. An explanation of how a problem was solved is often enough to help the teacher move the child in the right direction. I have never seen a good diagnostic instrument, although there are many of them out there.

Content/JP
Learning and being able to use mathematics effectively and efficiently is a merge of conceptual understanding and procedural knowledge. Algorithms, to me, are simple efficient ways of doing mathematics. The major concern that I have with algorithms is with the way in which they are developed. Children ought to have the opportunity to develop many algorithms on their own Using a constructivist approach (if you will). Other algorithms may need to be taught.
Sequencing/JP
Some degree of sequencing is probably necessary in order to lay a proper framework for some topics. Textbooks generally give one sequence which appears to be logical to the authors and/or editors. However, in many cases there are equally effective sequences. I like to teach volume before area in solid figures at the middle school level, for example, because for some, if not most, middle schoolers volume is an easier concept to understand. A summer institute at University of Chicago changed forever the way I taught trig at the high school and college level. In 1957 we used Dubisch's Trigonometry (Ronald Press) which developed trig functions using the arc length function. It was easy for those of us who had a good background in the trig ratios. From then on I taught the simple ratios first and then went through the algebraic trig. (Speaking of assessment, Northrop who taught the course developed trig through the wrapping function, wrapping a real line around a unit circle. His final used wrapping a real line around a unit square. You knew it or you didn't. It was an excellent example of a performance assessment.)
Applications/JP
In 1966 I wrote an NSF proposal that was funded to teach advanced mathematics using applications. It seemed to be a little before its time. Students learn better in context. If they understand when and how something is used they are better able to learn it. Not all mathematics needs to be or should be taught this way. I much prefer to look at the process as using connections à la the curriculum and evaluation standards. Mathematics is connected internally and it is connected to other disciplines and to the real world. I try to use one of the three as I teach mathematics. (Sorry for all the typos but I have a terrible editor)

I.24

Addressed Topics: assessment
Key Words: diagnostic testing, content knowledge, procedural knowledge

Subject: Assess/AT
Date: 7/17/95

Alba Thompson:

What kinds of assessment you ask? Any and all that are viable and get you (the teacher, or the district, or the state, or the nation, or the international community) the information you need to determine what the students are learning or whether or not the instructional program works. An oversimplification of the issues? Maybe.

I see too much evidence of passionate dogmatism and little common sense in discussions surrounding assessment issues. I've always approached assessment as an inquiry activity (not much different from the research I do in trying to understand/document children's mathematical thinking and quantitative reasoning). Unfortunately, I often encounter discussions on assessment that are cast in an advocacy mode. Portfolios, Yes! Multiple-choice items, No! Etcetera etcetera. When it comes to assessment, I prefer to adopt an attitude of inquiry over one of advocacy. To support such an attitude of inquiry one needs to be not only technically resourceful, ingenious, and skillful, but one needs to know quite a bit about the mathematical ideas being assessed, what students' thinking relative to the ideas tends to be, and how to access students' thinking. All these I view as necessary for diagnosing student's conceptions and understandings. Not quite the same, however, for diagnosing faulty use of calculational techniques and procedures which are more directly inferable from students' written work.

There is a big difference in what is involved in diagnosing conceptual understanding, calculational skills, and facility with mathematical thought processes (e.g., justifying, conjecturing, generalizing, etc.). To diagnose conceptual understanding one must have an image of the conceptual domain (à la Vergnaud) of which the mathematical ideas being assessed are a constitutive part. Without that image, we may find ourselves staring at a student's performance on a diagnostic test and not know what to make of it. An article that appeared in the May 1994 issue of *JRME* which I co-authored with Pat Thompson, provides an illustration of this point for the concept of rate—a concept that, as I have found, few undergraduates math majors have a good grasp of even after completing 16 semester hours of calculus/analysis!!

The desire to assess students' facility with mathematical thought processes provides the impetus for expanding traditional assessment practices beyond the restrictions imposed by the exclusive use of certain types of items/tasks. At least it has provided the impetus for our efforts to expand beyond multiple-choice items in the SAT II exams. But, in that case, the practical constraints are enormous. I won't go into that here since I'm sure everybody has a sense for what these are. Enough said for now.

Alba

I.25

Addressed Topics: content
Key Words: content knowledge, procedural knowledge

Subject: Content/AT

Date: 7/17/95

Alba Thompson:

I am having a terrible time dealing with this question. I guess I don't really understand it. I see the knowledge of procedural techniques as an integral part of what I consider mathematical content knowledge.

Alba

I.26

Addressed Topics: interests, sequencing, application
Key Words: motivation, problem solving, algebra, calculus

Subject: Interests/ME
Date: 7/17/95

Marj Enneking:

My name is Marj Enneking. Since finishing my Ph.D. in mathematics at Washington State University, I spent 2 years at the U. of Missouri-St. Louis and since 1968 have been in the Mathematics Department at Portland State University (PSU)—except for one year as a visiting professor in the Ohio State University Math. Dept. and 2 years as Program Director at NSF (where I had the pleasure of working with Liz Teles the second year). The past two years I've been Associate Vice Provost for Research and Sponsored Projects at PSU.

I told the "powers that be" that I was willing to do this administrative work for a year or two, but that I intended to go back to the department and teaching, where my heart is. So this next year I'll continue working 1/3 time in this office and 2/3 time back in the Math Dept., and the following year I'll happily be full time in mathematics again.

Since I've been able to read some statements already, I would like to share a personal reflection related to them—especially to sequencing and applications /motivation. Namely, about my daughter, Nancy, now working on her dissertation in Near East Studies (Egyptian Archeology) at Johns Hopkins, who managed to get through the standard high school mathematics courses and a semester of calculus at college, although she still has trouble with fractions. (So much for sequencing?) And she never would have stayed with mathematics that far, history clearly being her first love, except that a wonderful high school physics teacher got her interested enough in electronics and astrophysics physics that she wanted to keep that door open. (One semester of a traditional standard calculus course cured her of those thoughts.) This may be an example where applications were important both as a psychological and mathematical motivator—the desire to learn the mathematics in order to do something else, but also a framework to better understand the mathematics itself. So I hope we can think of applications not as those “applied problems” we assign after we “cover the material” but more often as the starting place from which we generate the mathematics we are teaching. And this from an algebraist who always avoided any mathematics courses which smacked of practical applications! In one of my favorite courses to teach I am struggling with whether to teach it more as I learned it and taught it for years, a theoretical approach to groups, rings, fields, etc., or whether I need to be approaching it from a whole different perspective. A struggle which is probably happening among teachers of mathematics at all levels.

At any rate, I have really enjoyed the statements so far and look forward to reading more.

Marj

I.27

Addressed Topics: interests, content/procedures, assessment
Key Words: curriculum, problem solving, precalculus, calculus

Subject: background info
Date: 7/19/95

Margaret DeArmond:

Hi Everyone,

I’m Margaret DeArmond, high school mathematics teacher of 24 years in the Kern High School District in Bakersfield, California. I have been fortunate to have many professional experiences in my tenure. I am currently president-elect of the California Mathematics Council and a past co-director of the San Joaquin Valley Mathematics Project (a professional development program for K-12 teachers). Most importantly, I have taught students of wide cultural and economic backgrounds. I thought for many years that teaching AP Calculus and math analysis was the highlight of high school teaching, but I now find the challenge in reaching all students. As we see changes in curriculum, instruction, and assessment, we also see many teachers like myself who have renewed excitement in their belief that all students can become mathematically powerful. For the past four years I have been teaching the Interactive Mathematics Program (four years of college prep curriculum with thematic, problem-solving based units of instruction—a curriculum that is open to all students). I have really changed my mind about what students can do and learn when questions are posed in a more open format. I am having a most rewarding teaching experience, and I know many of my colleagues are feeling the same way. I look forward to sharing more about content and assessment with all of you and hearing your comments. I am also having a summer of workshops and out-of-town meetings (without much computer access) so I'll try to stay in touch during this conference.

For more info about the Interactive Mathematics Program contact Linda Witnov, IMP, 6400 Hollis St. Suite #5, Emeryville, CA, 94608 510-658-6400.

I.28

Addressed Topics: sequencing
Key Words: content knowledge, curriculum, problem solving, algebra, geometry, trigonometry, calculus

Subject: Sequence/Musings on statements sent
Date: 7/19/95

Marj Enneking:

I've been thinking about the interplay of the sequencing, content, and assessment questions. I really liked Zal's list of different possible ways to sequence. Thanks, Zal! I can imagine two different sets of problem-based curriculum materials, such as the Interactive Mathematics Project, in which the same general content may be covered over a 3 or 4 year span, but in which the problems chosen dictate a quite different order of topics. One program may do lots of data analysis in year one, while another may get into trigonometry and geometry. Can we deal with that? What about kids who transfer from one school to another? Will they be repeating things already learned, and not knowing topics the rest of the class knows? (Ah, but that question assumes that they actually know the topics covered in the old school. But we all know how far the learned curriculum is from the taught curriculum. How many kids come into any class today having mastered the material taught in the preceding class?) How would we assess kids—especially on a statewide or national scale?

I agree with Alfred that algebraic skills and the concept of function are important for calculus students. We gripe about kids getting to our courses without them. But what about those kids who get to and all the way through our calculus courses without developing an understanding of the concept of rate, as Alba described? SDSU isn't the only place where that happens. We have seen anecdotal examples on our own campus: Colleagues of mine have developed a very interesting conceptual calculus course, Concepts of Calculus for Middle School Teachers. It has been particularly interesting that some of the people who had trouble in the course had completed both a full calculus sequence and advanced calculus. They were great with calculus computational skills, and were able to spout definitions and prove theorems, but faltered when confronted with really basic important ideas of calculus.

How do we take into account, when talking about sequencing, the differences of our ideal (very carefully sequenced) curriculum and the reality of what kids actually know when they come in to a class? I guess that's where the importance of good assessment plays a role.

The more I think about it, Walter's idea of a curriculum as an evolving web rather than a sequence of topics makes sense to me. Do we know—agree on—the big ideas? Do we need to? Can each teacher or each school or each district create their own sequence or web which fits their needs? I just read an interview with Amy Derby in a newsletter from the Northwest Regional Educational Laboratory, in which she commented, "Curriculum is a set of ideas, plans, and activities that students and teachers continually develop as their learning needs change. Curriculum ultimately resides in the minds and hearts of teachers and students . . . " I like that. And assessment for the purpose of helping to identify that curriculum as the classroom or school or district level would make sense. But I'm not sure how it would fit with assessment for other large-scale or political purposes.

I.29

Addressed Topics: application
Key Words: technology, algorithms

Subject: Apply/AES Statement/ABM
Date: 7/19/95

Alfred Manaster:

The changes in the meaning of computer literacy that Anita Solow (12 July) highlights as suggestive of similar changes in our understanding of mathematical literacy, both resulting in part from changes in technology, are very interesting and worthwhile. It seems that we have to keep changing our view of what constitutes thought as machines improve—once upon a time we thought checkers was an essentially human activity but now we worry about whether chess will suffice to play that role.

Anita's comments raised the question of the extent to which completing mathematical algorithms is still part of knowing and doing mathematics. While I agree that "the experience of applying mathematics in a meaningful environment . . . is important," I wonder about how much computation is usually needed to apply mathematics. Computation is intended to be understood in a broad sense. The question here in part is what makes the subject being applied mathematics rather than something else.

Reiterating this, when Anita says in her last paragraph that it is “not good enough for students to demonstrate that they can compute derivatives and integrals,” does this imply that they no longer need to be able to perform those computations? Or, does this mean that they need to be able to do them and more? Or, that they only need to be able to do some of the more straightforward computations, relying on computers to do more complex computations, while applying both the simpler and the more complex computations in realistic settings? Other options?

I hope many of us will comment on these questions.

Alfred

I.30

Addressed Topics: content/procedures, application
Key Words: representations, algebra

Subject: Content & Apply/ZU Statement/ABM
Date: 7/19/95

Alfred Manaster:

Re: ZU Content and Apply Statement

I also discovered the difficulty of responding to these two questions separately since comments about applications arose naturally in my response to the Content question.
One question which arose in Zal Usiskin’s (13 July) discussion of “Representations” is whether that category also explicitly includes formal symbolic representation, that is, using the formal language of mathematics as a representation of a situation.

