Dimensionality of NAEP Subscale Scores in Mathematics

CSE Technical Report 428

Jamal Abedi
CRESST/University of California, Los Angeles

August 1997

Center for the Study of Evaluation
National Center for Research on Evaluation, Standards, and Student Testing
Graduate School of Education & Information Studies
University of California, Los Angeles
Los Angeles, CA 90024-1522
(310) 206-1532

The work reported herein was supported in part under the National Center for Education Statistics Contract No. RS90159001 as administered by the U.S. Department of Education, Office of Educational Research and Improvement.

CONTENTS

DIMENSIONALITY OF NAEP SUBSCALE SCORES IN MATHEMATICS[1]

Jamal Abedi
CRESST/University of California, Los Angeles

Introduction and Purpose

The issue of dimensionality is an important consideration in the National Assessment of Educational Progress (NAEP) because it affects the administration, scoring, data analyses and reporting of the results. The subject matter areas assessed by NAEP are usually analyzed by content and process. For example, the reading assessment consists of three content areas—Information Text, Literary Text, and Documents—and two process areas—Constructs Meaning and Extends Meaning (see the NAEP 1990 Technical Report, Johnson & Allen, 1992, pp. 33-37). Similarly, the mathematics assessment framework consists of five content areas—Numbers and Operations; Measurement; Geometry; Data Analysis, Statistics and Probability; and Algebra and Functions—and three process areas—Problem Solving, Procedural Knowledge, and Conceptual Understanding (see the NAEP 1990 Technical Report, Johnson & Allen, 1992, p. 40). The scores in math, reading and science are reported at the subscale levels for Grades 4, 8, and 12.

Recent studies performed on the dimensionality of math, science and reading in NAEP (see, for example, Allen, 1992; Carlson & Jirele, 1992; Rock, 1991; Zwick, 1987) have shown that the subscale scores in the three curricular subject areas are highly correlated. These high correlations between the subscales could have implications for curriculum planning, teaching, and the reporting of the students' achievement scores in these subject areas.

For example, Rock (1991) in his study on NAEP math subscale dimensionality found very high correlations between the five subscales in math, which indicated a unidimensional trend at the subscale level. Based on his results, he concluded that "we are doing little damage in using composite scores in

---

[1] NAEP Technical Review Panel (TRP) Achievement Dimensionality Study: Report for Section A.

mathematics and science" (p. 2). Zwick (1987), in her study on NAEP reading items, concluded that "reporting several reading subscales would not add important information to reporting one reading scale" (p. 168).

However, what Rock and many others do not take into account is that in educational settings the environments consist of heterogeneous groups of individuals. Students' performance on NAEP assessments in math, science, reading, writing and other subject matter areas may be affected by instructional and background variables (see, for example, Muthén 1988, 1989a, 1989b; Muthén, Kao, & Burstein, 1991).

The Research Question

The main question in testing, scoring, and reporting mathematics test results in NAEP is whether the five subscales measure the same underlying math ability or whether they measure five different subject areas in mathematics. If the subscale items are measuring a general mathematics factor, as suggested by some of the studies done in this area, then one may not need to test and report by subscales; a test with general math questions may serve the purpose. However, if students' performance varies across instructional and background variables, then one should pay more attention to such variables when dealing with math test scores.

The Study

The dimensionality of math subscale scores was examined in two concurrent studies conducted at the National Center for Research on Evaluation, Standards, and Student Testing (CRESST) at UCLA. This paper reports the findings of Section A of that study. The data for Section A were obtained from the NAEP main administrations of 1990 and 1992. The analyses were done for the 1990 and 1992 data sets separately.

For the 1990 data, multiple discriminant analysis technique was used to identify those background variables that significantly discriminated groups of subjects on their math subscale scores. The background variables from the 1990 main administration were used as grouping variables and the math subscale scores as discriminating variables. The background variables that had significant effects on the math subscale scores were identified, and subgroups were formed based on the level of those background variables. Simple structure confirmatory factor analysis (FA) was performed on the math subscale scores, and the correlations between the subscale latent variables were compared across subgroups. To create subscale latent variables, item parcels were prepared. These item parcels consisted of items which were homogeneous with respect to their difficulty level and intercorrelations (see Cattell, 1956a, 1956b; Cattell & Burdsal, 1975, Cook, Dorans, Eignor, & Petersen, 1983). Two item parcels were constructed for each of the subscales and were used to create the subscale latent variable. The results of these analyses, which included correlations between the latent variables and indices of goodness of fit, were reported separately for each of the subgroups that were formed based on the level of the selected background variables.

In addition to structural models with five latent variables (for the five math subscales), the assumption of one general math factor was also tested. In a series of analyses, the math subscale item parcels were used to create one general factor based on the assumption that all items under different math subscales measure a general math ability. The indices of fit obtained from these analyses were then compared across subgroups. Furthermore, models with five subscale scores and one general math score were created. These models' indices of fit were compared with those of models that assume only one general factor and with those of models that assume five subscale factors and no general factor. The comparison of factor means and factor variances across subgroups of students formed by levels of background variables using multiple group factor analysis was also going to be examined. However, this type of analysis was to be performed in the sister dimensionality study being conducted at CRESST by a second team of researchers; therefore, it was not included in this study's methodology.

All of the analyses discussed above were performed on all of the math items in each subscale. Some of the math items were also selected based on their relationships with the background variables. Computer software, (Multi-Approach Correlation System [MACS]; Abedi, 1993) was specifically developed for this purpose. The correlation between each of the background variables with each of the math test items was computed using the appropriate techniques. Item parcels were then created from these items. The same analyses that were conducted on the parcels with the complete sets of items were performed on the parcels consisting of selected items, and the results were compared across the subgroups.

Because of the BIB spiraling nature of the NAEP data, analyses were conducted at the booklet level (see Beaton, Johnson, & Ferris, 1987; Carlson & Jirele, 1992; Zwick, 1987). For the 1990 data, there were 10 booklets. Analyses were performed on Booklets 8, 9 and 10, and because the test booklets were spiraled, the results of the analyses conducted on Booklets 8, 9 and 10 were considered replications and were used to cross-validate the results.

For the 1992 data, however, the structure of the test items in the booklets was different. There were more booklets in the 1992 administration than the 1990 administration—26 booklets for 1992 as compared with 10 for 1990. Consequently, in the 1992 administration there were fewer students answering items from the same booklet than in the 1990 administration. As a result, the students could not be divided into subgroups based on the background variables, especially those variables with more than two levels.

For the 1992 data, analyses were performed on the item parcels that were formulated based on the items that manifested higher correlations with the background variables. The same models that were used for the 1990 data were created for the 1992 data. Models with five subscale latent-variables were created as well as models with one general math latent-variable and models with five subscales and one general math latent-variable. Indices of fit were compared across all of these models.

Literature Review

This review summarizes the related literature in dimensionality. It also discusses the issue of dimensionality in NAEP curricular subject matters and describes a summary of studies conducted in an effort to clarify this issue. The purpose of the literature review is to provide a rationale for undertaking the present study.

The related literature is summarized into four different sections. In the first section, the concepts of dimensionality in educational testing are discussed. The second section describes the techniques and procedures used for assessing test items and subscale dimensionality in general. The third section summarizes the techniques/procedures used for assessing the dimensionality of NAEP test items in particular; and the last section describes the summary of results of the dimensionality studies in NAEP.

Assessing Dimensionality of Achievement Tests

The dimensionality of achievement tests is an important issue to educational assessment because it is the underlying assumption that many measurement techniques are based on (Cook et al., 1983; Hambleton & Rovinelli, 1986; Zwick, 1985). Zwick (1985) in her report summarizing the results of a dimensionality study conducted on the NAEP reading items indicated that “it was important to investigate the dimensionality issue because the validity of the item response theory (IRT) model used to estimate reading proficiency in the 1983-1984 NAEP survey rests on the assumption of unidimensionality” (p. 1).

A set of items is considered unidimensional if a single latent trait underlies the data. Hattie (1984) explains that unidimensionality

> is not defined in terms of unit rank, percentage of variance explained by the first component or factor, deviations from a perfect scale, the type of correlation, or the number of common factors. Although these have been used as methods to determine unidimensionality, they do not define it. A unidimensional test is not necessarily reliable, internally consistent, or homogeneous. Indeed a unidimensional test may be factorially complex in terms of the linear common-factor model. While the principle of local independence is fundamental to the definition of latent traits and therefore to the definition of dimensionality, it is not synonymous with dimensionality. (p. 50)

Perhaps the concept of unidimensionality has become the most apparent in item response theory. Unidimensionality has been identified as one of the most important assumptions underlying IRT (Cook et al., 1983; Zwick, 1985). Cook et al. (1983) have indicated that if the first-order factor variance (which is an indication of a general factor) is large, the data are unidimensional. On the other hand, they have also attested that a relatively large group factor would indicate a violation of unidimensionality. Zwick (1985) points out that, in practice, the unidimensionality assumption in IRT is always violated to some degree, and she stresses the need for more studies on the robustness of IRT estimation procedures to violations of the unidimensionality assumption.

Bejar (1980) discusses the issue of dimensionality based on content area. He contends that when a measure is based on the total-test concept, then the entire latent space is unidimensional and any other sources of variability are considered by definition "error." However, if the measure is constructed based on content-area, then the latent space is multidimensional.

Hambleton and Rovinelli (1986) maintain that despite the importance of the unidimensionality assumption in the currently popular item response model, there is confusion and controversy regarding the definition of dimensionality and the method for assessing the dimensionality of a set of test items. They cite, for example, a typical definition of dimensionality as it appears today in current psychometric literature. This particular definition—"a set of test items is unidimensional when a single trait can explain or account for examinee test performance"—is then referred to as "abstract and non-operational" (p. 287).

Procedures for Assessing Dimensionality in General

The concept of dimensionality in educational assessment has not been defined clearly in the literature. This lack of definitive clarity has caused problems in assessing indices of dimensionality. Jones, Sabera, and Trosset (1987) discuss the issues involved in the dimensionality of tests and conclude that there is neither a single technique nor a group of techniques most appropriate for assessing the dimensionality of a set of items. They refer to Lord (1980), who emphasized the need for a commonly acceptable test(s) for assessing dimensionality. Hattie (1985) pointed out that "unidimensionality has been confused and used interchangeably with other terms such as reliability, internal consistency, and homogeneity" (p.157). Due to the complex nature of this concept, and because of the lack of a commonly acceptable definition of dimensionality, many different procedures have been suggested in the literature for assessing the dimensionality of a set of achievement test items. Hattie (1984) has identified 87 indices suggested for determining the unidimensionality of a measure (see Table 1 in Hattie, 1984, for complete data). In addition, he provides rationale for each of the procedures and discusses their weaknesses and strengths. Hattie (1985) categorized these procedures into several groups and whenever possible gave references to the studies that employed these approaches. At this point in the review, the different procedures suggested for assessing dimensionality will be described. This description will be based on the format and categorization used by Hattie (1985).

Indices based on answer patterns. Under this category, Hattie briefly describes Guttman's reproducibility coefficient, which provides a method for testing a series of qualitative items for unidimensionality (Guttman 1944, 1950; Hattie, 1985). This index may be affected by the level of item difficulty. There are some approximations to Guttman's approach in the literature. For example, Jackson (1949) proposed the Plus Percentage Ratio (PPR) coefficient, which is not affected by item difficulties, and Green (1956) suggested a formula that is less time-consuming to compute.

Loevinger (1944) suggested an approach known as the index of homogeneity. This coefficient is 1 for a perfectly homogeneous test and departs from unity as the items within a test become more heterogeneous. Hattie (1985) comments on these indices and discusses the major criticisms of these approaches. Among the criticisms is Lumsden's (1959) remark, "These methods can only achieve their upper bounds if the strong assumption of scalability (i.e., a perfect scale) is made" (Lumsden, cited in Hattie, 1985, p. 143).

Indices based on reliability. The coefficient alpha is referred to in this category as one of the most widely used indexes of unidimensionality. However, the literature points out the various problems that can arise when one interprets Cronbach's alpha as an index of internal consistency or as an indication of the unidimensionality of a set of items. Cortina (1993) demonstrated that alpha is affected by (a) the number of items in the test; (b) the average item intercorrelation, and (c) the dimensionality of the items. He suggests using factor analysis techniques to assure that there is no large departure from unidimensionality. McDonald (1981) discusses the use of coefficient alpha as an index of reliability or dimensionality; he believes that this coefficient cannot be used as a reliability coefficient or as a coefficient of generalizability, nor can it be used as a criterion for assessing dimensionality. He indicated that "indeed, one might almost put the extreme view that alpha has not been shown to be a quantitative measure of any intelligible and useful psychometric concept, except when computed from items with equal covariances" (p. 111). Green, Lissitz, and Mulaik (1977) observed that a high internal consistency indexed by a high alpha maybe due to a general factor underlying the items but that it may not always be an indication of a general factor (see Hattie, 1985, p. 144). In their Monte Carlo simulation study, Green et al. (1977) concluded that "the chief defect of alpha as an index of dimensionality is its tendency to increase as the number of items increase" (Green et al., cited in Hattie, 1985, p. 144). Green et al. (1977) also found that the average inter-item correlation suggested by Cronbach to overcome the problem of effects of test length is influenced by the communalities of the items and by negative inter-item correlations. Hattie (1985) concludes that "despite its common usage as an index of unidimensionality, alpha is extremely suspect" (cited in Hattie, 1985, p. 145).

Also under this category of reliability indices, Hattie discusses "Index/Index-Max Formulas." These formulas are basically modifications of alpha. Among these formulas are Loevinger's (1944) *H* ratio and Horst's (1953) reconceptualization of Loevinger's index.

Indices based on correcting for the number of items. Because alpha is dependent on the length of a test, other indices of inter-item correlation or test homogeneity have been developed that claim to be independent of test length. Cronbach (1951) suggested estimating the mean correlation between items by applying the Spearman Brown formula to the alpha for the total test (Hattie, 1985). Armor (1974) suggested assessing the number of intercorrelations close to zero as a possibility for determining the number of dimensions in a test.

Indices based on principal components. Some researchers have used principal components analysis (PCA) to assess the dimensionality of a set of items or subscale scores. The idea is that if a large amount of variance is explained by the first component, then the set of items or subscales could be considered unidimensional. There are many questions and concerns regarding the appropriateness of the principal components technique as a tool for assessing the dimensionality of a set of items. Some of these concerns are about the applicability of PCA to dichotomously scored right or wrong (R/W) items; other concerns query the clearness of the criteria for judging unidimensionality based on the results of components analysis. We discuss these concerns in the following two sections.

1. Questions on the applicability of PCA to R/W items. Principal components analysis and factor analysis (FA) can be applied to a set of conditions in which each of the variables is a score on a multi-item test. However, when the principal components technique is applied to dichotomously scored items (phi or tetrachoric correlations), it may not produce valid information that could be used for judging the dimensionality of a set of items (see, for example, Bejar, 1980; Carroll, 1983; Cook & Eignor, 1984; Cook, Dorans, & Eignor, 1988; Hambleton &

Rovinelli, 1986; Hulin, Drasgow, & Parsons, 1983; Jones et al., 1987; McDonald & Ahlawat, 1974; Mislevy, 1986; Zwick, 1985).

When computing phi coefficient, one generally assumes that the items are truly dichotomous. However, this assumption causes major problems in computing component analysis and factor analysis for the matrices of phi coefficients. For example, the PCA or FA on the phi coefficients often produces a second factor that is related to item difficulty but has no relationship to any of the properties of the items (see, for example, Hambleton & Rovinelli, 1983; Lord & Novick, 1968; McDonald & Ahlawat, 1974; McDonald, 1967, 1981). Jones et al. (1987) found that "the magnitude of the phi coefficients is affected not only by the item difficulty but also by the strength of the relationships among the variables" (p. 3). Muthén and Christofferson (1981) indicated that factor analyses of phi coefficients produce "inconsistent and attenuated estimates in addition to incorrect standard errors of estimates and incorrect chi-square test[s] of model fit" (p. 407; also see Olsson, 1979). Hence, because these problems occur when factoring the phi coefficient, the literature has suggested the use of tetrachoric correlations as an alternative method.

Factor analysis on tetrachoric correlation matrices may solve some of these problems, but it also creates a new set of problems. For example, tetrachoric correlations are computed based on the assumption that the item responses are functions of underlying continuous variables that have a bivariate normal distribution (Zwick, 1985). If the assumption of bivariate normality is not met, then the tetrachoric correlations may not be good estimates of the relationships between the items (Jones et al., 1987).

Another problem with tetrachoric correlations is that they cannot be directly estimated. Simple approximation formulas may be accurate only in the neighborhood of $r = 0.5$ (Jones et al., 1987). More complex estimations (Castellan, 1966; Divgi, 1979; Kirk, 1973) "can become unstable when one or more cell proportion[s] of the pairwise item response table is extremely small" (Jones et al., 1987, p. 7). Hattie (1984) refers to this problem with tetrachoric correlation matrices as not being positive-definite and discusses the procedures for calculating tetrachoric correlations in order to overcome the problem of obtaining nonpositive-definite correlation matrices. Muthén (1978) indicated that the sample tetrachoric correlations have a larger covariance matrix than the Pearson correlation. He also observed (Muthén, 1981) that ordinary factor analysis of tetrachoric correlations may produce incorrect standard errors of estimates and chi-square test[s] of fit (see also Bock & Lieberman, 1970).

Furthermore, the factor analysis of tetrachoric correlations is problematic when guessing occurs. Factor analysis of tetrachoric correlations often produces spurious factors when respondents guess on most of their test items (see, for example, Carroll, 1983; Hulin et al., 1983; Zwick, 1985). In such cases, the magnitude of tetrachoric correlations is affected by item difficulty. There are, however, procedures reported in the literature for adjusting for this kind of guessing phenomena. Carroll (1945), for instance, suggests that the effects of guessing be removed from the table of item responses. Zwick (1985) applied Carroll's proposed solution to the item responses for the NAEP reading items in grade/age (grage) 13/VIII and found unsatisfactory results (Zwick, 1985). She indicated in her report that "16 percent of the tetrachoric coefficients were rendered incomputable because of negative adjusted cell frequencies" (p. 19). Despite these problems, Zwick (1985) applied the principal components of phi and tetrachoric correlations to her analyses of the three grades/age (grages) and found the results to be of the "worst case" (p. 15). She (1985) also reported the use of alternative procedures for correcting for guessed items and found that her results revealed nothing markedly significant. In addition, Reckase (1981) noted in his research that "over-or-under-correcting yields undesirable results" (p. 4).

Another alternative to factoring phi or tetrachoric correlation matrices that has often been mentioned in the literature is the factor analysis of image correlation. (For a definition of image correlation see Zwick's [1985] description of Guttman's [1953] version of image theory). Kaiser (1970) and Kaiser and Cerny (1979) indicated that principal components and factor analysis on the image correlation matrix would be a more appropriate overall analytical approach to dichotomous data. Zwick (1985) also noted that "for the three within-grade analyses, the first roots are between 14 and 47 percent larger than those for the Pearson matrix" (p. 23).

2. Criteria for judging unidimensionality of PCA. There are many different views in the literature on how small or large the variance explained by the first component should be in order to determine unidimensionality. Carmines and Zeller (1979), with no apparent rationale, postulated that if 40% of the total variance is explained by the first component, then the set of items is measuring a single dimension. Reckase (1979) believes that if the first order component explains only 20% of the variance of a set of items, then that set is unidimensional. One of the problems with the "percent of variance explained by the first component" is that this percent of variance depends on many different factors including the type of correlation coefficients. Zwick (1985), for example, found that when PCA was performed on phi matrices, between 17% to 25% of the variance was explained by the first component as compared to 30% to 40% of the variance that was explained by the first component when the analyses were performed on tetrachoric correlations on the same matrices. On another set of simulated data, Zwick (1985) found that the first component had about 25% of the variance explained by the first component as compared to 80% of the variance explained by the first component for the image correlation matrix. Clearly, there is not a set criterion for how small or large the variance explained by the first component should be to conclude that a set of items is in fact unidimensional.

Similarly, there is no set rule for determining how many factors should be extracted in a principal components analysis. Some researchers have suggested that the assessment of dimensionality be done based on the eigenvalues of the components. For example, Lumsden (1957, 1961) suggested using the ratio of the first to the second eigenvalue as an index of dimensionality. Hutten (1980) also suggested using the ratio of the first to the second eigenvalue of tetrachoric correlations as an index of dimensionality. Lord (1980) concurred that if the ratio of the first to second eigenvalues is large and the second eigenvalue is not larger than any of the others, then one can say the set of items may be unidimensional.

However, the eigenvalues of the components may not be a very reliable index to be used for assessing dimensionaltiy. Jones et al. (1987) with their simulated data found that "the magnitudes of the eigenvalues were affected by the number of dimensions in the data, the amount of random error present in the data, the difficulty level of the items, when an item loaded on multiple factors, and the discrepancies between the loadings" (p. 15). Hattie (1984) concluded that "on theoretical grounds some indices must fail (e.g., ratio of eigenvalues)" (p. 55).

Finally, the analysis of residuals has been suggested as a test for assessing dimensionality. Hattie (1985) proposed that the sum of the residuals, or the sum of the absolute values of the residuals after the first component was removed, can be used as an index of dimensionality. McDonald (1981, 1982) proposed the analysis of the residual items' covariances after fitting a one-factor IRT model as an index of departure from unidimensionality (see also Cook et al., 1983; Zwick, 1985). Hambleton and Rovinelli (1983) based on McDonald's (1981, 1982) procedure suggested analyzing the residual covariances after fitting a nonlinear single factor model (see also Hambleton & Swaminathan, 1985). Hattie (1981) applied McDonald's procedure on large-scale simulation data and found this approach to provide the best results. However, Hambleton and Rovinelli (1986) found the residual analysis to be "of limited value in addressing item dimensionality because large residuals may be due to the violations of several model assumptions, including unidimensionality" (p. 300).

Indices based on factor analysis. Common factor analysis has also been used for assessing the dimensionality of a set of items or subscale scores. Some researchers found principal components and common factor analysis to produce very similar results, whereas others may disagree on this finding (Jones et al., 1987). Velicer and Jackson (1990) compared component analysis with common factor analysis. They concluded on the basis of their review that

> only small differences existed in the numeric results produced by the two methods. In particular, we noted that numeric differences typically occurred only in the second decimal place and that decisions based on the patterns produced by alternative methods would be identical. (p. 99)

Hattie (1985), however, indicated that there are major differences between the two methods. He stated that he had "clearly demonstrated that contrary conclusions can result from using the two methods" (p. 147).

The same problems and limitations that were discussed for the application of PCA to dichotomously scored items arise in the application of common FA to test items. The hypothesis of unidimensionality can also be examined in a large sample by a chi-square test when using the maximum likelihood estimation method, assuming normality (see, for example, Bollen & Long, 1992; Cook et al., 1988; Gerbing & Anderson, 1992; Gold, 1990; Gold & Muthén, 1991; Marsh, Balla, & McDonald, 1988). A test of a fit of one factor versus two factors can be done also by testing the difference in chi-squares of the two models, (i.e., a one-factor model versus a two-factor model). Jöreskog (1978) indicated that the chi-square from the models is independently distributed as a chi-square with $(df_2 - df_1)$ degrees of freedom. Hattie (1985) cited McDonald's (1982) recommendation to use the residual covariance matrix as a very reasonable basis for the misfit of the model to the data. Hattie (1985) also offers Tucker and Lewis's (1973) suggestion of a goodness-of-fit test based on the ratio of variance explained by one factor to the total test variance.

Hattie (1985) briefly discusses the two coefficients based on factor analysis, (a) the maximized-alpha (Armor, 1974; Lord, 1958) and (b) omega (Heise & Bohrnstedt, 1970; McDonald, 1970), with the latter being a lower bound to reliability. Hattie (1985) also discusses the indices based on communality. In his discussion he refers to Green et al. (1977), who proposed two indices for assessing dimensionality. The first index, which was called u, is the sum of the absolute values of the correlations between all the possible pairs of items divided by the square root of the product of their respective communalities from a principal component analysis. The second index is one in which the correlations are first corrected for communality by dividing the correlations by the product of the square roots of their communalities. Such an index would also range between 0 to 1, 1 showing unidimensionality. Hattie points out that the main problem with these two approaches is that they lack the communalities of the items, which are contingent upon the knowledge of correct dimensionality.