Other questions arise from Zal’s Interests statement. It is fascinating to me that he tried for four years in the 1970s to develop for average students the content of elementary algebra through applications and that the short answer was “Yes (it is possible), but not all (of elementary algebra) is appropriate for such an approach.” First, why is this especially for average students? Secondly, (here I must confess to playing devil’s advocate) why include material if it cannot be developed through applications?

Again, let me invite many of you to comment. I think a discussion of these points could be helpful and instructive.

Alfred

I.31

Addressed Topics: sequence
Key Words: NCTM, curriculum

Subject: Sequence
Date: 7/20/95

Zal Usiskin:

Thank you, Marj, for your nice comments.

I would like to comment on two of your points. Integrated curricula exist in some (not all) countries, but in all those places there are national exams at the end of particular points of schooling. The exams serve to ensure that students are taking about the same things at the same time.

I disagree with Amy Derby's view, "Curriculum is a set of ideas, plans, and activities that students and teachers continually develop as their learning needs change. Curriculum ultimately resides in the minds and hearts of teachers and students . . . " Of course, a word can be defined in whatever way we wish, but the word curriculum has many years of usage with a quite different meanings. The views of the Second International Mathematics Study, which involved an international panel, seem to me to be cogent and clarifying: the ideal curriculum, that which is defined by goals and statements such as the NCTM Standards or the California Framework; the implemented curriculum, that which is taught; the achieved curriculum, that which is learned. One can easily add to that: the tested curriculum.

I.32

Addressed Topics: content/procedures, application
Key Words: curriculum, motivation, problem solving, representations, arithmetic, algebra, geometry, calculus

Subject: Re: Content & Apply/ABM reply to ZU/ZU
Date: 7/20/95

Zal Usiskin:

Let me try to respond to Alfred's questions.

First, regarding "average students": Mathematics courses today are descended from a time in which not all students were expected to need mathematics. For instance, traditional algebra is descended from a time in which the raison d'être for algebra was calculus. They also used to proceed from a notion that the student is self-motivated and thus does not need to be reminded, convinced, or taught of the uses of the subject.

When one decides that “average students” or “virtually all students” need mathematics, none of the other assumptions holds. The raison d’être for high school mathematics does not become college mathematics, and a student cannot be assumed to be self-motivated. Applications of mathematics to external situations, which are a primary reason one decides that all students need mathematics, now become a necessity for another reason: They provide a major motivation for taking the subject.

But this is not in any way to suggest that the students who were well served by traditional curricula (and there were many such students) do not need applications. It is merely to suggest that they were not the primary motivation for the movement that has taken place in the last generation.

Second, even though I am a zealot for applications, I believe strongly that in a curriculum for all students one should include mathematics that is not necessarily tied to real-world applications. Pure mathematics is an essential part of mathematics! (It is amazing that I have to state that.) Let us not play the pendulum game (with horrible logic) by assuming that the only alternative to a curriculum with no applications is one in which everything is applied.

There are many reasons for having pure mathematics. (1) Mathematics is in many ways a language. It enables us to describe many real-world phenomena, of course, but one can also speak and write in it without having to translate back and forth from the real world. Indeed, the person fluent in mathematics needs to be able to do that speaking and writing without the translation. Indeed, a major part of the power of mathematics—even when doing real-world problems—is that one operates within it without recourse to the situation that gave rise to it. For instance, one does not need to translate every line of the solution of an equation to the real world situation that gave rise to the solution.

(2) Mathematical truth is based on deduction (my apologies to those who wish to promote fuzzy logic or probabilistic proofs). Deduction is a fierce and relatively unyielding game in which one needs to deal in symbols and the logical relationships between propositions. Although reasoning from assumptions in real situations is exceedingly common and very valuable for teaching reasoning in mathematics classrooms, to limit oneself to such reasoning is to ignore over two millennia of mathematical history.

(3) Pure mathematics is exquisitely beautiful. From the fact that an integer is divisible by 9 if and only if the sum of its digits (in base 10) is divisible by 9, to Napoleon’s Theorem (if equilateral triangles are built outward on the three sides of any triangle and their centers are connected, an equilateral triangle is formed; if they are built inward, a second equilateral triangle is formed; the difference in areas of

the two equilateral triangles equals the area of the original triangle), to the Fundamental Theorem of Algebra (every polynomial equation over the complex numbers has at least one solution in the complex numbers)—mathematics is gorgeous. Applications are wonderful, and some applications are definitely beautiful (e.g., Escher drawings; scheduling teams by using regular polygons; fitting polynomial formulas to real-world data), but so is the pure stuff.

A good curriculum should endeavor to do both pure and applied mathematics well.

Third, and this is a little off the subject of the first two, by “representations” I mean any other language for dealing with a situation than the obvious one. For instance, if one is teaching (a+b)(c+d) and uses areas of rectangles to picture the distributive property, this is a geometric representation of the arithmetic and algebra. For another example, analytic geometry is an algebraic representation of geometry; but graphs are a geometric representation of algebra.

As to the question of whether the symbols of algebra provide a representation, I would like to say no. Although all language could be considered as representation, I would be uncomfortable with calling x a representation of something unless one decided that words too were representations.

Zal Usiskin

I.33

Addressed Topics: assessment, content/procedures, sequencing, application
Key Words: technology, diagnostic testing

Subject: Reform Conference
Date: 7/21/95

Gail Burrill:

Hi! This message is not from Kelly Fox but really from Gail Burrill. I am on the road and for some reason, I can only read messages on my e-mail. I cannot send or reply—the machines (three different software packages) just shut me out of e-mail when I try. So—it is a system thing but my home people don’t think so!!

So, I have read all of the messages and would like to comment but am borrowing someone else’s email number to even do this. I will continue to try to get on line.

I do have some comments on the original questions:
a. It seems as if we should think about assessment as a way to improve instruction, measure where you are in the learning plan and to look at what students can

actually do. Diagnostic assessment means that (to me) all of the results are used to improve instruction not just inform.

b. This one does not make sense to me? Is this the relation between symbols and technology?

c. Sequence is critical but, as has been pointed out, there really is no clear one way to sequence most of what we teach.

d. It seems critical to provide some context in which to learn mathematics, but that context can be the mathematics itself.

Sorry about the mess. I will continue to try to straighten it out. Gail

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I.34

Addressed Topics: interests, content/procedures
Key Words: technology, curriculum, algebra

Subject: Biographical information
Date: 7/24/95

John Harvey:

Dear Colleagues:

I know most of the participants in this conference and think that you know me. However, for the record, here is a short description of my background and interests.

B. S., Baylor University, 1955 (Mathematics and Education) M. S., Florida State University, 1957 (Mathematics) Ph.D., Tulane University (1961) (Mathematics; Specialty: Ordered Algebraic Structures; Major Professor: Paul F. Conrad)

1961-66, Instructor and Assistant Professor, Department of Mathematics, University of Illinois (Urbana/Champaign)
1966-75, Associate Professor, Departments of Mathematics (2/3) and of Curriculum and Instruction (1/3), University of Wisconsin-Madison
1975- , Professor, Departments of Mathematics (2/3) and of Curriculum and Instruction (1/3), University of Wisconsin- Madison

I came to the UW-Madison 29 years ago primarily because the Mathematics Department had decided to include mathematics education among the specialties (i.e., content areas) in which doctoral students could write their dissertations. Though I continue to regard myself as an algebraist, I have also specialized in mathematics education since 1966 and presently regard myself as a mathematical sciences educator since I am interested in curriculum, instruction, and learning in mathematics, statistics, and computer science.

I teach mathematics courses at both the freshman/sophomore and junior/senior levels. For example, in the fall I will teach one of the required courses for students majoring in elementary education and our introductory matrices and linear algebra course (where a TI-85 or HP-48G will be required). In the spring semester, 1995-96, I will teach the lower-level course in abstract algebra taken by juniors and seniors here.

In Curriculum and Instruction I teach graduate courses in mathematics education and undergraduate courses in mathematics education, computer science education, and computer education. In the spring of 1996 I will initiate a new course that, beginning in 1996-97 will be required of all undergraduate majors and minors in Secondary Education/Math. That course will be: The Uses of Technologies in Secondary School Mathematics.

Over time my interests in mathematics sciences education have been eclectic. I was one of the principal investigators of the project that developed Developing Mathematical Processes, a K-6 elementary school curriculum published in 1974-76 and developed with federal funds. I developed the teacher certification program in computer science education that leads to secondary school licensure for that subject. I have been a member of and chair of the MAA Committee on Testing. I have been a College Board AP reader for mathematics and computer science. I have served a term on the College Board Council on College-Level Services. At present I focus my attention on Grades 7-G and am particularly interested in: the uses of technology in mathematics curriculum, instruction, and technology, the development of “technology-based” tests, and the elimination of differences in achievement of students of diversity.

Subject: Romberg paper ref/jgh

July 25, 1995

From: John Harvey

Marj Enneking requested the Romberg reference; so, I thought that I’d send it to everyone. Here it is:

Romberg, T. A. (1992). Problematic features of the school mathematics curriculum. In P. W. Jackson (Ed.), *Handbook of Research on Curriculum*. New York: American Educational Research Association.

I know that Alfred is already familiar with this reference since they use it somewhere in their joint doctoral program in mathematics education.

I.35

Addressed Topics: assessment
Key Words: NCTM, constructivism, procedural knowledge, curriculum, performance tasks, problem solving

Subject: assess/statement/jgh
Date: 7/24/95

John Harvey:

First, let me apologize for joining the conference so late. There some "things" that were already on my calendar that could not be canceled or delayed when the invitation to participate arrive including the annual week-long visit of my (only) sister and brother-in-law. During that visit I cook a lot, chauffeur some, and entertain.

In making my statement on assessment I have the benefit of having read statements by Walter Denham, Alfred Manaster, Geoff Akst, Alba Thompson, Jack Price, and Norm Webb. So, I will try to make my own statement but at the same time to agree with and not repeat statements made by others.

In the past some have tended to use the terms "testing," "assessment," and "evaluation" interchangeably. And we have tended to believe that any data gathered about student knowledge could be used in (almost) any way; among those ways were: (a) a description of student achievement, (b) improvement of instruction, and (c) program evaluation. Fortunately, we now recognize that these terms need to have different definitions, that the relationships between the terms need to be explicated, and that the purposes of testing (assessment, evaluation) will often dictate the kind of data that we gather.

SOME DEFINITIONS

The Assessment Standards for School Mathematics (NCTM, 1995) define test, assessment, and evaluation in these ways:

assessment: The process of gathering evidence about a student's knowledge of, ability to use, and disposition toward mathematics and of making inferences about that evidence from a variety of purposes. . . . In this document, assessment is used as defined above to emphasize understanding and description of both qualitative and quantitative evidence in making judgments and decisions (NCTM, 1995, p. 87).

evaluation: The process of determining the worth of, or assigning a value to, something on the basis of careful examination and judgment. As used in this document, evaluation is one use of assessment information (NCTM, 1995, p. 88).

test: “A measuring instrument for assessing and documenting student learning. . . . ” (Hart, 1994, p. 114). “A formal, systematic procedure for obtaining a sample of [students’] behavior; the results of a test are used to make generalizations about how [students’] would have performed on similar but untested behaviors” (Airasian, 1991, p. 440).

I will assume in this conference that we are using these definitions of assessment, evaluation, and test, that we are not presently concerned with evaluation, and that we are concerned only with tests as one way of gathering evidence about student knowledge, etc.

The NCTM Assessment Standards for School Mathematics (1995, p. 27) also identifies four assessment purposes; they are: (a) evaluating student achievement, (b) making instructional decisions, (c) monitoring student progress, and (d) evaluating programs. (The implied order of importance is my own.)

WHAT DO WE WANT TO ASSESS?

What we want to assess is based upon our beliefs in at least these three areas: (a) What is mathematics? (b) How do students learn? (c) What mathematics do we want students to “know?” and (d) How do we want students to know the mathematics we think they should learn?