Nonlinear factor analysis. The literature has clearly shown the problems involved with using linear factor analysis on phi or tetrachoric matrices (see Hambleton & Rovinelli, 1983; Lord & Novick, 1968, McDonald, 1967, 1981; McDonald & Ahlawat, 1974). A linear factor analysis on phi or tetrachoric correlations could produce artifactual factors because of nonlinear relationships between the observed responses and the underlying trait. There are two possible alternatives to the use of linear factor analysis on dichotomously scored responses for assessing the dimensionality of test items. One approach could be to use the random regressor factor analysis model, which evaluates residual covariances after fitting a nonlinear single factor model. However, the literature is ambiguous on the effectiveness of nonlinear factor analysis as a method for assessing dimensionality.

For example, Hambleton and Rovinelli (1986) found nonlinear factor analysis to be the most promising tool for the assessment of dimensionality of dichotomous data. Hattie (1984), however, concluded that “the sum of absolute residual covariances from nonlinear factor analysis was not an effective index of dimensionality because the results from the unidimensional and multidimensional data set were not sufficiently distinct.”

Item parcel. Another alternative to using linear factor analysis on phi or tetrachoric matrices is to use item parcels, which are the collections of items measuring underlying dimensions. To linearize the nonlinear relationship between observed responses and underlying trait (which could occur because of the effects of item difficulty) one may use a small collection of non-overlapping items. These items are usually referred to as item parcels. Linear factor analysis could then be performed on the parcel scores. However, a serious problem may arise in analyzing factor item parcel scores when the items for a parcel are not selected carefully. For example, item parcels with different levels of difficulty could produce artifactual difficulty factors which could then introduce bias into the dimensionality assessment (see Cook et al., 1983; Cook et al., 1988).

Cook et al. (1988), in their study on the dimensionality of reading items, grouped items based on four item types. They created parcels of three to seven items with approximately equal mean difficulty based on the items' delta difficulty indices. They then used the correlations among the parcels in a linear factor analysis. Cattell (1956a, 1956b) suggested defining the general structure of underlying factors by items and then precisioning this structure with the parcels. There are, however, several different views on this strategy (Cattell & Burdsal, 1975). Evidently, some psychometricians believe that item-level factor analysis is the only method for factoring (see, for example, Eysenck & Eysenck, 1969; Howarth & Browne, 1971). There are others who maintain that item-level data may not be stable enough to be used for factoring. In these cases, homogeneous parcels of items are more desirable for factoring (see, for example, Nunnally, 1978).

Cattell and Burdsal (1975), for example, state:

> That items are unsatisfactory in several ways is obvious: (a) their repeat reliability (dependability coefficient) is poor, due to the effect of incidental events on a judgment of very short duration; (b) any one item, defining a specific situation, is more vulnerable to cultural localism (low transferability coefficient) than the means of a set of items; and (c) the rotation of items is less definitive because it presents relatively blurred hyperplanes. (pp. 165-166)

However, they conclude that "regardless of whether the first explorations are made by items or parcels, the best procedure for the ultimate, reliable, and precise definition of source traits seems to be the use of radial parceling" (Cattell & Burdsal, 1975, p. 167). To do item parceling according to the radial procedure, two factor analyses should be performed, one on the items and the other on the parcel scores. The first factor analysis provides information for constructing the parcels, and the second factor analysis helps define the dimensionality of the parcels.

Other factor analytic approaches. Recently, a procedure for assessing the dimensionality of dichotomous data, called full-information factor analysis (Bock, Gibbons, & Muraki, 1985), has been used (see also Jones et al., 1987; Zwick, 1985). This procedure uses the marginal maximum likelihood method (Bock & Aitkin, 1981) for estimating the parameters of the common factor model (Zwick, 1985).

Christofferson (1975) and Muthén (1978) proposed the generalized least square method. This technique provides a fit statistic that is asymptotically distributed as chi-square. But Jones et al. (1987) explain that

> the statistical test is based on distributional assumptions which may be too restrictive for the variables. Furthermore, for tests of moderate size, very large samples are required to insure the accuracy of the asymptotic approximation. In addition, restrictions are placed on the number of items which may be factor analyzed [according to Mislevy, 1986, 25 is an upper limit for the GLS procedure] or the number of factors in the solution [1-3 for tests with 60 items in the ML technique]. (p. 4)

Other approaches: Multidimensional scaling. Multidimensional scaling (MDS) has also been suggested as a technique for assessing the dimensionality of items. MDS does not need a full correlation or covariance matrix; it only requires that the similarity measures be ordered (see Jones et al., 1987, for more information). MDS is not a more general version of principal components analysis; rather, it is an alternative approach for assessing the dimensionality of items. Reckase (1981) used principal components analysis, factor analysis, MDS, item response theory, and cluster analysis to assess the dimensionality of a set of simulated data. He found MDS technique more effective in assessing dimensionality. On real data, however, Reckase found that the MDS technique was not satisfactory. Zwick (1986) applied MDS for assessing the dimensionality of a set of items and found the MDS analysis of the actual data not very clear.

Other approaches: Indices based on latent trait models. The most fundamental assumption underlying the latent model is the assumption of local independence (for a definition of local independence, see Hambleton, Swaminathan, & Rogers, 1991, pp. 10-12; see also Anderson, 1959, and

McDonald, 1981). Because the assumption of local independence may not hold in many educational test environments, Stout (1990) proposed a less restrictive assumption of “essential independence.” Under this assumption, major latent dimensions are considered while minor dimensions are ignored. Even though the assumption of essential independence is less restrictive, the dimensionality of the items is an important issue in item response theory. Therefore, different techniques have been suggested for assessing the dimensionality of items for different IRT models.

The one-parameter model, often referred to as the Rasch model, involves only the estimation of difficulty parameters. For this model, different indices for testing unidimensionality have been suggested (see Table 2 in Hattie, 1985, for complete data). Hattie (1985) concludes that most of these tests for assessing unidimensionality based on the Rasch model are insensitive to the violation of the unidimensionality assumption (see, for example, Gustafsson & Lindblad, 1978; Rogers, 1984; van den Wollenberg, 1982).

With the two-parameter model, which assumes no guessing, one can estimate difficulty and discrimination. The fit statistics summarized by Hattie (1985, Table 2) can be applied to the two- and three-parameter models with some modifications. Hattie (1985) reported that Rogers (1984) has proposed appropriate formulas for each of the indices listed by Hattie.

After reviewing the literature extensively and discussing the indices of dimensionality, Hattie (1985) concludes that “there are still no known satisfactory indices. None of the attempts to investigate unidimensionality have provided a clear decision criteria for determining it” (p. 158).

A Summary of Techniques for Assessing Dimensionality in NAEP

This part of the literature review will briefly describe the statistical techniques used to assess the dimensionality of NAEP items. In the next section of the review, the results obtained by these studies will be summarized.

Very few studies on the dimensionality of NAEP test items have been reported in the literature. One of the most comprehensive of these studies is that of Zwick (1985, 1987), who assessed the dimensionality of NAEP reading items. Zwick (1985) used the following methods: (a) principal components analysis (PCA) of the phi and tetrachoric correlations, (b) principal components analysis of the image correlation matrix, (c) Bock's full-information factor analysis implemented in the TESTFACT program, and (d) Rosenbaum's test of unidimensionality, monotonicity, and conditional independence using the Mantel-Hänszel procedure. Because guessing affected the results of the PCA on the tetrachoric correlations, Zwick (1985) used several different procedures for correcting the "guessing phenomenon." Among these was Carroll's (1945) procedure and modification of the guessing phenomenon. Zwick also applied principal components analysis of image correlation to simulated data sets.

Other studies conducted on the assessment of the dimensionality of NAEP items include that of Rock (1991), who used simple structure confirmatory factor analysis on item parcels to assess the dimensionality of the NAEP math items. He reported intercorrelations between five math latent variable subscales. Cook and Eigner (1984) also used the latent variable modeling approach to assess the dimensionality of the NAEP reading items. They created item parcels of two to five items each with approximately the same mean difficulties and then applied simple structure confirmatory factor analysis to the matrix of parcel correlations. First-order and second-order models were also applied to the data.

Carlson and Jirele's (1992) study on the dimensionality of the NAEP 1990 math items used item responses from four NAEP math booklets, two at Grade 4 and two at Grade 8. In addition to the NAEP math data, they analyzed four simulated one-dimensional data sets. Two different procedures were used—full information item factor analysis as implemented in the TESTFACT computer program, and the normal harmonic factor analysis as implemented in the NOHARM program. They reported the number of factors and chi-square and AIC index for each of the data sets. Yamamoto and Jenkins (1990) conducted a study to examine the dimensionality of the NAEP math tests for the 1990 main assessment in which they performed confirmatory factor analysis on the item parcels formed within each booklet at each grade level. They considered the results obtained from the different booklets as replications. They estimated the correlations between the five factors corresponding to the five math subscale latent variables; the reported interfactor correlations were very high. Allen (1992) conducted a study on the dimensionality of science test items for the 1990 main assessment. She used a three-latent-variable model that corresponded with the three science subscales. She then estimated the correlations between the three subscale latent variables, which were averaged over the booklets and used as indices of dimensionality (see Johnson & Allen, 1992).

A Summary of the Results of the Dimensionality Studies in NAEP

The results of Zwick's (1985) extensive study of the NAEP reading items, which used several procedures and examined subscale dimensionality from different views, indicated that the reading items in different subscales are measuring the same underlying dimensions. She reported sizable first roots obtained by principal components analyses of phi and tetrachoric correlations. She also reported roots that ranged from 17% to 25% of the trace for the phi matrices and 30% to 40% for the tetrachoric matrices. The first roots of PCA to image correlation matrices, as she reported, were even larger than those obtained from the application of PCA to phi and tetrachoric correlations. When Bock's full-information factor analysis was applied to a subset of the grage 13/VIII, she found that the first factor accounted for 29% of the total variance. In her report, she stated that "overall, the four dimensionality analyses of the NAEP reading items indicate that it is not unreasonable to treat the data as unidimensional" (p. 39).

Rock in his (1991) study of the dimensionality of math subscales found that there was little discriminant validity in the math subscales except for the geometry subscale at the 8th-grade level. He found near-perfect correlations between the subscale latent variables in math and science. For math subscales the average correlations were .94 for Grade 4, .91 for Grade 8, and .93 for Grade 12. The intercorrelations between the science subscales were high and very similar to the intercorrelations between the math subscales. The average correlations were .94 for Grade 4, .95 for Grade 8, and .95 for Grade 12. Based on these results, Rock (1991) concluded that "we are doing little damage in using a composite score in mathematics and science" (p. 2).

Due to the very large intercorrelations between the math subscales, Yamamoto and Jenkins (1992) in their study concluded that there is a general math factor that accounts for a large amount of variability in the math subscale scores. The average intercorrelations between the math subscale latent variables in their study were .94 for Grade 4, .91 for Grade 8, and .93 for Grade 12.

Finally, Allen (1992) found high correlations between the three latent variables corresponding to the three science subscales, indicating a unidimensional science test. In her study, the average intercorrelations between the subscale latent variables formed by item parcel scores were .95 for Grade 4, .95 for Grade 8, and .94 for Grade 12.

Method

The literature section of this paper summarizes the most commonly used techniques for assessing the dimensionality of a set of items and/or subscale scores. In these NAEP data sets no examinee has a complete set of items in any one subject area or in any one subscale of a subject area because NAEP uses a balanced incomplete block (BIB) spiraling design (Beaton et al., 1987; Zwick, 1987). Due to this limitation and because the previous studies on the dimensionality of NAEP test items (Zwick, 1987) revealed that the results of BIB spiraling were very similar to the results obtained from the complete data sets, we decided to conduct our dimensionality analysis at the booklet level.

It would have been possible to perform the dimensionality analysis at either the item level or the subscale level within each booklet. However, because the literature revealed that the results of principal components and (linear) factor analysis done on dichotomously scored items (phi or tetrachoric correlations) were much affected by factors such as item difficulty, item homogeneity, and guessing (see, for example, Bejar, 1980; Carroll, 1983: Hambleton & Rovinelli, 1983, 1986; Hulin et al., 1983; Jones et al., 1987; McDonald & Alhawat, 1974; Mislevy, 1986; Zwick, 1985), linear factor analysis was not performed on the item-level data. To linearize the nonlinear relationship between the observed responses and the underlying trait, we decided to use parcels of items and study the math subscale dimensionality based on the item parcels.