My beliefs in the first two areas are congruent with the Standards. It is difficult for me (anyone?) to say what mathematics IS. It is somewhat easier to say what mathematics IS NOT. It is not a well-defined system of rules, procedures, and outcomes (including theorems) but is a growing, changing, living discipline that intends to ??? Mathematics is not something to be learned by memorizing rules, procedures, definitions, and outcomes but is a way of thinking, communicating, solving problems, connecting different situations, etc. I strongly believe that each individual constructs his or her own mathematics understandings within the context of his or her environment (i.e., I am a social constructivist).

My beliefs about (c) will be expressed in my content position paper.

My beliefs about (d) will be expressed in my applications position paper.

KINDS AND FORMS OF ASSESSMENT

In both The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) (hereafter, the Standards) and the Assessment Standards for School Mathematics (NCTM, 1995) (hereafter, the A Standards), it is clearly stated that assessment must use multiple sources of evidence. Among the sources of evidence that I think we want are:

1. Tests (including multiple-choice tests)
2. Journals
3. Written and oral reports that describe student investigations and their results, including both cooperative group reports and individually authored ones
4. Homework
5. Informal and structured teacher observations of students at work
6. Portfolios

Teachers, mathematics educators, school administrators, etc. seem overwhelmed by this list. In reality there isn't much new here, except for portfolios and journals. What's new is the importance that we give to each of the items. I'm sure that each of us, while teaching mathematics, have used (1), (3), (4), and (5) to gather evidence to assess student progress and achievement and to assess our own instruction. And most of us have depended on tests to provide the evidence needed to make evaluations: grades, program effectiveness, etc. If we want assessment that is congruent with reformed curricula and instruction, then we must take into account and have systematic ways of making inferences about the evidence we gather using (2)–(6). In addition, we must diminish the importance (an evaluation) we assign to the evidence gathered by (1).

A FINAL REMARK

In what I've read from the conference participants on this topic there seems to be an emphasis on students and teachers. And I strongly endorse the cogent statement made by Alfred Manaster. BUT . . . We must not forget that we need to be talking about the kinds of evidence we need to gather in order to persuade the citizenry of this country that mathematics education has worthwhile goals and that it is (working toward) achieving those goals. The day is past when the mathematics program is a good one "because we say so." The time has come when agents of the citizenry are going to ask us to "prove" true anything we assert is true. We should not be discomfited by this; after all, it is what we ask our students to do on a day-by-day basis. But we must take into assessment for this purpose whenever we talk about what processes and instruments we use to gather assessment data.

Subject: assess/statement/addendum/jgh

July 24, 1995

From: John Harvey

RE: An addendum to my assessment statement (dated 7/24/95)

I think that Tom Romberg's paper from one of the recent BIG yearbooks (not NCTM) is a good discussion of "What is mathematics?" and "What is school mathematics?" Are you familiar with this paper. If not, let me know so that I can send you the reference. I use this paper in my doctoral mathematics curriculum course.

I.36

Addressed Topics: interests, assessment, content/procedures
Key Words: technology, diagnostic testing, calculus

Subject: Interests/LHB
Date: 7/24/95

Linda Boyd:

I apologize for joining in the conference so late, but I'll do my best to catch up. I had to make an unexpected trip with my mother to take care of some business connected with my father's estate. We came home just in time to dive into the final preparations for my niece's wedding on Saturday.

I have taught at DeKalb College (a two-year college on the outskirts of Atlanta) for 24 years. For many of those years I have included in-depth projects as part of student assessment. These are usually done in groups. Many of the projects require students to explore, form conjectures, and defend their conjectures. I have always encouraged my students to use appropriate technology in class, for homework, for projects, and on tests. At present I am directing an NSF grant that is supporting our efforts to incorporate labs into the calculus sequence. We are using Mathematica.

Changing my own assessment instruments and working with my colleagues to change the department assessment instruments has been at times exciting and at other times frustrating. We all have strongly held beliefs about what is crucial and compromise is frequently difficult.

On the national level, I served as a member of the MAA Committee on Testing for 6 years. During that time I directed a FIPSE project to produce a system that produces parallel versions of the items in the MAA Placement Test item bank. I continue to work with Mary McCammon of Penn State to produce the tests for the MAA. The items are multiple choice, and I share the concern about using multiple-choice items that some of you have mentioned. I do believe good items of this type can serve a valuable role in placement and perhaps in diagnostic testing.

I could go on and on, but I'd never catch up that way.

Linda Boyd

I.37

Addressed Topics: content/procedures
Key Words: technology, calculators, algorithms, procedural knowledge

Subject: Content/responses/AES
Date: 7/25/95

Anita Solow:

Mathematical procedures:

I would like to address some of the comments that have been made about the importance of mathematical procedures. Alfred Manaster questions "the extent to which students must master computational skills that computers and calculators can now execute faster and more accurately." He then goes on to say that there is value in students doing computations by hand and then seeing that the calculator gives the same answer since it helps build student confidence.

I, too, am very concerned with the question that Alfred raises, and although I agree with the value of using technology to verify answers that are arrived at in other ways, I am concerned that the procedures are stressed too much in the classroom. What is enough? Does one need the level of proficiency that was needed before technology? Or can one do less?

Jack Price wrote that "the major concern that I have with algorithms is with the way in which they are developed." He suggested that students should be developing them, not be handed them. I agree with this statement. There is much good mathematics in algorithms. But it is hard to find if the algorithms are handed down to the students as the word of God, to be used without questioning or thinking about what makes it work.

I have two children in school, and I have been watching what they learn in math in school. I think that the emphasis on procedures is too strong. And it continues, even after the students have demonstrated their ability to do the procedure well. Timed drill sheets do not make thinking mathematicians. I worry that procedures become synonymous with mathematics to too many people, rather than being (an important) part of mathematics.

Pure/applied mathematics:

Zal Usiskin wrote on the importance and beauty of pure mathematics. I could not agree more. One of my favorite areas of "applications" of mathematical ideas is in pure mathematics. Not all applications need to come from outside of mathematics.

I.38

Addressed Topics: application
Key Words: technology, calculators, trigonometry

Subject: Apply/AES reply to ABM/AES
Date: 7/25/95

Anita Solow:

Alfred, in responding to my first posting on applications, asked what level of proficiency is sufficient for students, given the availability and power of technology. He offered three alternatives: (1) students should be able to do everything by hand that they were expected to before, (2) students do not need to be able to do the computations by hand because the calculators/computers can do them, or (3) students should be able to do the simple computations by hand and leave the hard ones for the technology.

At the moment, I am leaning toward the third one. I am questioning the necessity of having students becoming whizzes at computing really messy derivatives of antiderivatives by hand. Asking the students to become proficient USERS of mathematics on top of proficient CALCULATORS of mathematics may be asking too much. (Also, we are asking them to become COMMUNICATORS.) So I am at the stage now where I am willing to ease up on one area to give them time to deal with other demands which I believe are more important.

But whatever decision I make today needs to be reevaluated regularly. Who knows, some day we may think it is silly to teach long division by hand, or antidifferentiation via trig substitution, just as today we no longer teach students how to compute square roots by hand.

I.39

Addressed Topics: sequencing
Key Words: curriculum

Subject: Sequence/ZU Statement/ABM
Date: 7/25/95

Alfred Manaster:

Let me start with an apology for my delays in responding to statements. I want to comment on many of the statements made and part of the subsequent conversations, but am simply behind at the moment. I will try to continue making comments more or less in the order in which the statements were made. It seems to me that there are many opportunities for further discussion.

The variety of possible sequences that could structure a mathematics curriculum that Zal outlined (13Jul) were very interesting to me. It seems clear that, as he said, the more structural frameworks, the greater the opportunity to engage students and help them learn substance. The principal question raised for me is how to select the dominant sequence to use in structuring a curriculum. It seems necessary, given the differences among the ways Zal listed, to choose one as the organizing basis and then try to incorporate as many of the others as possible at various times. A discussion about these points would be helpful.

Alfred

I.40

Addressed Topics: assessment, content/procedures
Key Words: procedural knowledge

Subject: Assess and Content/NLW Opening Remarks/ABM
Date: 7/25/95

Alfred Manaster:

The juxtaposition of Norman Webb's comments (15Jul) on Assessment and Knowledge made me eager to learn more of his and others' views about the kinds of assessments that are called for by the reform effort. What role does, should, must, procedures play in those assessments, especially in the context of Norman's assertion that "Procedures are essential to mathematics?"

Alfred

I.41

Addressed Topics: assessment
Key Words: NCTM, diagnostic testing

Subject: Assess/ET Opening Statement/ABM
Date: 7/25/95

Alfred Manaster:

I hope we can focus our discussion, as Elizabeth Teles assumed (17Jul) on student learning rather than program evaluation even though the two are often intertwined. Her last sentence provides a good discussion point, about which I would again urge comment from many of us. I don't see why diagnostic assessment should be limited to measuring only what skills and knowledge (certainly broadly defined) students bring to the classroom. Such measurements can be usefully made at many stages in a course. The broader question raised by that sentence, what student knowledge can we assess and then use to enrich student learning, seems like a very fundamental one. (This seems different to me than using tests for program evaluation, as clarified by the NCTM Assessment Standards that John Harvey quoted yesterday.)

Alfred

I.42

Addressed Topics: assessment
Key Words: diagnostic testing, motivation

Subject: Assess/GA Statement/ABM
Date: 7/25/95

Alfred Manaster:

Here is my answer to Geoff's question (16Jul) about what we mean by "diagnostic assessment." It seems to me that we could choose either of his suggestions. I would prefer what he called the stricter interpretation so that the goal of diagnostic assessment would be to identify specific areas where individual students or classes of students have strengths and weaknesses. This is the primary intent of the California Mathematics Diagnostic Testing Project's outreach to the schools. On the other hand, many diagnostic tests are also used as a factor in determining student placement. One reason may be that such tests are often validated by correlation with student success and failure in later courses.

A helpful contrast comes from the prognostic testing program developed at Ohio State University and used in Ohio. My understanding is that these tests, which are essentially placement tests for entering students at OSU, are offered to Ohio students at the end of their junior year. The students are then told what courses they could take at Ohio State with their current background and knowledge. This is a successful way of motivating many students to take more mathematics their senior year in order to be better prepared when they do enter college. I wonder whether the test results are also used for diagnostic purposes. Does anyone in this conversation know? It certainly seems that they could be used in that way also, strengthening Geoff's suggestion that the two purposes are often linked.

Alfred

I.43

Addressed Topics: content/procedures
Key Words: content knowledge, procedural knowledge

Subject: Content/AT Statement/ABM
Date: 7/25/95

Alfred Manaster:

Is Alba "having a terrible time" with the Content question because her answer is simply that content knowledge and procedural techniques are both essential components of mathematical education?

Alfred

I.44

Addressed Topics: assessment
Key Words: diagnostic testing, calculus

Subject: Assess/AT Interests and Assess/ABM
Date: 7/25/95

Alfred Manaster:

Why are Alba's insights (17Jul) into how students know the mathematics they have studied a source of great concern to her? Is this question partially answered by her description of the poor grasp many students have of the concept of rate, even after two years of calculus? It is distressing that this does not come as a surprise to Marj Enneking or me or, probably, any of us. Do we have any evidence that reform calculus is helping? Certainly my anecdotal experience suggests it might be.

Let me ask a related question, probably to Alba. Is it possible to explain briefly in this discussion what is meant by "an image of a conceptual domain?" Can this then lead to insights about diagnostic assessment?

Alfred

I.45

Addressed Topics: content/procedures, application
Key Words: technology, calculators, algorithms, constructivism, procedural knowledge, curriculum, problem solving, representations, interdiscipline, calculus

Subject: Content & Applications/Statement/jgh
Date: 7/25/95

John Harvey:

I apologize for the ragged, incomplete lines in my assessment statement; WordPerfect did something to me that I didn't expect, even though I've been using it since 1984!

This message will contain my statements related to questions (b) and (d).

THE WORLD HAS CHANGED (IS CHANGING)

I'm sure it is no surprise that I strongly advocate the appropriate uses of technologies in the teaching and learning of mathematics. BUT this does not mean that students will no longer need procedural knowledge. However, our students will be unlike us—or, at least, unlike the people we were before we began to use technologies and relied solely on mental and paper-and-pencil procedures. (Yes, I still know how to approximate the square root of a number using a paper-and-pencil algorithm, know the squares of many numbers, etc.) Our students will know and use some paper-and-pencil procedures, they will have procedural knowledge about their calculators and computers (that is probably not mathematical procedural knowledge), and they will have calculator- and computer-based procedural knowledge.

Our students' procedural knowledge will also differ from our own when they learn in a (social) constructivist environment where the teacher guides instead of telling. In that environment students will invent, test, and debug their own procedures and algorithms. One of our roles will be that of giving students sets of "test data" that will help them discover the errors and shortcomings in their procedures. Another role will be guiding them to discover correct algorithms and procedures. Another role will be sharing with them the theorems that lets them organize and consolidate their knowledge so as to produce efficient, accurate schema and maps of their mathematical knowledge.

In short, technologies have changed or are changing the procedures that students will develop and learn, but procedures are still important. However, I think that the current reform movement has correctly identified that procedural knowledge and its acquisition can no longer be the primary content that is taught in school and collegiate (e.g., calculus) mathematics. Concept knowledge, the uses of mathematics as tools for problem solving, the connections within mathematics and to other disciplines, and the applications of mathematics to other disciplines must have "equal billing" with procedures. Once again, the newness here is the emphasis that we give to facets of mathematics instruction and learning that have been around for a long, long time.

MATHEMATICS IS MATHEMATICS

On July 20, Zalman argued convincingly that there is a (growing) body of knowledge that we call mathematics and that knowledge of that body of knowledge can be important for its own sake. Thus, I see questions (b) and (d) as really being these two questions:

1. What is school (collegiate) mathematics?
2. What is the role of applications in mathematics instruction?

Both questions are highly relevant. The first is relevant because so much of what we have taught can be eliminated or the emphasis on it diminished because of the technological tools we have. On 22 July Anita argued (using calculus as an example) that "[s]ince available technology can do many of the manipulations traditionally taught in calculus, it is not good enough for students to demonstrate that they can compute derivatives and integrals. They need to demonstrate that they can apply the IDEAS of calculus to situations they have not seen before." [By the way I would add limits and continuity to Anita's list of essential calculus topics.]

The second question is highly relevant because the audience for mathematics knowledge grows on a daily basis. This audience is much like my knowledge of computers and computer science. Basically I have a master's degree in computer science acquired in the late 70's and early 80's here. But in talking about what I needed to know, my CS mentor, Larry Travis, commented that while I need to know something about how the computer worked, I was really interested in using the computer as an applications tool. Thus, in those days, I needed to know computer programming languages, data structures, and the like. The same is true of most of our mathematics students at the school level and in many of our undergraduate college courses. They need a sound knowledge of mathematics so that they can use it to solve problems in their chosen profession (for lack of a better word). If we don't satisfy this audience of "mathematics consumers," others will. I feel sure that those of us who teach at the collegiate level have departments within our institutions that teach their own brand of statistics. If we don't respond, they'll teach their own brand of calculus, linear algebra, . . . And the mathematics department will shrink to the size of the classics department (11 faculty on my campus). Mathematics is too vital and necessary a discipline to leave its instruction to "amateurs!"

However, my answer to Alfred's last question, "Why include material if it cannot be developed through applications?" is that not everything included in the mathematics curriculum has to be related to an application. (I won't repeat Zalman's arguments here but will endorse them again.) I regard applications and modeling as only one way to teach mathematics. We must use this way, but we must also find other ways to interest students in the mathematics we think they need to know; other ways that come instantly to mind are: games, puzzles, paradoxes, and art and music. Whatever we do must not be superficial or shallow. I was a school student (I started to school in 1940) during the "practical applications" era when one computed, for example, how many hours it would take to cook a roast that weighs x pounds if you cook it y minutes per pound. At one time I could tell you exactly how the $600 income tax deduction was computed for a median annual salary!

So, we need to think carefully about the mathematics that we think students will need, the mathematics that they need in order to acquire the mathematics they will need, and the mathematics that should be a part of their "liberal education." We should, as often as we can, include applications in our curriculum and in our instruction.

SYMBOLS AND REPRESENTATIONS

As a footnote I want to disagree with Zal's statement that symbols aren't representations. Maybe symbols are representations when they are injected into the student's environment. But I would assert that they become just as "real" as the objects which they represent. This symbol "2" is just as real to me as two marbles, two dollars, two cars, etc. And a lot more real than two billion dollars!

Regards,
John Harvey

I.46

Addressed Topics: content/procedures, application
Key Words: content knowledge, representations, arithmetic, algebra, geometry

Subject: Content & Applic/Reply to JGH/ZU
Date: 7/25/95

Zal Usiskin:

I did not understand John Harvey's remarks regarding symbols and representations. Perhaps he would elucidate them.

But I think we will get off the track if we worry about whether symbols are representations or not. My point was that I prefer not to think about letters of the alphabet and words as representations. If one wishes to consider them as representations, then I believe mathematical symbols must also be considered as representations. But usually we do not think of letters and words as representations . . .

BUT sometimes we think of various ways of representing a concept, such as function. This is a broader use of the word "represent." Then virtually anything can be a representation. For instance, for functions we commonly teach algebraic representations (through equations or mapping rules), arithmetic representations (through lists of values or tables), geometric representations (through graphs).

Zalman Usiskin

I.47

Addressed Topics: content/procedures, application
Key Words: technology, calculators, algorithms, procedural knowledge, curriculum, problem solving, representations

Zal Usiskin:

Subject: Content & Applications/AES Reply to ABM/ZU
Date: 7/25/95

The question of skills is extremely complex. Last year I wrote a paper on this issue, entitled "Paper and Pencil Skills in a Calculator/Computer Age." It was published in UCSMP Newsletter No. 16 (Winter 1995). It is too long to reproduce here, but a quick and dirty summary may be useful for our discussion.

The use of a calculator or computer to get an answer is a skill just like the use of paper and pencil or the use of mental (spoken or otherwise unwritten) procedures. Each of these involves algorithms.

Five principles:
1. Technology changes the relative importance of algorithms. Some algorithms become more important, some less, and some do not change.
2. For a given task, there are three kinds of algorithms: those you do in your head, those you keep track of with paper and pencil, and those you do with technology.
3. No matter what algorithm is taught, students will process it in a variety of ways.
4. In order to use an algorithm, you must have the necessary tools for that algorithm and you must know how to use the tools to carry it out.
5. To be worth teaching, the purpose of the algorithm must be worthwhile.

Reasons for choosing one algorithm over another:
Power
Reliability
Accuracy
Speed
Provides a record
Provides a mental image
Instructiveness

Provides a proof
Used in later algorithms
Interesting object of study in its own right

Dangers inherent in all kinds of algorithms:
blind acceptance of results
overzealous application
belief that algorithms train the mind
helplessness if the technology for the algorithm is not available

Regarding the choice of paper-and-pencil algorithms (P) vs. calculator/computer algorithms (C), we have (logically) four choices: PC (teach both), P'C (teach calculator/computer only), PC' (teach paper-and-pencil only), and P'C'. One choice will not do for all the tasks in school mathematics. The tendency over time is a shift in the direction towards P'C. But history tells us that just as some mental work has survived the 500-year onslaught of paper-and-pencil algorithms, so some paper-and-pencil algorithms will survive in a calculator/computer age. Those that remain will be in our curriculum not because they are curiosities and not because they train the mind, but because they provide some of the qualities that good algorithms provide.

Allow me to finish by quoting the last paragraph of the paper: "These four facets of mathematics—procedures, reasoning, problem solving, and communication—remind us continually of the breadth and wide-ranging importance of mathematics. They are somewhat related to the four dimensions of understanding that we emphasize in UCSMP materials—skills, properties, uses, and representations. These categorizations reflect the belief that procedures are important but they constitute only a part of mathematics. Procedures are a means by which we solve problems, by which we explore and represent relationships, and through which we can explain to each other how we have arrived at conclusions. They are not the ends of mathematics, but mathematics cannot be done without them. Because of this, we need careful discussion of procedures, and I hope that my remarks today have contributed to that discussion."

Zalman Usiskin

I.48

Addressed Topics: sequencing
Key Words: curriculum

Subject: Sequence/ABM Response/ZU
Date: 7/25/95

Zal Usiskin:

In response to my statement about sequencing the curriculum, Alfred stated, “The principal question raised for me is how to select the dominant sequence to use in structuring a curriculum. It seems necessary, given the differences among the ways Zal listed, to choose one as the organizing basis and then try to incorporate as many of the others as possible at various times. A discussion about these points would be helpful.”

It may be useful to bring in a different concept, namely that of the “size of the curriculum.” Curriculum exists in at least five different sizes:

the episode, or individual problem
the lesson
the unit
the year
the entire schooling experience

Each of these sizes after the first can be thought of as a set of 6-24 of the previous sizes. For instance, a lesson may consist of 6-24 episodes, a unit consists of 6-24 lessons, etc. The specific numbers aren’t important, but it is interesting that there is some consistency, and what is important is that each size tends to be an order of magnitude larger than the previous.

Because of the difference in order of magnitude, what is a good general organizing principle for one size of curriculum may not necessarily be a good organizing principle for another. For instance, it may be quite reasonable to sequence a unit historically, but it would be madness to sequence the entire schooling experience that way.

All this is to say that I believe there is no single or simple answer to Alfred’s question.

Zalman Usiskin

I.49

Addressed Topics: sequencing
Key Words: technology, calculators, content knowledge, curriculum, problem solving, representations, algebra, geometry, precalculus, calculus

Subject: Sequence/Statement/jgh
Date: 7/25/95

John Harvey:

It seems to me that question (c) is the trickiest of them all.

It is clear that some sequence is necessary since we cannot teach everything simultaneously. But . . .

THERE IS NO UNIQUE SEQUENCE

In my opening statement I mentioned that during 1968-76, I participated in the developing of Developing Mathematical Processes (DMP), a K-6 mathematics program. For the past couple of years, Tom Romberg and I (along with others) have been working on a revision of DMP so as to bring it into the 1990's. In general, we identify units that embrace related sets of objectives or goals (e.g., Inventing Algorithms for the Addition and Subtraction of Whole Numbers) and should take two, three or four weeks to teach. Once these have been identified we consider them as blocks that can be moved around within the grade level on which we are working. You would not believe how many different arrangements of 10 blocks (potentially 100!, of course) make sense! Naturally there are some restrictions. For example, you don't put my example unit before one that, say, develops understanding of the concepts of addition and subtraction using manipulative and iconic representations. Nor do you put such a unit before one that develops knowledge of large numbers, including place value.

To me there are some big ideas that seem essential to the school mathematics program. Here is my list; it is probably incomplete:

1. Number
2. Variable
3. Function
4. Continuous mathematics
5. Discrete mathematics
6. Quantitative literacy
7. Informal and formal geometry

It seems to me that within each of these big ideas you can identify some of the things (i.e., units) that need to come first, second, . . . and that you can identify the dependencies of these units on units from the other big ideas. After that it is matter of "taste and style" and, possibly, of expediency (we have to teach so-and-so in Grade 3 because . . .). For example, some knowledge of number and of number relationships (for example, 1 x 3 = 3, 2 x 3 = 6, ...) is needed before one can introduce the idea of variable and some ideas about variable are needed before functions can be introduced. But there is no reason that number needs to be completely developed before variable or variable before function. I'm with Walter and Marj when they advocate that we need to develop "a web" instead of a sequence.

BUT WHAT ABOUT TEACHERS?

There is a clear difference between the designed curriculum, the published curriculum, the taught curriculum, the learned curriculum, and the tested curriculum. (Though I expect that teachers perceive that they test what they teach since I strongly subscribe to that old truism: What is tested is what is taught.) Webs may sound fine to us, but what about the teacher who on a day-by-day basis must teach students (whether as a sage or as a guide doesn't matter). My experience with the original DMP tells me that giving teachers a web and telling them to choose their own paths through it will not be very successful. In DMP for each grade level we gave teachers a directed graph showing the dependencies of units on one another and told them to choose a path. You can probably guess what they did instead; most of them started with unit 1. Then they taught unit 2, unit 3, etc. So, even if the web idea of curriculum development proves to be viable, we will have to give teachers a (very) small number of paths through the web that will work and tell them to choose the one they like. By giving teachers a list of paths we could control for some of the problems identified by Marj.