It must be noted, however, that the literature cautioned that serious problems could arise when factor analyzing parcel scores if the selection of the items for a parcel is not done properly (see Cook et al., 1983). For example, item parcels with different levels of difficulty could produce difficulty factors that introduce bias into the dimensionality assessment process (see, for example, Swinton & Powers, 1980). In response to this potential problem, Cattell and Burdsal (1975) suggested the use of radial parceling. To do item parceling according to the radial procedure, two factor analyses should be performed, one on the items and another on the parcel scores. The first factor analysis would provide information for constructing the parcels and the second factor analysis would help define the dimensionality of the parcels.

We took Cattell and Burdsal's suggestion into consideration and tried to construct item parcels in such a way that the items within the parcels were homogeneous and the parcels had approximately equal variance and equal means. Because background variables could have an impact on subscale math dimensionality (see, for example, Muthén, Kao, & Burstein, 1991), we used these item parcels in factor analytic models to study the effects of background variables on math dimensionality.

NAEP collects a substantial number of instructional and non-instructional (cognitive and noncognitive) background variables. Because it was very difficult and time-consuming to use all of these variables in our dimensionality analysis, we decided to select a small set of the background variables that had a significant impact on the math subscale scores. We ultimately used all of the cognitive (instructional) variables and some of the noncognitive background variables that seemed to be somehow related to student mathematics ability. These variables were then used as grouping variables in a multiple discriminant analysis in which the math subscale scores were used as discriminating variables. Based on the results of the discriminant analysis, the cognitive and noncognitive variables that had a significant impact on the math subscale scores were further identified and selected to be used as a basis for forming student subgroups. We had planned on comparing the factor means and factor variances across the subgroups of students that were formed based on the levels of background variables using the multiple group factor analysis procedure. However, because of the small number of subjects in the subgroups we decided to use the MIMIC (Multiple Indicators and Multiple Causes) approach.

We applied the MIMIC model to the 1990 and 1992 data. The application of the MIMIC model to the 1992 data was particularly useful because of the small number of subjects per booklet for the 1992 data. However, because a sister dimensionality study was being conducted at CRESST/UCLA by a second team of researchers who would perform the same analysis on the same data file, we decided to report only the simple structure factor analysis results.

For the 1990 data, all the students who answered the items in a given booklet were divided into subgroups based on their responses to the selected background variables (questions). Once the subgroups were formed, item parcels were created for each of the subscales within each subgroup. These parcels were used in three different latent variable models: Model 1, a one-factor model that assumes one general mathematics factor; Model 2, a five-subscale-factor model that assumes five mathematics subscales; and, Model 3, a hierarchical model (a five-subscale-one-general-factor model that assumes one general math factor plus five math subscales). Indices of fit for the three models were obtained and were compared within each subgroup as well as across subgroups. For Model 2 (five subscale factors with no general math factor), correlations between the five subscales or five latent variables were estimated and were compared across the subgroups. We did this to see if more discrimination would appear (i.e., lower correlations) between the five math subscale latent variables for the subgroups that were formed based on the variables with more discriminating power. All of these analyses were performed on all of the items within each of the booklets.

For the second phase of analyses on the 1990 data, correlations between each individual item and the background variables were computed using the Multi-Approach Correlation System (MACS) (Abedi, 1993). Items that had high and low correlations with the background variables were selected and two different sets of item parcels were created. One set consisted of items that had higher correlations with the background variables, and the other set consisted of items that had lower correlations with the background variables. The item parcels were then used in three different latent variable models (i.e., the one-factor model, the five-subscale-factor model, and the five-subscale-one-general-factor model or hierarchical model). The second-phase analyses were performed for all of the subjects, and the results of the three models were compared. It must be noted, however, that on some of the booklets and for some of the subscales, there were not enough items to create item parcels of high and low correlations.

The 1992 math test item data were different from the 1990 data in at least two aspects. First there were more open-ended questions in the 1992 administration than in the 1990 administration, and second, the math items were distributed into more booklets in the 1992 administration than in the 1990 administration (there were approximately two and a half times more booklets in 1992 than in 1990). Because of these differences, the analyses performed on the 1992 data were different from those performed on the 1990 data.

Because we did our 1992 data analyses at the booklet level, we found that there were not enough subjects per booklet to group into subgroups based on the subject responses to the background variables (questions) especially for those variables (questions) that had more than two responses/categories. Therefore, for the 1992 data, we selected items based on their relationships with the cognitive and noncognitive background variables. Once these items were selected, we compared the intercorrelations between the five math subscale latent variables of the parcels that contained items that had high correlations with the background variables and the parcels that contained items that had low correlations with the background variables.

Results

We discuss the results of the discriminant analysis (DA) and factor analysis (FA) in the next sections. All tables appear in the Appendix, in the following order:

Discriminant Analysis 1990 Data:	Tables A1–A27
Discriminant Analysis 1992 Data:	Tables B1–B18
Factor Analysis 1990 Data:	Tables TA1–TA9
	Tables 4A1–4A15
	Tables 8A1–8A12
	Tables 12A1–12A9
	Tables IA1–IA9
Factor Analysis 1992 Data:	Tables TB1–TB8
	Tables IB1–IB8

Results of the Discriminant Analysis

Using a multiple discriminant analysis, we selected a small set of cognitive and noncognitive background variables that had a significant effect on the math subscale scores. Discriminant analysis (DA) was employed because of "(1) parsimony of description; and, (2) clarity of interpretation" (Stevens, 1992). A discriminant analysis is parsimonious because out of the five subscale scores the subgroups may differ on only one or two subscales. For example, some of our analyses performed on the total group of students have revealed that even though the subscales were highly correlated, the geometry subscale was more distinct among the others (see also Rock, 1991). Discriminant analysis has clarity of interpretation because the discrimination of groups on one function is quite independent from the discrimination of groups on the other functions.

The discriminant analyses that yielded the cognitive and noncognitive background variables used in our study satisfied the following conditions:

1. At least one discriminant function was significant with the following statistics:
   a. The canonical correlation was .25 or greater.
   b. χ^2 was significant at the .05 level.
2. The univariate F-ratios for all or most of the subscales were significant at the .05 level.
3. The structure coefficients of all or most of the subscales on the discriminant functions were greater than .30.

(Pedhazur, 1982)

We next discuss the results of the multiple discriminant analysis to show how the background variables were selected for our analyses. We will present only the results of those analyses that yielded significant results. We first discuss the results for the 1990 data and then those for the 1992 data.

Results of the Discriminant Analysis for the 1990 Data

Results for Grade 4. For Grade 4, three different groups of subjects who answered items from Booklets 11, 12, and 14 were used. Tables A1 through A3 present the results of the DA for Booklet 11. Table A1 summarizes the results of the DA for the subgroup that responded to the background variable (question) "Home Environment—Reading Materials." As Table A1 indicates, only one function was statistically significant, χ^2 = 121.18, df = 10, p <. 00. For this function, the F-ratios showing significant differences in the subscales across subgroups were all significant beyond the .01 nominal level.

Table A2 presents the results of the DA for the subgroup that responded to the background variable (question) "How Do You Feel About This Statement: I Am Good in Math." In this table there was also only one function that was significant, χ^2 = 129.53, df = 10, p < .00. In addition, the F-ratios for all the subscale scores were significant beyond the .01 nominal level indicating that the subgroups performed differently on the five subscales.

Table A3 presents similar DA results for the subgroup that responded to the background variable (question) "In Math Class How Often Do You Work With Rulers, Blocks, Shapes." In this table, one significant function, $\chi^2 = 83.52$, $df = 20$, $p < .00$, emerged as well. The F-ratios that demonstrate the significant discriminating power of the five subscale scores were all significant.

As we explained earlier, the analyses were done on the booklet level due to NAEP's BIB spiraling design, and the results obtained from the different booklets were used as cross validation data. Tables A4, A5, and A6 present the results of the DA on Booklet 12. These results are similar to the results that are presented in Tables A1, A2, and A3 respectively. Tables A7, A8, and A9 also present similar results for Booklet 14.

The results of the DA that was conducted on the three independent groups of subjects were consistent across booklets. For instance, compare Table A1 with Tables A4 and A7. Then compare Table A2 with Tables A5 and A8, and Table A3 with Tables A6 and A9 to see how consistent the results are across the three different groups of students who answered items in Booklets 11, 12, and 14.

Results for Grade 8. For Grade 8, subjects who answered math items in Booklets 8, 9, and 10 were chosen for the DA. Table A10 presents the results of the DA for the subgroup that responded to the background variable (question) "Home Environment—Reading Materials." As Table 10 indicates, only one significant function discriminated between the groups that were formed based on the above background variable (question), $\chi^2 = 123.05$, $df = 10$, $p < .00$. The five subscale scores had all significant mean differences across the subgroups and the F-ratios corresponding to the five subscale mean differences were all significant beyond the .01 nominal level.

Table A11 summarizes the results of the DA for the subgroup that responded to the background variable (question) "Do You Agree: I Am Good in Math" for Booklet 8. Once again, only one significant function emerged, $\chi^2 = 156.31$, $df = 20$, $p < .00$. The F-ratios for the subscale scores were all significant above the .01 nominal level which indicated that there were differences in all of the subscale scores across the subgroups.

Table A12 presents similar results for the DA for the subgroup that responded to the background variable (question) "What Kind of Math Class Are You Taking." This analysis yielded two significant functions: $\chi^2 = 303.28$, $df = 20$,

$p < .00$ for function 1, and $\chi^2 = 33.23$, $df = 12$, $p < .00$ for function 2. The F-ratios for all of the subscale scores were significant.

Tables A13, A14, and A15 present the results of the DA that was conducted on the data from Booklet 9. These results are comparable with Tables A10, A11, and A12 respectively and cross validate each other. Tables A16, A17, and A18 present results that are comparable with Tables A10, A11, and A12. Tables A10, A11, and A12 are also comparable with Tables A13, A14, and A15 respectively.

After comparing the results presented in these tables it is quite evident that there is a consistent trend in the results of the DA of the different groups of subjects that answered different sets of math items. The only major difference between the results obtained from the data in the three booklets is that the DA for the subgroup responding to the background variable (question) "What Kind of Math Class Are You Taking" in Booklet 8 yielded two significant discriminant functions. In this analysis, some of the subscale scores had significant structure coefficients on two of the functions. For example, the Numbers and Operations subscale had a structure coefficient of .79 on function 1, and .58 on function 2. The same analysis on Booklet 10 (including the same background question/ response series) also yielded two functions. But for Booklet 10, all of the subscale scores had high structure coefficients on the first function and a few moderate structure coefficients on the second function. The analysis of Booklet 9 however, resulted in only one significant function; therefore the subscale scores had only high structure coefficients on the first function.

Results for Grade 12. For Grade 12, DA(s) were performed on the data obtained from Booklets 8, 9, and 10. Tables A19 through A21 present the DA results for Booklet 8. Tables A22 through A24 present the DA results for Booklet 9, and Tables A25 through A27 present the DA results for Booklet 10.

Table A19 summarizes the DA results for the subgroup that responded to the background variable (question) "Home Environment—Reading Materials." As the table indicates, the first function was significant, $\chi^2 = 79.85$, $df = 10$, $p < .00$, and all of the five subscale scores were significantly different across the subgroups.

Table A20 presents similar DA results for the subgroup that responded to the background variable (question) "Do You Agree: I Am Good in Math." This analysis yielded two significant functions: $\chi^2 = 196.36$, $df = 20$, $p < .00$ for the first function, and $\chi^2 = 25.09$, $df = 12$, $p < .01$ for the second function. The subscale scores were significantly different across the subgroups with F-ratios that were significant above the .01 nominal level.

Table A21 presents the DA results for the subgroup that responded to the background variable (question) "In Math Class How Often Do Problems on Worksheet." One function was significant in this analysis, $\chi^2 = 77.12$, $df = 10$, $p <$ .00. The significant F-ratios indicated that all the subscale scores were different across the subgroups.