CONNECTIONS

One of the problems with the curriculum that has evolved since the New Math era is that it is too linear and too bounded. For example, there seems to be an assumption that students must have mastered (nearly) all of "number" before proceeding to "algebra." And that one can't teach geometry at the same time one is teaching about function or variable or whatever. My (incomplete) list of big ideas needs to be subdivided into units and those units arranged so that the seven ideas are intertwined and connected to each other. For example, there should be problems that can be solved equally well using continuous mathematics, discrete mathematics, and geometric representations. It is likely that technologies will help to break down the boundaries and make it possible to move away from linear development. At least this is what Frank Demana, Bert Waits, and I argue in a recent paper in the Journal of Mathematical Behavior (March 1995) about algebra, precalculus, and calculus. And I suspect that almost all of us has seen some of this merging as we have used graphing calculators to teach algebra or precalculus. As we assure ourselves that our kids are learning good mathematics with these technologies these barriers and paths will further erode.

AN APOLOGY

I know that I've added considerably to your mail in the last couple of days. And I apologize for making my statements so long. I guess I'm a windbag. I just hope that I've said something worth saying and that I have been too repetitive either within my statements or of your statements. For the moment, I'M DONE!

I.50

Addressed Topics: sequencing
Key Words: curriculum

Subject: Sequence/ZU Response to ABM/ABM
Date: 7/25/95

Alfred Manaster:

Zal's response was instructive and helpful. If I could narrow the question, it would be to ask about criteria for selecting a sequencing approach for at least a year's worth of mathematics, if not the high school curriculum.

Alfred

I.51

Addressed Topics: content/procedures, application
Key Words: calculators, motivation, problem solving, interdiscipline

Subject: Content and Applications/Statement/EJT
Date: 7/26/95

Liz Teles:

One of the content and applications areas about which I am very concerned is the relationships between applications and working closely with other disciplines to create interdisciplinary applications that can be used both in the mathematics classrooms and other classes. Here are a few questions and musings:

(a) What are exciting, interesting examples for some are totally unknown by others. It is very difficult to find applications which are truly meaningful to the whole class unless the applications are in some sense created within the classroom. Thus I support at times having students simulate work experiences and collect data and then work with applications that they have in some sense created. I know this does not work all the time, but mathematics as a "laboratory course" can be extremely motivating and exciting. (Two examples over the years that I thought would work that bombed when working with foreign students including asking "When will the balloon POP? being asked if that was POP as in 'Daddy' or 'soda'?" and giving a problem about bowling to find out at least half the class did not know the game at all.)

(b) To do problems and applications well, I think students truly need time for problem solving. Thus I think we need to consider methods that give problems of varying lengths during a course (5 minutes, 30 minutes, 1 hour, 3 hours, 1 to 5 days, 1 to 5 weeks, etc.) Creating problems that students can do in those time frames is a real challenge to the community. Problems that can be done alone, in teams of varying sizes, same problem for whole class with varying parts contributing to a whole, etc. is another challenge. In addition, what tools are needed and how will students assess those tools is something else to address.

(c) Problem-solving structures are also important. All students but particularly those in courses that are more traditionally for students entering technical careers (technicians as well as scientists and engineers) need to know trouble shooting, thinking, many methods for attaching, tool choosing, etc.

(d) Many examples of "mathematics" created by those without mathematics backgrounds can just be wrong. Just in the last week, I have seen problems created by other disciplines that define the tangent line as a "line that touches the graph one time and never touches it again." (A panelist from a discipline other than mathematics saw nothing wrong with that because "all the examples given fit that definition.")

(e) Many students find pure mathematics and games (like Tetris, Rubik's cube, etc.) really fun, exciting, and interesting. Are those applications?

(f) Finally, NYNEX has created a joint program with 15 community colleges for a telecommunications associate degree (in case you are wondering if students sign up for this program, if you are accepted, you get one day off a week to be in the program and a $250 a week raise in pay—9000 applications for 400 slots).Two conditions they put on were that mathematics was basic and should be taught using tools (computers and calculators) and that alternative pedagogical approaches including long-term problem solving be used.

I.52

Addressed Topics: assessment
Key Words: technology, constructivism, content knowledge, procedural knowledge, problem solving, calculus

Subject: Assess/LHB
Date: 7/26/95

Linda Boyd:

I agree with Anita that far too much time is still being spent on procedures, and I suggest that one of the main causes is assessment. Most of the presentations that I see at conferences are related to uses of technology or ways to have students work in groups. Very few of the sessions are devoted to assessment. As I look at assessment instruments constructed by my colleagues, I see a majority of questions dealing with procedures. I'm sure that most of us were assessed with these types of instruments and until faculty members are comfortable with other forms of assessment, I don't see much change in content, sequence, or applications.

In my own classes I now do the very things that Liz described in her content and applications statement. In the calculus I class this quarter students have completed 2 group projects, each of length 2.5 weeks (these with one other will comprise 1/4 of the grade). In one of the projects they investigated limits graphically, numerically, and analytically. The other project required that they construct a function model, approximate the maximum value of the function using graphs and tables, confirm their results using calculus, and then build a physical model. They have also turned in individual work for my comments only. At almost every class meeting groups are assigned parts of a problem and one member of the group presents the results at the board. The other students must evaluate the presentation. Almost all of these activities involve some use of technology, but most of the tasks could be done without it. The students select a method and are encouraged to use different methods to verify their results. I am constantly assessing these activities and formulating questions and activities to help them develop concepts. (I agree with John that students construct their own mathematical understandings. The reason they don't have understanding is that they have attempted to memorize procedures.)

Assessing these activities requires much more of my time than assessing procedural knowledge or skill only. It also requires that I behave in a different way in class. At first I thought that veteran teachers with more confidence would be willing to change they way they assess. Then I thought that new teachers (without the baggage of years of doing things a certain way) would be willing to use a variety of assessment strategies. Now I'm not sure, and at times I am discouraged. One of my students this quarter is a middle school teacher who is conducting workshops this summer for middle school teachers. She is facing the same problems and even more open resistance. I told her that all we can do is continue to lead by example. Change is coming, but it will not arrive tomorrow.

I.53

Addressed Topics: application
Key Words: problem solving, interdiscipline, calculus

Subject: Applications/LHB
Date: 7/26/95

Linda Boyd:

Good applications that are accessible to students are very hard to come by. That does not mean we shouldn't try to find them. And we should definitely share them when we find good ones.

On the other hand, I think we can do some very interesting things with some of the contrived applications we do have. Tonight I had my class examine the classic ladder problem in a different way. (I got the original idea from Ivan Niven.) For dx/dt=k, dy/dt is -k*x/y. I had the students tell me whether they thought they would be hurt if they were on the top of the ladder. Most of them had done the problem in the book and decided they would be OK. Then I had them make a table and assigned different students different parts of the table. After a while they all decided something was wrong with the model and we had a good discussion about the flaws. In the past I had let them do the problem in the book and I had shown them that there was a problem as y approached 0. I'm sure some of them appreciated it, but most did not. I am fairly sure that spending the extra time tonight helped most of them.

What Liz said about the dangers of having people without mathematical backgrounds construct applications is true, but we still need to get ideas from them. We are pairing science and business faculty with math faculty to write applications. I worked with a physics teacher. At first his problems were too simple for the level of calculus I needed for my students. We kept working until he found a problem that was at the correct level and I was able to help him state it properly. Also I showed him how to use Mathematica to draw the 3-dimensional graph. He was as thrilled as a child with a new toy, and I have a great application for multivariable calculus.

I.54

Addressed Topics: content/procedures, application
Key Words: problem solving, representations

Subject: Re: Content & Applic/Reply to ZU/jgh
Date: 7/27/95

John Harvey:

I don't want to sidetrack the discussion either, Zal. In one of your earlier statements you said that, to you, symbols weren't representations. I would remark that sometimes they are—and sometimes they're not. Long ago I bought into Bruner's representational scheme: physical ==> iconic ==> symbolic. We used it throughout DMP. Thus, to me when you have proceeded through this sequence and have arrived at a symbol or symbols to represent what you have been doing with manipulatives and pictures, that symbol is a representation and it has meaning. For example, suppose that students have been grouping or partitioning sets of objects using, say, Unifix cubes. If the set has 27 elements and it is grouped by 4's, then the child would tell you that he/she has 6 groups of 4 and 3 leftover. He or she would say the same thing after drawing rings around a pictured set of 27. And so, when the symbolic expression 4(6) + 3 is introduced, this will, in time, equally represent the solution of the problem: How many groups of 4 are there? And how many leftovers are there when you have a set with 27 elements?

But back to the main point. Let's not get off on this tangent. We want to talk about content and applications and not about what symbols mean to us.

Regards,
John Harvey

I.55

Addressed Topics: content/procedures, application
Key Words: algorithms, calculators, constructivism, problem solving

Subject: Re: Content & Applications/AES Reply to ABM/ZU/jgh reply to ZU
Date: 7/27/95

John Harvey:

I agree with what Zal is saying here. Being a constructivist I would like, in many places, to replace the word "teach" with the word "learn" or "construct." One place especially is point (5) where Zal says something about teaching an algorithm if and only if (?) it is worthwhile. If we use the learn or construct word, then it becomes "Students will construct algorithms that are worthwhile to them." Of course, this change in language doesn't mean that we can't introduce questions/problems/situations/application/ . . . that will make an algorithm worthwhile we want students to develop. Nor does it mean that after students have invented or tried to invent an algorithms we can't work with them to develop efficient, accurate algorithms that may resemble those we have. (I say "may resemble" because I continue to assert that our students are unlike us if they are permitted free use of calculators and computers while learning and doing mathematics.)

I agree strongly with Zal that mental computation (including estimation and approximation) and paper-and-pencil algorithms are going to survive. I like his description that mental computation has survived for 500 years even though we've had paper-and-pencil algorithms for at least that long.

Regards,
John Harvey

I.56

Addressed Topics: sequencing
Key Words: curriculum

Subject: Re: Sequence/ABM Response/ZU/jgh response to ZU
Date: 7/27/95

John Harvey:

Zal, I agree with your ideas about sequencing. But from your original list, I pay a great deal of attention to sequencing based on our knowledge of learning. Many, many years ago Pat Suppes suggested that our logical sequence of curriculum may be wrong even though it is the way we see that mathematics “goes together.” A recent example of starting where the child is and progressing through the curriculum in the way children’s thinking develops is Tom Carpenter’s model for the solving of addition and subtraction sentence. He pretty well nailed that one down. And he showed me that the way we had been doing things wasn’t the best way. This caused us to change the sequence in which we teach the solution of addition and subtraction sentences in DMP2 (the revision of which I spoke earlier).

Regards,
John Harvey

I.57

Addressed Topics: application
Key Words: NCTM, problem solving, calculus

Subject: Re: Applications/Reply to LHB/jgh
Date: 7/27/95

John Harvey:

I agree with you, Linda, that good applications are hard to find. However, it is worth looking at what some people have done and are doing. The NCTM volume on applications/modeling is a good one. The work from the North Carolina School of Science and Mathematics is worth considering. The MAA “calculus volumes” contain some good examples. I have been told repeatedly that the Harvard calculus books contain some interesting applications. Two of my favorites are Dan Teague’s elevator problem and Dan Teague’s/Tom Tucker’s irrigation problem. The toothpick problem is also a good one when it is placed in a good context. (The toothpick problem is in one of the NCTM Standards Addenda books.)

There are some good hints for applied problems in the last few TMC videotapes (QL, Modeling, Discrete Mathematics, Teaching Strategies). I don’t quite know how many people are participating in the conference, but I think I have enough copies of the two tapes (four shows) to send everyone a copy by “snail mail” if they don’t already have one.

Regards,
John Harvey

I.58

Addressed Topics: moderator
Key Words: none

Subject: E-Conf/Longevity/AJC
Date: 7/27/95

Dear colleagues:

The discussions in all four areas of the conference are going very well—thank you for all of your input.

Next week we are looking to conclude the electronic conference. Please include any provocative comments that you would like responses to as soon as possible. You may conclude your participation in the discussions any time between now and August 6th with closing statements regarding any of the four issues that are being discussed in our forum on mathematics education reform.

Upon the conclusion of all discussions, each panel member will again receive all the messages that were sent throughout the conference (at that time you can eliminate the many messages that you’ve been saving on your e-mail system). You will be asked to go over you contributions and make any changes that you deem necessary (including spelling, grammar, or content). The changes that you will make will go into the final transcript of the electronic conference.

At the conclusion you all will also be asked to evaluate this electronic conference (both the content and the procedures). All suggestions and comments will be highly valued!

Right after you receive this message you will get 5 messages that include the transcript of the entire electronic conference so far. Please use those messages to help finish the discussions in all four areas.

Thank you very much for all your efforts through out the conference!

Sincerely,
Alex

<u>I.59</u>

Addressed Topics: sequencing
Key Words: curriculum, geometry

Subject: Sequence/JH Response/ZU
Date: 7/28/95

Zal Usiskin:

It is impossible to disagree with John Harvey's comment, "of starting where the child is and progressing through the curriculum in the way children's thinking develops". The question is to find the questions that the child wants to ask at a particular time, because the thinking can be led to develop in a variety of ways.

As an example, in some of my curricular materials, 2 x 2 matrices are introduced to represent certain geometric transformations. That is, in general,

$$\begin{matrix} a & b \\ c & d \end{matrix}$$

stands for the transformation that maps (x,y) onto (ax + by, cx + dy). (I don't know how to put a matrix on e-mail. I hope this comes through!)

On the first day of introduction, we give some specific examples. That is, we graph polygons (they are 2 x n matrices) and find their images when multiplied by one of these matrices. Perhaps the matrix

$$\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$$

or the matrix

$$\begin{matrix} 3 & -2 \\ 4 & 5 \end{matrix}$$

is used as examples. Students can make up their own. At first, these matrices are just a curiosity, an application of matrix multiplication and they are so different that they are rather captivating. An immediate natural question arises in many (not all) students' minds: Which transformations have these matrices? Do any of the simple transformations students have dealt with (reflections, rotations, translations, size transformations) have these matrices?

One can go through reflections over the x-axis, y-axis, and the line x = y, and then size changes with center at the origin. Quickly (through multiplying these matrices) one gets to rotations with center at the origin and magnitude 90o, 180o, and 270o.

Now there is a second natural question. Are these the only rotations that can be represented by such matrices? The answer is: No. But in order to represent other rotations, we need to introduce cosines and sines. In our materials, we define (cos x, sin x) to be the image of (1,0) under a rotation of x about the origin. This is after students have seen sines and cosines as ratios of sides of right triangles. So there is now a question of whether these are the same sines and cosines. We go on to derive the matrix for a general rotation. There are lots of directions one can go to from here; we use different directions depending on the level of the student. Sometimes we stop and graph the sine and cosine functions. Sometimes we derive the formulas for cos(x+y) and sin(x+y); with this approach it takes only a couple of lines!

A question that arises for students during this discussion is how they can remember the matrices for the various transformations. And there is a nice theorem that can be discussed at this time: If (a,b) is the image of (1,0) and (c,d) is the image of (0,1) under a transformation with a 2 x 2 matrix, then the matrix is
a c
b d.

There are three points I wish to make about this. First of all, this sequence did not arise just because it is mathematically logical; there are many logical orders. It arose from actual teaching, from the first couple of times that we taught this content—when we ourselves were not sure of what should come next—from questions that students asked. Second, although I believe one could couch the sequence as moving from specific to general, of taking advantage of the student's curiosity to know about new stuff, it is not a sequence that could be predetermined from a psychological theory. Third, we would like to think that the teacher, by the choice of this sequence, is helping the student to think in particular ways—to think ["can] it be generalized?" When does it hold? When does it not hold? These are some of the questions that one hopes for in a sequence that follows a logical framework.

Zalman Usiskin

I.60

Addressed Topics: application
Key Words: problem solving

Subject: Applications/Reply to LHB/ZU
Date: 7/28/95

Zal Usiskin:

Let me add to John Harvey's list of sources for applications: almost anything done by COMAP, ranging from the book For All Practical Purposes to their UMAP and HIMAP modules, to the newsletter The Elementary Mathematician.

We'd like to think that in UCSMP texts we have a great number of applications, ranging from brief, one-line, straightforward questions to problems that require students to come up with mathematical models to fit and analyze data.

Zalman Usiskin

I.61

Addressed Topics: content/procedures, application
Key Words: abstraction, geometry

Subject: Content & Apply/ABM question about deduction/ZU
Date: 7/29/95

Zal Usiskin:

I am responding to Alfred's comments on my comments on his comments on . . .

There is no question that an abstract situation can be "cleaner" than a real one. But it does not necessarily follow that deduction within the situation is easier, because the need for deduction may not be as clear, and the "givens" of the situation may seem arbitrary.

For instance, suppose we wish to estimate the population of California in 2020. What is the given information? A choice has to be made. We could take only one data point (e.g., the population in 1990) and some growth rate. We could take that data point and a constant increase per year. We could take several data points and fit a line and then extrapolate. We could take several data points and fit an exponential curve and then extrapolate. In each case, the given information has been selected overtly and then deductions made from it. That input into the process of deduction is often missing from theoretical treatments.

So the complexity of the situation actually can be of assistance in teaching the student about deduction.

But we still need abstract deduction, because of the surety of the results it gives. When one is modeling in the population situation above, some students get the opinion that virtually any estimate could be made and justified for the 2020 population, so the mathematics has not helped at all! Number theory and synthetic geometry have wonderful advantages in this regard, because the results one obtains can be checked by calculation or by drawing.

Zal Usiskin

I.62

Addressed Topics: sequencing
Key Words: content knowledge, procedural knowledge, curriculum, arithmetic, algebra, geometry

Subject: Re: Sequence/ ME Musings and Interests/ABM
Date: 7/29/95

Alfred Manaster:

Marj Enneking's statements (17 Jul) about sequencing were very thought-provoking for me. I certainly agree that a student like her daughter should not be prohibited from learning mathematics because of difficulties with fractions. Still, this does not seem to me to require that we stop giving students the opportunity to learn how to manipulate fractions and understand rational arithmetic, in part in the context of proportional reasoning. The difficult question for me is finding a balance between emphasizing the importance of these understandings and procedural skills, thereby giving the greatest possible number of students the best possible chance to learn and develop proficiency, and so over-emphasizing them that they do become barriers for many students. This is a request for comments or insights about how to find such a balance.

Marj's "Musings" (19 Jul) and some of the other discussion about sequences has given me some helpful insights. Sequencing does not have to be the rigid categorizing of content in seemingly discrete topics (e.g., algebra, geometry, discrete mathematics) often for at least a semester. Instead sequencing might mean understanding which—fairly detailed and specific—understandings, skills, and approaches need to come before others. Thus, the web of connections has some one-way edges. John Harvey suggested this perspective when he mentioned that he had used directed graphs to outline a curriculum. Of course, a problem then arose since teaching and learning are done in time, which is linear and sequential.

Finally, for now, Marj's concern about how a webbed curriculum, rather than a sequenced one, "would fit with assessment for other large-scale or political purposes" raises some questions about broad-scale curricula. It seems to me that another important issue comes from the mobility of our society. Each year many children move not only from one school to another, but from one district to another and often from one state to another. While a national curriculum seems unobtainable for a number of good reasons, including respect for the tremendous variety of ways that students learn, how do we resolve the competing needs of children who move often and the desire of some for effective national standards with the recognition that many approaches will often be effective, but different ones for different students and different teachers? These seem to me to be other aspects of the sequencing issue.

Alfred

I.63

Addressed Topics: content/procedures
Key Words: constructivism, procedural knowledge, curriculum

Subject: Re: Content/Should Procedures Ever Be Directly Taught?/ABM
Date: 7/29/95

Alfred Manaster:

Several of the earlier comments suggest that students should essentially always construct the mathematical procedures that we want them to learn. This conclusion that teachers should almost never simply tell students what is true or how to proceed appears to be based in part on an overemphasis on "drill and kill" in recent curricula in this country. Another basis for this conclusion is our perception, aligned well with the constructivist philosophy, that students learn and understand more effectively when they are allowed to create their own knowledge.

Doesn't this conclusion represent an over-reaction? Shouldn't we help students learn how to take advantage of knowledge that our predecessors developed? Indeed, isn't one of the benefits of being human the ability to benefit from the knowledge of others and then build upon it to create better understanding and new knowledge? Isn't it too hard for each individual to reconstruct all the (even relevant) discoveries of the past?

Pretty clearly, I have overstated my point. In spite of this, I think it would be helpful if more moderate versions of these questions could be addressed in this discussion. How do we balance the desire to let students build deep understanding through constructing it themselves with the goal of efficiently advancing their knowledge by using observations of others? Indeed, is part of learning how to learn from others?

Alfred

I.64

Addressed Topics: content/procedures, sequencing

Key Words: content knowledge, curriculum, motivation, problem solving, arithmetic, algebra

Subject: content & sequence/comments/md
Date: 7/30/95

Margaret DeArmond:

I apologize for not participating in this conference as I had planned. I have been on the road most of this month. Since it has been quite awhile since I said hello, let me remind you that I teach high school mathematics and that I am interested in reforming high school curriculum, assessment, and instruction.

I was most interested in Zal's list of ways to sequence the curriculum. I feel that many (or most) teachers believe there is only one way to sequence the mathematics curriculum—that is the way that they were presented the mathematics. As Zal stated, this method is the "historic" approach (whole numbers before fractions, arithmetic before algebra, etc.). Although I feel that any curriculum should have structure, I also think that the curriculum organized around problem solving will do more to motivate students to learn mathematics. Ask any teacher what is the most common question they are asked by their mathematics students and they will undoubtedly say "What are we ever going to use this stuff for?" When students are presented large, interesting problems first and then presented a need to study the mathematics necessary to solve them, the students are highly motivated and interested. I would argue that Zal's "problem-oriented" method of sequencing the curriculum should take top priority.

It is also true that we live in a very mobile society. Students do move from school to school. I have often wondered why it seems to be only the mathematics teachers that worry so about this issue. What if a student had just studied Hamlet and then transfers to another school that is just beginning the unit on Hamlet? Again, our problem is the view that mathematics is only a subject of sequential steps and a hierarchy of topics. How could we open the curriculum to accommodate for these issues? What are the major concepts and strands (areas) of mathematics that all students should study? I hope all teachers will begin to get involved in this discussion.

Margaret DeArmond

I.65

Addressed Topics: content/procedures
Key Words: algorithms, constructivism, content knowledge, procedural knowledge, curriculum, problem solving

Subject: Re: Content/ABM question about constructivism/ZU
Date: 7/31/95

Zal Usiskin:

I agree with Alfred that the notion that students should construct everything is an overreaction. And I think he has not at all overstated his point.

I have neither the time nor the space to respond to this issue in detail. Here are just a few comments.

The movement for constructivism, which, as you will see, I think, is based on some questionable tenets, has had at least one very positive outcome. It has focused our attention once again on the importance of the learner in learning, and on the importance of active learning. This last point is not new; it has always been advantageous for a child to be involved in learning, and this is a fine goal. Learning can be active with or without a constructivist perspective. But in our concentration on newer curricular topics and on the roles of the school and teacher in setting up an environment for learning, it is important to pay attention to the roles of the student.

We have been through this before. In the 1960s, there was a strong movement for teaching and learning "by discovery" in mathematics. At that time, the advocates realized that there was a difference between "pure discovery" and "guided discovery." Very little discovery is pure; it is almost all guided. It was found that discovery teaching was quite helpful for the learning of concepts but not as helpful for skills. It was also found that it took a great deal of time to learn by discovery, which had practical consequences in that teachers felt they could not teach everything this way.