Tables A22, A23, and A24 present the results of the DA for Booklet 9. These results are comparable to the results in Tables A19, A20, and A21. Tables A25, A26, and A27 present the results of the DA for Booklet 10. These results are comparable with the results in Tables A19, A20, and A21, and the results in Tables A22, A23, and A24 respectively.

When these results were compared, consistencies were found across the three different groups that answered different sets of items from the three separate booklets. There was only one disparity in the results across the three groups. The results of the DA on Booklet 8 yielded two significant functions for the subgroup(s) that were formed based on the selected response choice to the background variable (question) "Do You Agree: I Am Good in Math." The results of the analyses performed on Booklets 9 and 10 revealed only one significant function.

Based on the series of DAs that were run on the 1990 Math data, the following set of cognitive and noncognitive background variables were found to have significant effects on the math subscale scores:

- Grade 4
  - Home Environment—Reading Materials
  - How Do You Feel About This Statement: I Am Good in Math
  - In Math Class How Often Do Work With Rulers, Shapes, Blocks
- Grade 8
  - Home-Environment-Reading Materials
  - Do You Agree: I Am Good in Math
  - What Kind of Math Class Are You Taking
- Grade 12
  - Home Environment—Reading Materials
  - Do You Agree: I Am Good in Math
  - In Math Class How Often Do Problems on Worksheet

Results of the Discriminant Analysis for the 1992 Data

As mentioned earlier, the 1992 math items were distributed into 26 booklets as compared to the 1990 items which were placed into 10 booklets. Thus, there were fewer students per booklet in the 1992 administration than in the 1990 administration. Because of this limitation, for the 1992 analyses, we had some difficulty grouping the students based on the background variables, especially for those variables that had a large number of responses/choices. We will discuss the results of discriminant analysis on the 1992 data set for each grade separately.

Results for Grade 4. Tables B1 through B3 summarize the results of the DA for the three selected background questions that were used on the data in Booklet 15. Table B1 presents the results of the DA for the subjects that responded to the background variable (question) "Agree/Disagree: I Am Good at Math" for Grade 4, Booklet 15. As Table B1 indicates, only one function, $\chi^2 = 19.57$, $df = 10$, $p < .03$, significantly discriminated between the subgroups that were formed based on the range of responses to the above background variable (question). All of the subscale scores except those for the Measurement and Algebra subscales yielded different means across the subgroups.

Table B2 presents similar DA results for the subjects that responded to the background variable (question) "Agree/Disagree: I Like Math." In this analysis, one function also emerged, $\chi^2 = 20.09$, $df = 10$, $p < .03$. The subscale scores were all different with the exception of scores for the Measurement and Algebra subscales across all of the subgroups.

Table B3 presents the DA results for the subjects that responded to the background variable (question) "How Much Time Spent Each Day on Math Homework." One significant function emerged, $\chi^2 = 45.8$, $df = 30$, $p < .03$. The Numbers and Operations and the Statistics subscales yielded different means across the subgroups.

Tables B4, B5, and B6 present very similar results for the DA analyses that were performed on the data from Booklet 17. These tables' results are parallel to those reported in Tables B1, B2, and B3 respectively. When comparing these two sets of tables, one can see the consistency in results obtained based on two different groups of students.

Results of DA for Grade 8. The results of the DA that was performed on the Grade 8 1992 data, in which two selected background variables (questions) were used, are reported here. Tables B7 and B8 present the results of the DA for Booklet 1. Tables B9 and B10 present the results of the DA for Booklet 2, and Tables B11 and B12 summarize the results of the DA for Booklet 15.

Table B7 presents the DA results for the subjects that responded to the background variable (question) "Do You Agree: I Am Good in Math" for Booklet 1. There was only one significant function that emerged in this analysis, $\chi^2 = 60.01$, $df = 20$, $p < .00$. As the F-ratios indicate, all of the subscale scores yielded significant differences across the subgroups except for the Statistics subscale score.

Table B8 summarizes the results of the DA for the subjects that responded to the background variable (question) "Agree/Disagree: Math Is Mostly Memorizing Facts." This analysis also yielded one function that was significant, $\chi^2 = 31.87$, $df = 20$, $p < .05$. The F-ratios in this table indicated that all of the subscale scores had different means across the subgroups.

Table B9 reports the results of the DA of the first background variable, "Do You Agree: I Am Good in Math," for Booklet 2, and Table B10 reports similar results for the second background variable, "Agree/Disagree: Math Is Mostly Memorizing Facts," for the same booklet. Similarly, Table B11 summarizes the results of the DA for the first background variable that was applied to the data in Booklet 15, and Table B12 summarizes the results of the DA for the second background variable for the same booklet.

The results presented in Tables B7 and B8 are comparable with the results presented in Tables B9 and B10. These two sets of tables are also comparable with Tables B11 and B12 respectively. When comparing the sets of tables that display the first background variable, "Do You Agree: I Am Good in Math," there is only one disparity in the results of the three different groups of students that answered the math items in the three different booklets. The dissimilarity appears in the F-ratio reporting for the Statistics subscale. The results for Booklet 1 yielded no significant differences in the mean score for this subscale across the subgroups, whereas for the other two booklets, there were significant mean differences in this subscale across the subgroups. Except for this one instance, the results of the analyses for the three booklets were consistent.

Results for Grade 12. We applied DA on the data from the three booklets (1, 15, and 17) for Grade 12. Due to space constraints, we will report only the summary results of the DA for two background variables that were applied to the data in each of the three booklets. These two variables were "Do You Agree: I Am Good in Math" and "Agree/Disagree: Math Is Mostly Memorizing Facts."

Table B13 presents the results of the DA for the first background variable, "Do You Agree: I Am Good in Math," for Booklet 1. As Table B13 indicates, only one function significantly discriminated between the subgroups, $\chi^2 = 97.8$, $df = 20$, $p < .00$. The F-ratios in this table were all significant, which indicated that the subscale scores were all different across the subgroups.

Table B14 presents the DA results for the second background variable, "Agree/Disagree: Math Is Mostly Memorizing Facts," for Booklet 1. Consistent with the data that were presented in Table B13 and prior DA analyses, one significant function was yielded, $\chi^2 = 97.4$, $df = 20$, $p < .00$. All of the F-ratios that corresponded to the subscale scores were also significant.

Similar results were obtained when the two background variables were used in the data from Booklets 15 and 17. Tables B15 and B16 report the results of the DA for Booklet 15, and Tables B17 and B18 present the results of the DA for Booklet 17. Tables B13 and B14 can be compared with Tables B15 and B16 and Tables B17 and B18 respectively. In all of these analyses, one significant function emerged and all of the subscale scores were significantly different across the subgroups.

Based on the series of DAs that were run on the 1992 math data, the following set of cognitive and noncognitive background variables were found to have significant effects on the math subscale scores:

Grade 4
- Agree/Disagree: I Am Good in Math
- Agree/Disagree: I Like Math
- How Much Time Spent Each Day on Math Homework

Grade 8
- Do You Agree: I Am Good in Math
- Agree/Disagree: Math Is Mostly Memorizing Facts

Grade 12
- Do You Agree: I Am Good in Math
- Agree/Disagree: Math Is Mostly Memorizing Facts

Results of the Factor Analysis

Discriminant analysis helped us identify those background variables that had a greater impact on student math performance. After the background variables that had a higher discriminating power were identified and selected, we then performed confirmatory factor analysis on the subgroups that were formed based on the level of responses to the selected background variables.

In approaching confirmatory factor analysis, we set out to test the following three hypotheses:

1. that the five math subscales are highly correlated because they are actually measuring one general math factor;

2. that the test items in the five math subscales are measuring five distinct math abilities; and,

3. that while the five subscales are measuring a common math ability, each subscale in turn may measure a distinct math ability.

From these three hypotheses, three entirely separate latent variable models were created. Each latent variable model was derived from and corresponded with one of the above hypotheses. We then applied these three new models to the data sets that were obtained from the different booklets and different subgroups. Indices of fit from the three different models were compared to see which model exhibited the best fit to the data.

We once again ran into the problem of not having enough subjects per booklet to group into fairly equable subgroups. The BIB spiraling design of NAEP and the occurrence of background variables (questions) that had a higher number of responses/categories caused many of the subgroups to have far too few subjects. Due to this complication, in some of our analyses, rather than grouping the subjects into subgroups, we decided to use the items that had high correlations with the background variables. In other words, in any given booklet, we used all of the subjects' responses to the items that had high correlations with the background variables. We then applied the three models to the composite scores of the selected items.

As we mentioned in the literature section of this report, there are several studies that have been conducted on the NAEP math items that used the total group of students. These studies mainly revealed that the math items were generally unidimensional. In our study, we examined the dimensionality of the math items on the total group of students as well as on the subgroups of students that were formed based on the range of responses to the selected background variables (questions) that we found, in our DA, to affect math performance. We next report our findings for the total group of students and then for the subgroups of students. We will discuss these results individually for each of the three grades (4th, 8th, and 12th).

Results of the Factor Analysis for the 1990 Data

Results for Grade 4. Table TA1 summarizes the results of the confirmatory factor analysis that was done on the data from Booklet 11. The top portion of the table lists the indices of fit including chi-square, degrees of freedom, chi-square ratio, normed fit index (NFI), non-normed index (NNFI), and comparative fit index (CFI) for the following three models: (a) Model 1, which assumes that a general math factor is underlying all of the five math subscale scores and that there is no subscale variation; (b) Model 2, which assumes that the five subscales are measuring five different areas in math—thus, the math items could be categorized under five different factors (or subscales); and (c) Model 3, which assumes that in addition to a general math factor, there are also some subscale factors that cannot be explained by the general math factor. The lower half of Table TA1 presents the estimated correlations between the five subscale latent variables. As Table TA1 indicates, all three models fit the data based on the fit indices. However, Models 2 and 3 (which assume more than a general math factor) seem to display a better fit to the data. The chi-square ratio for Model 1 was 2.17 whereas the chi-square ratios for Models 2 and 3 were 1.79 and 1.90 respectively. The correlations between the five subscale latent variables (as shown in the lower half of the table) were all extremely high except for the Geometry subscale. These high correlations were all indicative of unidimensionality of the five subscales.

Table TA2 presents the results of the confirmatory factor analysis for the data from Booklet 12. These results are very similar to the results that were presented in Table TA1 for Booklet 11. The three models fit the data, but Models 2 and 3 once again seem to exhibit a better fit to the data. The chi-square ratio for Model 1 (which assumes only one general math factor) was 1.46 whereas the chi-square ratios for Models 2 and 3 (which assume subscale factors as well) were .81

and .71 respectively. In this table, the correlations between the subscale latent variables were also extremely high with the exception of the Geometry subscale. This once again indicates unidimensionality of the math subscale scores.

Table TA3 reports the results of the analyses for Booklet 14. Consistent with the results presented in Tables TA1 and TA2, the indices of fit in this table indicate that the data fit the three models. However, Models 2 and 3 exhibit a better fit. The chi-square ratio for Model 1 was 2.64. For Model 2, the chi-square ratio was 1.89, and for Model 3, 1.85. The subscale latent variable correlations were all extremely high and indicative of unidimensional subscale scores.

We now turn our discussion to the results of the analysis that was conducted separately on each of the subgroups that were formed based on the levels of response to the selected background variables. Table 4A1 presents the results of the confirmatory factor analysis on the group of students who indicated that they were *undecided* [2] when responding to the background variable (question) "How Do You Feel About This Statement: I Am Good in Math." The fit indices in this table indicate that the three models had a satisfactory fit. However, based on the chi-square ratios, one may conclude that Model 2 had a slightly better fit than the other two models. The lower half of the table lists the correlations between the five math subscale latent variables. When comparing the subscale correlations that were obtained from the total group analyses to the subscale correlations from the subgroup analyses, the subscale correlations were smaller for the subgroups. These smaller correlations indicate that there is evidence of multidimensionality in the subscale scores for the subgroups. For example, in the analyses that were performed on the subgroups, the correlation between the Geometry and the Numbers and Operations subscales was .68, and between the Statistics and Geometry subscales the correlation was .81.

Table 4A2 presents the results of the analyses for those students who responded to the background variable (question) "How Do You Feel About This Statement: I Am Good in Math" by selecting either *agree* [1] or *disagree* [3]. The indices of fit for both of these subgroups (displayed in the top and middle portions of the table) indicate that the three models fit the data at approximately the same level. However, the lower half of the table reveals a different trend in the correlations between the subscale latent variables. For example, for the *disagree* [3] subgroup, the subscale correlations were smaller than for the *agree* [1] subgroup, which is indicative of multidimensionality in the math subscales. The correlations for the *disagree* [3] subgroup between the Statistics subscale with the Numbers and Operations, Measurement, and Geometry subscales were .91, .77, and .87 respectively, whereas the correlations for the *agree* [1] subgroup between the Statistics subscale with the Numbers and Operations, Measurement, and Geometry subscales were .95, 1.0, and .98 respectively.

Table 4A3 summarizes similar results to those that were presented in Table 4A2 for Booklet 12. The top portion of this table indicates that all of the three models fit the data for the subgroup that indicated *agree* [1] to the background variable (question) "How Do You Feel About This Statement: I Am Good in Math." But Models 2 and 3 exhibited a slightly better fit than Model 1. The chi-square ratio for Model 1 was 1.73, for Model 2 it was 1.25, and for Model 3 it was 1.08. The middle section of the table lists the indices of fit for the subgroup that selected the *disagree* [3] response to the background variable (question) listed above. For this group, the three models fit the data at approximately the same level. However, the lower half of the table, which lists the correlations between the subscale latent variables, indicates that there is more evidence of dimensionality for the second subgroup than for the other subgroups. For example, the correlations between the Geometry subscale with the Numbers and Operations and Measurement subscales were .79 and .84 respectively for the second subgroup whereas the correlations for the first subgroup between the same subscales were .86 and .87 respectively.

These results were consistent with the results that were obtained for Booklet 11. Table 4A4 summarizes the results for those subjects that selected the *undecided* [2] response/category to the "How Do You Feel About This Statement: I Am Good in Math" background variable (question). In this analysis, the three models fit the data at about the same level. The chi-square ratios were .63 for Model 1, .53 for Model 2, and, .57 for Model 3. There were high correlations between the subscale latent variables, but some of these correlations were not as high as the correlations that were found for the same background variable/response choice in the total group analyses.

Table 4A5 presents the results of the analyses for the "I Am Good in Math" background variable (question) for Booklet 14. For this analysis, all three of the models fit the data at approximately the same rate. In Model 1, the students who chose *agree* [1] for this background variable (question) had a chi-square ratio of 1.73. In Model 2 the chi-square ratio was 1.31, and for Model 3, it was 1.32. For the *disagree* [3] subgroup, the chi-square ratios were .75, .71, and .68 for the three models respectively.

Table 4A6 presents the results for those subjects that selected the *undecided* [2] response/choice to the background variable (question) “How Do You Feel About This Statement: I Am Good in Math.” Again, the three models fit the data at about the same rate. There were, however, lower correlations in the subscale latent variables’ correlation matrix, which indicates evidence of multidimensionality in the subscales.

Tables 4A7 and 4A8 summarize the results of the confirmatory factor analyses on the three subgroups of students that selected one of the three response choices to the background variable question, “Home Environment—Reading Materials” for Booklet 11. Tables 4A9 and 4A10 present the results for the same background variable (question) and set of answer response choices for Booklet 12, and Tables 4A11 and 4A12 report the results for the same background variable/answer choice sequence for Booklet 14.

Table 4A13 presents the results for the two subgroups that selected either one of two responses (1 = *almost every day*; or 5 = *never*) to the background variable (question) “In Math Class How Often Do You Work With Rulers, Blocks, Shapes” for Booklet 11. There were originally five selection responses (1 = *almost every day*; 2 = *several times a week*; 3 = *almost once a week*; 4 = *less than once a week*; and 5 = *never*) for this background variable (question). However, we used the data from only those subjects who selected response 1 or 5 because we wanted to examine the differences in subscale scores between the two extreme range choices. Table 4A14 reports the results for the same background variable (question) and answer choices for Booklet 12, and Table 4A15 summarizes the results for the same background variable (question) sequence for Booklet 14.

The results that were reported for the “I Am Good in Math” background variable (question) were very similar to those found for the “Home Environment—Reading Materials” and “In Math Class How Often Do You Work With Rulers, Blocks, Shapes” background variables (questions). For instance, the three models that were created for the “Home Environment—Reading Materials” and the “In Math Class How Often Do You Work With Rulers, Blocks, Shapes” background variables (questions) fit the data at approximately the same level as the three models that were created for the “I Am Good in Math” background variable (question) sequence. However, there was a trend of a better fit for Models 2 and 3 in the “Home Environment—Reading Materials” and the “In Math Class How Often Do You Work With Rulers, Blocks, Shapes” background variables (questions). In these tables (for “Home Environment—Reading Materials” and “In Math Class How Often Do You Work With Rulers, Blocks, Shapes”), one can see that correlations between the subscale latent variables were lower than those observed for the same two background variable (question) sequences in the total group of students analyses (see Tables TA1, TA2, and TA3 for the appropriate comparisons).

As mentioned earlier, we also computed the correlations between the individual math test items and the individual background variables using special software that was prepared solely for this purpose. Based on the item-background correlation results, we selected certain items and applied our latent variable approach to these newly selected items

Table IA1 presents the results of the factor analysis of the selected items based on the item-background correlations. As the top portion of Table IA1 indicates, the three models fit the data, but Models 2 and 3 once again displayed a slightly better fit. The bottom portion of the table presents the correlations between the subscale latent variables. In some cases the correlations between the subscale latent variables were considerably lower than those correlations that were obtained from the total group analyses. For example, the correlations between the Numbers and Operations subscale with the Measurement, Geometry and Statistics subscales were .571, .725, and .727 respectively as compared to the total group correlations for Booklet 11 which were .96, .83, and 1.0 respectively (see Table TA1).

Table IA2 presents the results for the data in Booklet 12. Again, in this table, the three models exhibited approximately the same level of fit. The chi-square ratio for Model 1 was 1.22, for Model 2 it was 1.20, and for Model 3 it was 1.11. The subscale latent variable correlations were lower in comparison to the correlations in the total group analyses (Tables TA1, TA2, and TA3).

Table IA3 summarizes the results of the analysis that was conducted on the selected items from Booklet 14. The top portion of the table indicates that the three models displayed a fit to the data similar to the previous model fits. The bottom section of the table lists the subscale latent variable correlations, which seem to be somewhat lower than those that were obtained from the total group analyses (Tables TA1, TA2, and TA3).

Results for Grade 8. We will report some of the results of the latent variable analyses that were conducted on the total group of subjects using all of the items, and then we will report the results of the analysis by subgroups that were formed based on the response choices to the selected background variables (questions). Finally, we will report the results of the analyses that were conducted on the total group of subjects using the selected items that were chosen based on the item-background correlation analyses.

Table TA4 summarizes the results of the analyses for Grade 8 for the subjects that answered all of the items in Booklet 8. The top portion of this table lists the indices of fit for the three models (Model 1, which assumes one general factor; Model 2, which assumes five subscale factors; and Model 3, which assumes five subscale factors plus one general factor). The indices of fit indicate a relatively good fit of the data to Models 2 and 3 but a poor fit for Model 1. For example, the chi-square ratio was 3.47 for Model 1, 1.41 for Model 2, and 1.86 for Model 3. The correlations between the five subscale latent variables were all high and indicative of unidimensional subscales.

Table TA5 presents the same series of results for Grade 8, Booklet 9. Models 2 and 3 displayed a better fit to the data and were similar to the model fits that were reported for Booklet 8. The chi-square ratios for Booklet 9 were 3.61, 1.82, and 2.29 for the three models respectively. The subscale latent variable correlations were also relatively high for Booklet 9.

Table TA6 summarizes the results of the analyses conducted on Booklet 10. The indices of fit at the top portion of the table indicate that Models 2 and 3 once again exhibited a much better fit to the data than Model 1. The chi-square ratio for the Model 1 was 4.90, for Model 2 it was 1.18, and for Model 3 it was 1.69. The size of the correlations between the subscale latent variables, however, indicated that the subscales were unidimensional.

We will now discuss the results of the latent variable analysis conducted on the subgroups that were formed based on the levels of responses/choices to the background variables. For each subgroup, we will present the indices of fit for the three models and discuss the correlations between the subscale latent variables.

Table 8A1 presents the results (fit indices and subscale correlations) for the subjects who indicated *strongly agree* [1] or *disagree* [4] as a response to the background variable (question) “Do You Agree: I Am Good in Math” for Booklet 8, Grade 8. For the first subgroup (those subjects who selected *strongly agree* [1]), Models 2 and 3 exhibited a much better fit than Model 1. Thus, multidimensional models seem to fit the data better than unidimensional models. For the second subgroup (those subjects who selected *disagree* [4]), all three models displayed a good fit, but Models 2 and 3 exhibited a slightly better fit. The chi-square ratios for Models 1, 2, and 3 were .80, .37, and .45 respectively. The subscale correlations located in the lower portion of the table were relatively high, which indicated unidimensionality of the math items.

Table 8A2 reports results that are similar to those in Table 8A1 for the subjects who selected *0-2 types* [1] or *4 types* [3] as their response to the background variable (question) “Home Environment—Reading Materials.” For the first subgroup (those students who selected *0-2 types* [1] as their answer), the indices of fit indicated that all three models fit the data at approximately the same level. For example, the chi-square ratios for Models 1, 2, and 3 were 1.70, 1.37, and 1.22 respectively. For the second subgroup (those students who selected *4 types* [3] as their answer choice), the indices of fit indicated that all three models had a relatively good fit to the data, but again Models 2 and 3 exhibited a slightly better fit. The chi-square ratios were 2.31, 1.53, and 1.57 for the three models respectively.

However, the lower part of this table does indicate that the correlations between the subscale latent variables for the first subgroup [1] were lower than the collateral correlations that were obtained from the analyses on the total group(s) of subjects for Booklet 8 (see Table TA4). (Results of analysis in Table 8A3 are not discussed.)

Table 8A4 presents the results of the analyses conducted on the subjects who responded by selecting *pre-algebra* [3] to the “What Kind of Math Class Are You Taking This Year” background variable (question) for Grade 8, Booklet 8. The fit indices in this table indicate that Models 2 and 3 exhibited a better fit to the data than Model 1. The chi-square ratios were 2.00, 1.54, and 1.66 for the three models respectively. The subscale correlations, reported at the bottom section of the table, reveal relatively lower correlations than those that were reported for the corresponding total subject analyses.

Table 8A5 summarizes the results of those students who selected *strongly agree* [1] or *agree* [2] to the "Do You Agree: I Am Good in Math" background variable (question) for Booklet 9. For the first subgroup (those students who chose *strongly agree* [1]), the indices of fit indicated that Models 2 and 3 exhibited a better fit to the data. The chi-square ratios were 3.20, 1.96, and 2.03 for the three models respectively. For the second subgroup (those students who chose *agree* [2]), the indices of fit indicated that the three models fit the data as well. The chi-square ratios were 1.39, 1.31, and 1.29 for the three models respectively. The subscale latent variable correlations for the two subgroups, reported in the bottom section of the table, are all relatively high but are slightly lower than the corresponding correlations that were reported for the total group analyses (see Table TA5).

The results of the analyses reported in this table are comparable with Table 8A1, which summarizes the results of the analysis that was conducted on the same background variable (questions/answer choices) for Booklet 8. The comparison of these two tables reveals a consistency in the results across independent samples of students. Likewise, the results of the analyses reported in Table 8A6 are comparable with the results reported in Table 8A2; the results in Table 8A7 are comparable with the results presented in Table 8A3; and the data presented in Table 8A8 are comparable with those in Table 8A4.

Tables 8A9, 8A10, 8A11, and 8A12 report the results of the analyses that were conducted on the data from Booklet 10. Table 8A9 can be compared with Tables 8A1, and 8A5; Table 8A10 is comparable with Tables 8A2, and 8A6; Table 8A11 can be compared with Tables 8A3, and 8A7; and finally, Table 8A12 can be compared with Tables 8A4 and 8A8. These comparisons also reveal a consistency in the findings between the different groups of 8th-grade students who answered math items in Booklets 8, 9, and 10.

We can now turn to the results of the analyses on the items that were selected based on their correlations with the background variables.