The basic tenets of constructivism—e.g., that every child learns an idea differently, that knowledge exists because of connections within each person's brain—are of course true. (Some constructivists call these tenets trivial, but they are used as the essence of the argument for having children construct virtually all knowledge.) But there is knowledge out there that is inaccessible to a child without a child being instructed. Clinton is President; Bosnia is in disarray; the volume of a sphere; the quadratic formula; Earth goes around the sun in an elliptical orbit; and so on.

Among some of the espousers of constructivism, there seems to be the notion that if a child discusses a mathematical concept with a classmate, then it is constructed knowledge, but if a child discusses the concept with the teacher, then it is not constructed. This is illogical. The constructivist paradigm is also rooted in a strange paradox about the abilities of children: children are considered quite capable of constructing a great deal of the knowledge that they need, but they are considered quite incapable of learning from someone else, particularly if that someone else is a teacher!

While it is clear that many students have not learned mathematics well in traditional classrooms, it is also clear that some have learned mathematics well under these conditions. In the zeal towards exploring new classroom arrangements, there has been a (willful?) neglect of the traditional conditions that have led to success. These successes, perhaps more than any other factor, keep teachers teaching the way they do, and reluctant to change. And because the successes occur in virtually every classroom of every teacher, they make instructional change hard to come by.

Not everything can be constructed in the sense of the constructivists. You cannot construct the history of a subject; you must read about it or be told. You can construct a definition, but you cannot construct a standard definition, because definitions are arbitrary. You cannot construct what results are significant and which are not; you can only determine significance after a great amount of experience. Only geniuses make conceptual leaps, and even then only after much study. Some algorithms in school mathematics have developed over centuries; we cannot expect students to construct them. Even Ramanujan studied books in detail.

Going one step further, from constructivism to radical constructivism, the radical constructivist position is in my opinion a philosophical position that is contrary to that put forth by virtually all doers of mathematics and all writers on mathematics. As I understand the position, it argues that mathematics has no objective reality; it exists only as it is constructed by the user and that the theorems of mathematics have no existence by themselves. That is, there is no one Pythagorean Theorem, but different versions of the theorem as we have constructed them ourselves. If this were the case, surely we would not have had generations of mathematicians trying to solve such problems as the four-color problem or Fermat's last or the trisection of an angle. I think most mathematicians believe they are discovering rather than constructing mathematics.

I have doctoral students from China. I put the following on the chalkboard one day: In dABC, tan A + tan B + ... We had never discussed anything like this result, but one of the Chinese students immediately finished the statement: In dABC, tan A + tan B + tan C = tan A o tan B o tan C. There is a huge body of knowledge we call mathematics. It is a worldwide enterprise. It exists out there and we learn about it in various types of activities: by reading books; by going to lectures; by listening and talking; by playing with the ideas. To suggest that one only "really" learns by the last two of these types of activities is simply not the case. And, as I have said above, it cannot be the case.

I believe that some (not all) of constructivism is rooted in a dangerous anti-intellectualism, a nihilism that denies the knowledge that has been developed by previous generations and our present one, a nihilism that denies that an adult might be able to transmit knowledge directly to a child, a nihilism that considers books as evil. For instance, I have heard teachers boast that they do not use a textbook. Is this really something to boast about? It is likely that some of the adherents do not realize the implications of the positions they have taken.

Consequently, Alfred, I think you understated your point.

Zalman Usiskin

I.66

Addressed Topics: assessment, content/procedures, sequencing, application
Key Words: technology, calculators, algorithms, NCTM, diagnostic testing, content knowledge, procedural knowledge, curriculum, performance tasks, abstraction, algebra, trigonometry

Subject: All/WFD
Date: 7/31/95

Walter Denham:

Participating in this forum (my first) has been quite an experience. It shows me that a number of people can, indeed, be in a collective conversation through e-mail. I suspect that at least some other participants, however, also feel a bit overwhelmed at the pace. I travel a lot and have, on the average, read the messages about every other day. By the time I've made a few notes for a response, more messages have come in, and I'm reluctant to refer back to specific messages from a few days before. I'd therefore hope that future forums would have a ten to fifteen week duration rather than just five. But even more time wouldn't cure the scope problem. The four original questions encompass a very wide range of mathematics teaching and learning issues. How can anyone synthesize or even summarize the forum contributions?

The following may be considered either my collected Phase II responses or my Phase III statements. The first week of August is quite full and I don't expect to be able to write much more before August 6.

The most gratifying aspect of reading the comments is to see so much serious and sensible reflection and commentary. I see a great deal of agreement about the issues imbedded in the four questions. Indeed, disagreements are either minor or subtle. What matters to me even more, of course, is that I agree with the great majority of the points made. In "court opinion" terms, I would be glad to add my signature to many of the statements. In a few places I would rather write a concurring opinion.

In education generally, and in mathematics in particular, we have what amounts to a large pretense. We talk about what students are supposed to learn, and we think and plan hard so we can teach (all of it) to them. We in the business know how little understanding or skill students at fifth, ninth, or fourteenth grade have, but we sort of keep it "in the family." We do this largely by using norm referenced measures for reporting results. We also let grading norms slide so that, no matter how poor the achievement, few students "fail." Grade point averages, in fact, have risen dramatically since 1960, but college entrance grade point requirements have barely changed. Many students with B's in mathematics are so weak they wind up taking high school (remedial) math again in college, even though "only" the top thirty per cent enroll in four year colleges. In other words, the eightieth percentile student is ill prepared for college. So who's kidding whom about Zal's average student? [His historical explanation on that point is one of his particularly salient, and he has an endless supply.]

A couple of months ago I reviewed the entire multiple-choice test bank in mathematics for secondary mathematics credential candidates in California. What struck me is that most of the items could be done correctly by an entering college freshman who was in pretty fair shape against the NCTM Standards. (These candidates are supposed to be comparable the college graduates with at least a strong minor in mathematics.) The obvious inference is that either we're not serious about our expectations for entering freshmen, or we don't expect much net gain in mathematics knowledge from ten to twelve college math courses.

The post script to my observation at the time is that an esteemed university colleague I spoke with thought it might be reasonable—or at least accurate—to have such a small "added value" (my term) for the college course work. I taught university mathematics for five years some time ago, and I can testify that student understanding is generally dismal, but I know even better after my years with K-12 teachers that the stated expectations for high school students (and earlier) are not real expectations.

This relates to the origin of the forum in its significance for "diagnostic" testing. It's one thing to talk about relative strengths and weaknesses. It would quite another thing to talk about "absolute" capabilities, or the student's meeting of performance standards, if you will.

At least three of you made some mention of big ideas, and John Harvey provided a fine list of big ideas with very large grain size. We'd have little problem agreeing, in substance at least, to a list of fifteen or twenty of medium grain size. Not to be disrespectful, but so what? What would we be able to do after we hammered out the wording more than we are able to do now? We don't have disagreements about the big ideas; we have some disagreements about the relative emphases, or the instructional approaches, or the assortment of tests to be used. But in this forum, the differences are small. And what we share strongly is frustration or dismay at how little net change, at least in student learning, we have been able to effect, even as we have come to understand more, and to have more promising curricula.

Alfred and I are members of a committee working on the next edition of the joint (CA) university systems' Competencies Expected of Entering (College) Freshmen. We've discussed at great length what particular "unaided" skills should be expected. We are in essentially the same place as those of you who have spoken most directly to this question. Anita, for example, "leans toward" the alternative of wanting students "to be able to do the simple computations by hand and leave the hard ones for the technology." Our committee has identified solving simple linear equations and multiplying powers as skills entering freshmen should have down cold. Analogous to fifth graders being automatic with the times table, and with multiplying or dividing by powers of ten.

In the rest of "Apply/AES reply to ABM/AES," Anita notes that "some day we may think it is silly to teach long division by hand." That day has already come! (The same thing for trig substitution.) And while I find "silly" a highly appropriate label, it's more important to note how counterproductive teaching the (or would some of you let the student develop and practice his/her own?) algorithm is. (I am ambivalent about teaching the area model of multiplying two digit numbers, because that seems so useful in later approaching (a + b)(c + d). But I see no advantage in a teacher-provided long division algorithm, and it uses valuable time and stifles interest. Worse, of course, proficiency demands produce defeatism and despair in students.) At the high school level I see little sense to having students trying to develop proficiency with rational polynomial expressions.

Procedures. Well, of course mathematics involves procedures. I urge all who haven't thought about it lately to look at the evaluation standards in the NCTM Curriculum and Evaluation Standards, especially the one about procedural knowledge. I wish that had gotten more play since 1989, because the authors worked hard to distinguish simple (or even complicated) mechanical skills from "procedural knowledge." It's too bad that so many (not in this forum, of course) continue to believe or suggest that procedures are the non-thinking part of mathematics.

The really big issue in elementary mathematics concerns proficiency with paper-and-pencil computational algorithms with multidigit numbers. It's a foolish objective, but very sensitive with the public. Broadly speaking, we have made little headway in describing our great expectations for number sense, including producing numerical results in a wide variety of situations, and including knowing how the accuracy and precision of calculation or measurement needed depends on the purpose at hand. We, at least in California, are still beleaguered by those who say "It [the traditional program] worked for me." The successful in this group, of course, don't rely on paper-and-pencil computations. [Yes, I know some of us do it because we have the skills and it's nostalgic at times, etc. But few adults have proficiency enough to rely on, and their children simply don't need proficiency with the paper-and-pencil algorithms we had to learn before calculators.] I don't work with college math enough these days to know the software, but I'm sure that no practicing engineers or scientists do elaborate algebraic manipulations. [I wonder what an ideal course involving ordinary differential equations is like these days.]

On "diagnosing," I most want to agree with Alba where she says, "There's a big difference in what is involved in diagnosing conceptual understanding, calculational skills, and facility with mathematical thought processes, . . . Without an image of a conceptual domain . . . we may find ourselves staring at a student's performance on a diagnostic test and not know what to make of it." As I suggested in my initial statement, a diagnostic assessment that is worth doing would have to involve assessment of the student's understanding, etc. To have a test that focuses primarily on routine computations is to, in effect, devalue the more important aspects of mathematical capability. I continue to reject the argument that tests of a limited range of mathematical knowledge are better than no tests because at least that allows us to rank students, or at least to get feedback on a part of the curriculum. Limited range feedback has a distorting influence on the teacher's adjustments in teaching, adding more emphasis on the area tested at the expense of the areas not tested.

Several have talked about the role of applications, in ways that helped my understanding. I do have two comments. First, I much prefer to talk about the contexts for mathematical tasks, rather than to use dichotomizing language that suggests one is either doing mathematics or doing applications. The word "applications" suggests a by-product, not of direct interest to mathematics learning. But as some of you discussed, learning mathematics in general depends on the student being engaged in a context he/she recognizes, and to some extent identifies with.

My second point responds to what some of you said about mathematics itself being a perfectly fine context. I certainly agree that many, perhaps most students will find some abstract mathematical questions sufficiently interesting that the "engagement" criterion is satisfactorily met. I believe, however, that the proportion of mathematics-itself-as-context should be clearly under a third.

On a related note, I came to see, over the last ten years, that there are far fewer real contexts that provide worthwhile mathematical experiences for children that I had hoped and expected. I've actually come to believe that a substantial majority of productive elementary tasks, and perhaps even of middle school tasks, will be "imaginary" or "contrived." I still have hopes that real contexts can predominate in high school.

I've enjoyed and appreciated almost all that's been said about sequencing. Thanks to Zal for describing five different "sizes" of curriculum. And in John Harvey's THERE IS NO UNIQUE SEQUENCE he notes that after identifying dependencies of units on big ideas preceding, that sequencing "is a matter of 'taste and style' and, possibly, of expediency." Regarding his concern about teachers making sequence choices, I believe that, in most cases, it is hopelessly romantic to expect teachers to create the flow (sequence) of the curriculum. That is done by curriculum developers. Providing a "directed graph" is exactly what the developer should be responsible for.