Table IA4 presents the results of the latent variable analysis that was conducted on the selected items for Grade 8, Booklet 8. As this table indicates, all three of the models fit the data at approximately the same level, but Models 2 and 3 exhibited a slightly better fit. The chi-square ratio was 1.65 for Model 1, 1.17 for Model 2, and 1.15 for Model 3. The lower half of the table presents the correlations between the subscale latent variables. These correlations were considerably lower than the correlations that were obtained from the corresponding analyses on the total group (see Table TA4).

Table IA5 summarizes the results of the analyses that were conducted on the selected items from Booklet 9. The three models in this analysis fit the data at about the same level. The chi-square ratios were 2.65 for the first model, 2.65 for the second model, and 2.33 for the third model. The correlations between the subscale latent variables were lower in comparison to the corresponding correlations that were obtained from the analyses conducted on the total group (see Table TA5).

Table IA6 presents the results of the analyses for the selected items in Booklet 10. These results were consistent with the results that were presented in Table IA4 for Booklet 8 and in Table IA5 for Booklet 9.

Results for Grade 12. The results that were obtained for Grade 12 were similar to those that were obtained for Grades 4 and 8. Latent variable modeling was performed on the data from Booklets 8, 9 and 10. The three hypotheses—(1) a general math factor underlying the five subscales; (2) the five subscale factors; and (3) the general math factor plus five subscale factors—were examined. This was accomplished by applying the three different models (each representing one of the above hypotheses) to the data that included all of the students' answers to all of the items in any of the given booklets. We also applied the three models to data that consisted of the subjects who were grouped into subgroups based on their chosen responses to the selected background variables. And the three models were applied to the items that were selected based on their correlations with the background variables' subscale scores.

First, we will discuss the results of the analyses that were performed on the total groups of subjects who answered all the items in a given booklet. Table TA7 summarizes the results of the analyses for Grade 12 Booklet 8. The upper half of this table presents the indices of fit for the three models (Model 1, which assumes one general math factor; Model 2, which assumes five subscale factors; and Model 3, which assumes five subscale factors plus one general factor). The indices of fit indicated that Model 1 does not fit the data as well as Models 2 and 3. The chi-square ratio for Model 1 was 3.83, 1.56 for Model 2, and 1.95 for Model 3. The correlations between the subscale latent variables (listed in the lower section of the table) were, however, all high indicating a unidimensionality of the math items.

Table TA8 presents similar results for Grade 12, Booklet 9. Consistent with the results presented in Table TA7, Models 2 and 3 exhibit a slightly better fit than Model 1. The chi-square ratios were 5.31, 3.19 and 3.97 for the three models respectively. The correlations between the subscale latent variables (listed in the lower half of the table) were all extremely high and indicated unidimensionality of the math items.

Table TA9 summarizes the results of the analyses conducted on the data from Booklet 10. The results in this table are very consistent with those results that were reported in Table A7 and Table A8. The indices of fit in this analysis reveal that Models 2 and 3 clearly fit the data better than Model 1. The chi-square ratios were 4.62 for Model 1, 2.08 for Model 2, and 2.77 for Model 3. The very high correlations between the five subscale latent variables, however, indicated a unidimensionality of the math items.

We then applied latent variable analyses to the subgroups that were formed based on the responses to the selected background variables (questions) for Grade 12. The same background variables that were used in the previous 4th- and 8th-grade analyses were applied to the data in Booklets 8, 9 and 10.

Table 12A1 presents the results of the latent variable modeling analysis that was conducted on the subjects who indicated that they *strongly agree* [1] or *agree* [2] to the background variable (question) "Do You Agree: I Am Good in Math" for Booklet 8, Grade 12. The top section of the table lists the indices of fit for the first subgroup (those subjects who selected *strongly agree* [1] to the above background variable [question]). The middle section of the table lists the indices of fit for the second subgroup (those subjects who selected *agree* [2]). Finally, the bottom section of the table lists the correlations between the subscale latent variables for the two subgroups that are defined above.

The indices of fit for the first subgroup indicated that all three models fit the data but, Models 2 and 3 exhibited a much better fit to the data than Model 1. The chi-square ratios for subgroup 1 were 2.08, .84, and 1.08 and the chi-square ratios for subgroup 2 were .81, .33, and .41. The correlations between the subscale latent variables were relatively high, but they were not as high as those that were obtained for the corresponding total group analysis (see Table TA7).

Table 12A2 summarizes the results of the analyses that were performed on the subgroups that were formed based on the response choices to the selected background variable “Home Environment—Reading Materials.” Two subgroups were formed based on the subjects who selected either *0-2 types* [1] or *3 types* [2] as their response to the above background variable (question). The top section of the table reports the fit indices for the first subgroup (those subjects who selected *0-2 types* [1]). The middle section of the table reports the fit indices for the second subgroup (those subjects who selected *3 types* [2]). Finally, the lower section lists the correlations between the subscale latent variables for the two subgroups.

For the first subgroup, the indices of fit indicated that the three models fit the data at about approximately the same level. The chi-square ratios were 1.51, 1.53, and 1.55 for the three models. In the second subgroup, the three models also fit the data at approximately the same level, but Models 2 and 3 exhibited a better fit. The ratios for the second subgroup were 2.54, 1.44, and 1.29. Consistent with the results that were presented in Table 12A1, the correlations between the subscale latent variables were all high but not as high as the correlations that were obtained for the corresponding analysis for the total group (compare this table with Table TA7).

Table 12A3 presents the results of the latent variable modeling for the subgroups that were formed based on the selected answer choices to the “In Math Class How Often Do Problems on Worksheet” background variable (question) for Booklet 8. Two subgroups were formed based on the students who chose either *almost every day* [1] or *several times a week* [2] as their response the above background variable (question). For the first subgroup (students who selected *almost every day* [1]), the fit indices indicated that all three models performed about the same, but Models 2 and 3 did slightly better. The chi-square ratios were 1.98, 1.56, and 1.56 for the three models respectively. Similar fit indices were obtained for the second subgroup (students who selected *several times a week* [2]). The three models in the second subgroup fit the data at about the same level, and again, Models 2 and 3 did slightly better. The chi-square ratios for the three models were: 1.55, 1.19, and 1.08 respectively. The correlations between the subscale latent variables were all high, but they were slightly lower than the corresponding correlations that were obtained for the total group analysis (see Table TA7).

Tables 12A4, 12A5, and 12A6 present the results of the analyses for Booklet 9 and are similar to the results that were reported in Tables 12A1, 12A2, and 12A3 for Booklet 8. Likewise, Tables 12A7, 12A8, and 12A9 summarize concordant results for Booklet 10. Thus, one could compare the data in Table

12A1 with the data in Table 12A4 and 12A7 or compare the data in Tables 12A2, 12A5, and 12A8. Also Tables 12A3, 12A6, and 12A9 can be compared. These highly comparable results can be used as a cross-validation for the results that have been obtained throughout the study.

The results of the analyses on the items parcels that consisted of the items that were selected based on their item-background correlations are reported in Tables IA7, IA8, and IA9. Table IA7 summarizes the results of the analysis that was conducted on the selected items in Booklet 8. The upper section of the table lists the fit indices for the three models and the lower section lists the correlations between the subscale latent variables. The indices of fit indicate that the three models performed very similarly. For instance, the chi-square ratios of the three models were 2.56, 2.68, and 2.84, and the correlations between the subscale scores ranged from a low of .506 to a high of 1.0. These correlations were lower than those that were reported in the total group's corresponding analysis.

Table IA8 presents the results for Booklet 9. The fit indices in this table indicate that the three models performed about the same. The chi-square ratios were 3.00, 2.54, and 2.37 for the three models respectively. Consistent with the data reported for Booklet 8, the correlations between the subscale latent variables were considerably lower than those that were observed in the corresponding analysis of the total group.

Finally, Table IA9 summarizes the results for Booklet 10. When comparing Tables IA7, IA8, and IA9 (which all report the results of the analyses of the same background variable [question/answer] sequence for the three different booklets) a consistency in the results can be clearly determined.

Results of the Factor Analysis for the 1992 Data

Because there were fewer subjects per booklet in the 1992 administration than in the 1990 administration we could not successfully divide the subjects into equitable subgroups based on their response patterns to the background variables (questions). Therefore, we performed our analyses on the items that displayed different levels of correlations with the background variables.

For the 1992 analyses, we used Booklets 15 and 17 for Grade 4; Booklets 1, 2, and 15 for Grade 8; and Booklets 1, 15, and 17 for Grade 12. We applied the same three models that we used in the 1990 data to the 1992 data: Model 1, a one-general-factor model; Model 2, a five-subscale factor model; and Model 3, a five-subscale-plus-one general-factor model. We performed our structural equation modeling on the total groups of subjects that answered all of the items in one of the given booklets. We also applied our structural models to the item parcels that consisted of the items that were selected based on the item-background correlations. We will discuss the results of our analyses separately for each of the three grades. For each grade, we will discuss the results of the analyses that were conducted on all of the items and on the selected items.

Results for Grade 4. Table TB1 summarizes the results of the analyses that were conducted on the total groups of 4th-grade students who answered the math items in Booklet 15. The upper section of the table lists the indices of fit for the three models, and the lower section lists the correlations between the subscale latent variables. The indices of fit for the three models indicated that all of the models fit the data at approximately the same level. For example, the chi-square ratios for the three models were 1.45, 1.77, and 1.67 respectively, and the correlations between the subscale latent variables were all very high indicating a unidimensionality of the math items.

Table TB2 presents the results of the analyses done on Booklet 17. Consistent with the results presented in Table TB1, the indices of fit indicated that all three models equally fit the data. The chi-square ratios were 1.32, 1.36, and 1.40 for the three models, and the intercorrelations between the subscales were also high and indicative of unidimensionality.

Table IB1 presents the results of the analyses that were conducted on the selected items for Grade 4, Booklet 15. The indices of fit for the three models indicated that all they all fit the data equally, but the subscale latent variable correlations were lower than those reported for the total items (see Table TB1).

Table IB2 summarizes the results of the analyses that were conducted on the selected items from Booklet 17. The indices of fit in this table also indicate that the three models performed about the same level. The chi-square ratios were 1.08, 1.34, and 1.23 for the three models respectively, and the subscale correlations were also all high.

Results for Grade 8. Table TB3 reports the results of the analyses on the total groups of 8th-grade students who answered the math items in Booklet 1. The indices of fit indicated that Models 2 and 3 performed much better than Model 1.

The chi-square ratios were 3.3, 1.2, and 1.1 for the three models respectively. The correlations between the subscale latent variables also indicated that some of the subscales were not highly related. For example, the correlations between the Statistics subscale with the Numbers and Operations, Measurement, and Geometry subscales were .78, .78, and .82. These correlations were relatively lower than those that were reported in the 1990 data.

Table TB4 reports similar results for Booklet 2, Grade 8. Consistent with the results that were reported in Table TB3, the indices of fit in this table indicate that Models 2 and 3 performed better than Model 1. The chi-square ratios were 1.15, .61, and .65 for the three models respectively.

Table IB3 summarizes the results of the structural modeling that was conducted on the selected items from Booklet 1, Grade 8. The indices of fit are reported in the upper section of the table and indicate that all of the three models fit the data but that Models 2 and 3 fit the data better than Model 1. The subscale correlations, reported in the lower half of the table, are considerably lower than those that were reported for the parcels containing all the items (see Table TB3). For example the correlations between the Numbers and Operations subscale with the other four subscales were .82, .57, .53, and .64 and were lower than the corresponding subscale correlations (.93, .97, .78, and .95) in the total item analysis.

Table IB4 presents similar results for the selected items from Booklet 2, Grade 8. The indices of fit in this table indicate that all three models performed at about the same level. The chi-square ratios were 1.27, 1.70, and 1.46 for the three models respectively, and the subscale correlations were relatively high as well.

Table IB5 presents the results of the analyses done on Booklet 15, and the results are similar to those that are reported in Tables IB3, and IB4. The indices of fit in this analysis indicated that all three of the models fit the data, but Models 2 and 3 performed slightly better. The chi-square ratios were 2.25, 1.23, and 1.24 for the three models respectively. The subscale correlations were considerably lower than those that were reported for the cases in which all of the items were used in the parcels (see Table TB5).

Results for Grade 12. Table TB6 presents the results of the analyses that were conducted on all of the items from Grade 12, Booklet 1. The models appeared to perform about the same based on the indices of fit that are reported in this table. The chi-square ratios were 1.64, 1.36, and 1.22 for the three models respectively. The subscale correlations were relatively high, although not as high as those that were reported in the 1990 data.