It's also not just a single year. I subscribe to the view that the quality of the instructional units matters much more than their topics or sequence. Still, on at least an empirical basis, I expect some collections of units, and some sequences, to be more effective generally than others, and I am glad that modern curriculum developers have paid so much attention. Rather than name a couple that I know about, let me just say that most of the NSF-sponsored developers have paid considerable attention to the flow of units, coming to see that many sequences are possible and that not all can be tried. There is an inevitable arbitrariness to a given fourth or seventh or tenth grade, but there should be a consciousness, a describable rationale, for the choices that are made.

Walter Denham

I.67

Addressed Topics: sequencing
Key Words: curriculum

Subject: Sequence/ABM on ME/WFD
Date: 7/31/95

Walter Denham:

Conditionally, I accept Alfred's statement that "the web of connections has some one-way edges." I can readily agree that if one has a set of five or ten or twenty units, that for several there would be a one-way relationship. But if one were saying that a set/web of mathematical ideas had one-way edges I'd disagree in most instances because mathematical ideas are not learned/ mastered/nailed down at points in time, whereas each class is (we hope) a learning event. Ratio/rate understanding certainly develops over several years. Part/whole understanding perhaps begins earlier, but it is not finished when ratio begins. My overall point is that "web" is actually too discrete an image for the connected learnings of a student.

I.68

Addressed Topics: none
Key Words:

Subject: Catching up
Date: 7/31/95

Geoff Akst:

Back from two weeks of fun in Canada. Is there anything left to say?

Geoff

I.69

Addressed Topics: sequencing
Key Words: algorithms, calculators, content knowledge, curriculum, problem solving, representations, arithmetic, algebra, geometry, calculus

Subject: Re: content & sequence/md comments/zu
Date: 8/1/95

Zal Usiskin:

Reading Margaret's comments about sequence, I feel a need to clarify some of the comments I made about sequence.

I think it can be said that the K-12 curriculum has traditionally followed one of four ways to sequence.

The curriculum as a whole is not historically sequenced; if it were, geometry would come first, then arithmetic (Hindu-Arabic numerals came after Greek geometry), then algebra. Overall, the curriculum has a sort of developmental sequence, i.e., one based on psychological principles, where algebra and geometry are delayed because of the traditional view that most students cannot understand the formal aspects of these subjects earlier (a view with which many people now disagree).

The concepts of arithmetic have been sequenced historically (whole number, then fraction, then decimal, then negative number), but within these concepts the skills of arithmetic have been sequenced algorithmically. For example, we teach division of whole numbers after subtraction and multiplication because the long division algorithm requires that a student be able to subtract and multiply. With calculators, the algorithmic sequence can be changed, since, e.g., a student can get an answer to a division problem without doing either multiplication or subtraction.

The skills of algebra have also been sequenced algorithmically, based on paper and pencil algorithms. For instance, we have to solve linear sentences before quadratic sentences because the latter requires the former when done with paper and pencil. Again, if one has calculators, this could be changed.

In contrast, traditional geometry has been sequenced logically. This is how we can explain why the theorems about base angles of an isosceles triangle, which are rather obvious and not particularly important in the long run of things, come early in geometry, whereas the Pythagorean theorem comes late.

Problem solving has been used as an organizer, but usually not for more than a day or two at a time. For instance, a 7th-grade course might have a day or two on problems involving simple interest. And there has seldom been any connection from one year to the next. For instance, students are typically given "mixture problems" involving percentages in algebra without ever being given the corresponding arithmetic questions in earlier years.

I do not know of a curriculum of more than a year's length in any other country that is based on problems. The problem is not movement from one school to another; as Margaret pointed out, students are quite malleable and the adjustment would take place. The problem is efficiency and interest. Children need new concepts every year both to maintain their interest and to enable them to grow. Problems have not (yet) been analyzed in enough detail to give us a sequence that suggests which ones come first and how we build from one to the next.

In this regard, one has to view some of the newer NSF curricula based on problem-solving units as research enterprises, studying whether in fact multi-year curricula can be structured in this way.

From 1979-1983, Max Bell and I worked on an alternate structure for arithmetic based on meanings of number and operation. We think it has some value, and to some extent it is now found in a few sets of materials (the UCSMP elementary curriculum, the UCSMP secondary curriculum, the Scott Foresman elementary curriculum). The significance of this in relation to Margaret's remarks is that it constituted an attempt to create a framework for the arithmetic curriculum that would suggest how to sequence real-world problems.

The work was never published commercially. The structure is described in three volumes with a total of over 500 pages, only available through ERIC as ED 264 087, ED 264 088, ED 264 089. Given its length, it is impossible to even summarize it here. But here is an example.

One of the use meanings of subtraction is comparison. (This is currently taught early, perhaps as early as 1st or 2nd grade, but most teachers do not spend as much time on it as they do on the take-away use meaning.) A special case of comparison is change, which can be taught almost from the start. (Change is typically not taught explicitly.) One of the use meanings of division is rate (now just beginning to enter the curriculum as a topic). This could surely be taught in the middle elementary grades. After these have been taught, then it is a natural thing to discuss rate of change. That naturally leads to the concept of slope, found in algebra. (In today's curriculum, slope is sometimes taught without mention of these previous ideas. Thus students have no idea why the slope formula involves subtraction and division.) After the slope of a line has been taught, one can extend this to the slope function for a curve, i.e., the derivative, and later to the partial derivative, as one usually sees in calculus. At each stage, there are appropriate problems to be asked.

A second example: The first and most basic use meaning for addition is "putting together". One context for putting together is with lengths; i.e., the length of the segment formed by putting together two segments end-to-end is the sum of the individual lengths of the segments. This is obvious to us, and easy for most students with arithmetic, but when students get to algebra they sometimes think that if a segment with length 3 is next to a segment with length x, the total length is 3x. A basic use meaning for multiplication is area of a rectangle. The traditional representation of the distributive property using areas of rectangles can be interpreted as the natural consequence of combining these two use meanings. The representation of multiplication of binomials (and polynomials, more generally) by areas of rectangles follows. Thus a representation that seems to many students to come out of the blue can be viewed as something that follows from basic principles.

Zalman Usiskin

I.70

Addressed Topics: content/procedures
Key Words: constructivism, content knowledge, procedural knowledge, problem solving, representations

Subject: Content/Zal re Constructivism/WFD
Date: 8/1/95

Walter Denham:

I doubt anyone would dispute that "you cannot construct the history of a subject; you must read about it or be told. You can construct a definition, but you cannot construct a standard definition, because definitions are arbitrary. You cannot construct what results are significant and which are not; you can only determine significance after a great amount of experience." The practical issue for high school and college teachers is the degree to which (or just which) concepts or procedures—in contrast to history and definitions and useful representational forms—should be presented (lecturing) by the teacher, as opposed to giving the students more or less proven problem-solving assignments in which they will encounter . . . etc. Zal, perhaps unintentionally, sidesteps this issue when he talks of the teacher "discussing" with the student. I'd say that it's inevitable and desirable for teachers to discuss a little with individual students, but that the great bulk of discussion, if it is discussion, has to be among students. Having 30 students listen while the teacher "discusses" with one student is not the way to go.

Then there's the argument that some students have learned mathematics well in traditional (lecture style) programs. I grant that, although it's a pitifully small proportion. I'm confident that the great majority of those would have learned more mathematics in classrooms with a teacher who understood constructivism well.

I.71

Addressed Topics: content/procedures
Key Words: geometry

Subject: Content/ZU reply to WD/ZU
Date: 8/2/95

Zal Usiskin:

Walter seems to be reading the word "discussion" as if it is the word "lecture." Let us not compare a theoretical best of a new idea with a practical worst of an old; it is not fair to either.

Exactly what is the "pitifully small proportion" who learned well under traditional teaching is not clear. One could probably assume that future engineering science, and mathematics majors of the past learned well. That gives a low end value. In Senk's 1981 study of proof competence after a year of geometry, the fraction of all students (at the grade level) who were deemed to be successful was about 1/6, which is about in line with the proportion that schools put into "honors" classes. (Which came first, the honors classes or the proportion, is an interesting question.) Whatever the exact value, the proportion is large enough not to be ignored.

Zalman Usiskin

I.72

Addressed Topics: content/procedures
Key Words: geometry

Subject: Content(?)/ZU re WD/WD
Date: 8/2/95

Walter Denham:

Zal, I said you sidestepped the lecturing issue by using the word discuss. I then commented on discussing, which I did not imply was like lecturing.

On the proportion of successful students, you note "In Senk's 1981 study of proof competence after a year of geometry, the fraction of all students (at the grade level) who were deemed to be successful was about 1/6, which is about in line with the proportion that schools put into 'honors' classes." The key phrase is "deemed to be." In my long commentary a few days ago I argued that the actual performance capabilities of students has been overstated for years now. There aren't 1/6 of the students in really good shape at the end of high school. One-sixth, in fact, is the fraction of HS graduates who do not have to take remedial high school math when they enter college in CA, i.e., over 40% of the top 30% of our HS graduate do have to take remedial courses. (Yes, I know that our achievement is a bit below other states', but not dramatically so.) It's not that the Passing level on the state university system's Entry Level Mathematics test is set so high, either.

Although there is precious little information at the high school level, we have consistent data from Vermont, Kentucky, California, even from NAEP, that the percentage of "proficient," not to be equated with honors, students at eighth grade is ten to twelve. How can you say that 1/6 of the students are qualified for "honors" courses?

I wonder what those of you who teach freshmen at the selective schools, like the Univ. of CA, believe the mathematical competence is of the median freshmen. Only the top 1/7 of our graduates are admitted to UC.

I.73

Addressed Topics: none
Key Words: none

Subject: Sign off for JP
Date: 8/2/95

Jack Price:

I am going to have to leave the discussion. I've just run out of time. The last pull down resulted in 125 pages of printed materials. I have 53 messages unread. Sorry. It was interesting but too time consuming. Jack

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I.74

Addressed Topics: none
Key Words: none

Subject: good bye
Date: 8/2/95

Anita Solow:

I have found this electronic conference interesting, but frustrating.

Every time I log on, I find numerous messages. By the time that I read them and formulate a response, the discussion has moved on to a new topic.

I can't help thinking that this would have been more interesting in person. But I am glad that I was part of this conference.

I leave tomorrow, so this is good-bye.

Anita Solow

I.75

Addressed Topics: none
Key Words: none

Subject: Re: good bye
Date: 8/3/95

John Harvey:

I think that the conference would have not been so frustrating to you, Anita, had it been conducted at a more leisurely pace.

Two or three (or more?) years ago I participated in a teleconference conducted by the MAA. We did that conference over a three-month period, as I recall. It was much easier to keep up there.

Have a good time in Burlington. I assume that's where you are
going.

Regards,
John

I.76

Addressed Topics: moderator
Key Words: none

Subject: EConf/Procedure/JRN
Date: 8/4/95

Greetings all,

It looks as if our conference is winding to a close. Alex and I would like to thank you all for your participation and for the stimulating and informative discussions that resulted. I would like to invite you to submit any final statements on the various topic areas, and we would like to get those statements by the end of next week.

We will be leaving the list server active, and if any of you would like to continue to discuss these or any other topics related to mathematics education, please feel free to use this avenue. We would also like your input regarding the question of opening up this list server to the general mathematics education community.

Once again, thank you very much for your participation and input.

John Novak

I.77

Addressed Topics: none
Key Words: none

Subject: Re: E-Conf/Procedure/AJC
Date: 8/8/95

Gail Burrill:

Sorry that I was actually unable to participate in the dialogue. As I stated in the one message I was able to get out, my e-mail was not cooperating—and is still not fixed. The marvels of electronic communication!

I do have some observations, however, about the procedure, some of which I notice were shared by others.

i. The initial tasks seemed in retrospect to be too many and too broad. Thus, the conversation grew exponentially but not focused on any one aspect. Perhaps it would be better to do a sequence of discussions on a specific topic rather than all at once.

ii. The time span was short and the amount of reading enormous. I found it hard to concentrate on all of the material. It did not seem to be coming to any consensus??

iii. The appearance of new people, while great because it gave new perspectives, added to the overwhelming amount of information and to the difficulty of seeing where the discussion was (and /or should be) going to be productive.

Thanks for giving me the opportunity to participate, even though I was primarily a bystander.

Gail Burrill