Table TB7 presents the results of the analyses that were conducted on Booklet 15. The three models performed about the same in this analysis, and these findings were consistent with the results that were reported in Table TB6. The subscale correlations were all very high indicating unidimensionality of the math items.

Table TB8 reports the results of the analyses conducted on Booklet 17. All of the three models in this analysis fit the data, but Models 2 and 3 performed better once again. The correlations between the subscale latent variables were all relatively high but not as high as those that were reported in the 1990 data.

Table IB6 summarizes the results of the analyses that were conducted on the selected items from Booklet 1. The indices of fit indicated that the three models fit the data very well. However, the correlations between the subscale latent variables were drastically lower than those that were reported for the item parcels that were constructed from the total items (see Table TB6). For example, the correlations between the Numbers and Operations subscale with the other four subscales were .510, .313, .346 and .557 whereas the same series of comparative correlations for the total item parcels were .91, .81, .79, and .87.

Tables IB7 and IB8 present the results of the analyses that were conducted on the selected items for Booklet 15 and 17 respectively. These tables are comparable with Table IB6, which reports almost identical data for Booklet 1. The indices of fit in Tables IB7 and IB8 indicate that all three models performed equally well. The correlations between the subscales reported in both tables were considerably lower than those correlations obtained from the item parcels created from the total item set.

Discussion

To test the dimensionality of the math items, we examined the following three hypotheses for the 1990 and 1992 NAEP math data:

1. all NAEP math items measure the same underlying math ability;

2. the NAEP math items measure five different subscales (i.e., Numbers and Operations, Measurement, Algebra, Geometry, and Data Analysis); and

3. in addition to the five different subscales, a general math ability factor underlies the math items.

We created three different latent variable models that corresponded to each of the three hypotheses. The results of our analyses that were performed on the data for the total groups of subjects that answered the math items in the given booklets supported the first hypothesis. The math subscale scores were highly correlated and were primarily unidimensional.

The results were consistent across all three grade levels and for both the 1990 and 1992 data sets. However, when the background variables that had significant relationships with students' math performance were used in connection with the math subscale scores, more discrimination among the math subscale scores was found. The results of the analyses that were performed on the subgroups (that were formed based on the responses/choices to the selected background variables) supported Hypotheses 2 and 3. These results indicated that the NAEP math items are multidimensional. For example, for the 1990 data, in all three grade levels, when the analyses were performed on the subgroups, lower correlations were obtained and more evidence of multidimensionality was observed.

Similarly, the results of the analyses that were conducted on the item parcels consisting of items that were correlated with the selected background variables for both the 1990 and 1992 data indicated multidimensionality of the NAEP math items. Evidence of multidimensionality was more visible in Grades 8 and 12 than in Grade 4.

Caution should be used when comparing the results of the analyses obtained on the total groups of subjects for each of the booklets versus the results that were obtained from the subgroups. Several factors could have contributed to the differences between the statistics that were obtained for the total group and the statistics that were obtained for the subgroups. The number of subjects, within-group homogeneity, and student ability/level are just a few of the factors that could produce some of these differences.

Discretion should also be applied when comparing the results of the analyses that were performed on the item parcels that included all the items versus the results that were obtained from the item parcels comprised of only selected items (based on the item-background correlations). Factors such as the number of items, item characteristics, differences in the homogeneity of the items within the parcels, and differences in the parcel means and variances across the parcels could account for some of the differences in the statistics across parcel group analyses.

The findings in this study also reveal that a one-general-math-factor model may not be the most effective way to describe the NAEP math subscale scores. The statistics that were obtained across the three grade levels/ages indicate that the five-factor model and the five-subscale-one-general-factor model exhibited better fits to the data than the unidimensional one-factor model.

REFERENCES

Abedi, J. (1993). *Multi-Approach Correlation System (MACS), a computer program*. Los Angeles: University of California, Center for the Study of Evaluation.

Abedi, J. (1996). Interrater/Test Reliability System (ITRS). *Multivariate Behavioral Research, 31*, 409-417.

Allen, N. L. (1992). Data analysis for the science assessment. In E. G. Johnson & N. L. Allen (Eds.), *The NAEP 1990 technical report* (Report No. 21-TR-20, pp. 275-301). Washington, DC: National Center for Education Statistics.

Anderson, T. W. (1959). Some scaling models and estimation procedures in the latest class model. In U. Grenander (Ed.), *Probability and statistics: The Harold Cramer volume*. New York: Wiley.

Armor, D. J. (1974). The reliability and factor scaling. In H. L. Costner (Ed.), *Sociological methodology* (pp. 17-50). San Francisco CA: Jossey-Bass.

Beaton, A. E., & Johnson, E. G. (1992). Overview of the scaling methodology used in the national assessment. *Journal of Educational Measurement*, *29*, 163-175.

Beaton, A. E., Johnson, E. G., & Ferris, J. J. (1987). The assignment of exercises to students. In A. E. Beaton (Ed.), *Implementing the new design: The NAEP 1983-84 technical report* (pp. 97-118). Princeton, NJ: Educational Testing Service.

Bejar, I. (1980). A procedure for investigating the unidimensionality of achievement tests based on item parameter estimates. *Journal of Educational Measurement*, *7*, 283-296.

Bock, R. D., & Aitkin, M. (1981). Marginal maximum likelihood estimation of item parameters: An application of the EM algorithm. *Psychometrika*, *46*, 443-459.

Bock, R. D., Gibbons, R. D., & Muraki, E. (1985). *Full-information item factor analysis* (MRC Rep. No. 85-1). Chicago, IL: National Opinion Research Center.

Bock, R. D., & Lieberman, M. (1970). Fitting a response model for *n* dichotomously scored items. *Psychometrika*, *35*, 179-197.

Bollen, K. A., & Long, J. S. (1992). Tests for structural equation models. *Sociological Methods and Research, 21*, 123-131.

Carlson, J., & Jirele, T. (1992, April). *Dimensionality of 1990 NAEP mathematics data*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Carmines, E. G., & Zeller, R. A. (1979). *Reliability and validity assessment*. Beverly Hills, CA: Sage.

Carroll, J. B. (1945). The effect of difficulty and chance success on correlation between items or between tests. *Psychometrika*, *10*, 1-19.

Carroll, J. B. (1983). The difficulty of a test and its factor composition revisited. In H. Wainer & S. Messick (Eds.), *Principals of modern psychological measurement: A festschrift for Frederic M. Lord* (pp. 257-282). Hillsdale, NJ: Lawrence Erlbaum Associates.

Castellan, N. J. (1966). On the estimation of the tetrachoric correlation coefficient. *Psychometrika*, *31*, 67-73.

Cattell, R. B. (1956a). A shortened "basic English" version (Form C) of the 16 PF Questionnaire. *Journal of Social Psychology, 44,* 257-278.

Cattell, R. B. (1956b). Validation and intensification of the Sixteen Personality Factor Questionnaire. *Journal of Clinical Psychology, 12,* 205-214.

Cattell, R. B., & Burdsal, A. C. (1975). The radial parcel double factoring design: A solution to the item-vs-parcel controversy. *Multivariate Behavioral Research*, *10*, 165-179.

Christofferson, A. (1975). Factor analysis of dichotomized variables. *Psychometrika*, *40*, 5-32.

Cook. L. L., Dorans, N., & Eignor, D. (1988). An assessment of the dimensionality of three SAT-verbal test editions. *Journal of Educational Statistics*, *13*, 19-43.

Cook. L. L., Dorans, N., Eignor, D., & Petersen, N. (1983). *An assessment of the relationship between the assumption of unidimensionality and the quality of IRT true-score equating*. Paper presented at the annual meeting of the American Educational Research Association, Montreal, Canada.

Cook, L. L., & Eignor, D. (1984, April). *Assessing the dimensionality of NAEP reading test items: Confirmatory factor analysis of item parcel data.* Paper presented at the annual meeting of the American Educational Research Association, New Orleans.

Cortina, J. M. (1993). What is coefficient alpha? An examination of theory and applications. *Journal of Applied Psychology*, *78*, 98-104.

Cronbach, L. J. (1951) Coefficient alpha and the internal structure of tests. *Psychometrika*, *16*, 297-334.

Divgi, D. R. (1979). Calculation of the tetrachoric correlation coefficient. *Psychometrika*, *44*, 169-172.

Drasgow, F., & Parsons, C. K. (1983). Application of unidimensional item response theory models to multidimensional data. *Applied Psychological Measurement, 7*, 189-199.

Eysenck, H. J., & Eysenck, S. (1969). *Personality structure and measurement*. San Diego, CA: R. R. Knapp.

Gerbing, D. W., & Anderson, J. C. (1992). Monte Carlo evaluations of goodness of fit indices for structural equation models. *Sociological Methods and Research 21*, 132-160.

Gold, Karen F. (1990). *Applications of hierarchical confirmatory factor models: Assessment of structure and integration of knowledge exhibited in mathematics achievement data*. Unpublished doctoral dissertation, Los Angeles, University of California.

Gold, K. F., & Muthén, B. (1991, April). *Extensions of covariance structure analysis: hierarchical modeling of multidimensional achievement data*. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.

Green, B. F. (1956). A method of scalogram analysis using summary statistics. *Psychometrika*, *21*, 79-88.

Green, S. B., Lissitz, R. W., & Mulaik, S. A. (1977). Limitations of coefficient alpha as an index of test unidimensionality. *Educational and Psychological Measurement*, *37*, 827-838.

Gustafsson, J. E., & Lindblad, T. (1978). *The Rasch model for dichotomous items: A solution of the conditional estimation problem for long tests and some thoughts about item screening procedures* (Rep. No. 67). Göteborg, Sweden: University of Göteborg, Institute of Education.

Guttman, L. (1944). A basis for scaling qualitative data. *American Sociological Review*, *80*, 139-150.

Guttman, L. (1950). The principal components of scale analysis. In S. S. Stouffer (Ed.), *Measurement and prediction* (pp. 312-361). Princeton NJ: Princeton University Press.

Guttman, L. (1953). Image theory for the structure of quantitative variates. *Psychometrika*, *18*, 277-296.

Hambleton, R., & Rovinelli, R. (1983). *Assessing the dimensionality of a set of test items*. Paper presented at the annual meeting of the American Educational Research Association, Montreal, Canada.

Hambleton, R., & Rovinelli, R. (1986) Assessing the dimensionality of a set of test items. *Applied Psychological Measurement, 10*, 287-302.

Hambleton, R. K., & Swaminathan, H. (1985). *Item response theory: Principles and applications*. Boston: Kluwer-Nijhoff.

Hambleton, R. K., Swaminathan, H., & Rogers, H. J. (1991). *Fundamentals of item response theory* (Measurement Methods for the Social Sciences series, Vol. 2). Newbury Park, CA: Sage Publications

Hattie, J. (1981). A four stage factor analytic approach to studying behavioral domains. *Applied Psychological Measurement, 5*, 77-88.

Hattie, J. (1984). An empirical study of various indices for determining unidimensionality. *Multivariate Behavioral Research, 19,* 49-78.

Hattie, J. (1985). Methodology review: assessing unidimensionality of tests and items. *Applied Psychological Measurement*, *9*, 139-164.

Heise, D. R., & Bohrnstedt, G. W. (1970). Validity, invalidity, and reliability. In E. F. Borgatta & G. W. Bohrnstedt (Eds.), *Sociological methodology* (pp. 104-129). San Francisco, CA: Jossey-Bass.

Horst, P. (1953). Correcting the Kuder-Richardson reliability for dispersion of item difficulties. *Psychological Bulletin*, *50*, 371-374.

Howarth, E., & Browne, J. A. (1971). An item analysis of the 16 of personality questionnaire. *Personality: An International Journal*, *2*, 117-139.

Hulin, C. L., Drasgow, F., & Parsons, C. (1983). *Item response theory: Applications to psychological measurement*. Homewood, IL: Dow Jones–Irwin.

Hutten, L. (1980, April). *Some empirical evidence for latent trait model selection*. Paper presented at the annual meeting of the American Educational Research Association, Boston, MA.

Jackson, J. M. (1949). A simple and more rigorous technique for scale analysis. In *A manual for scale analysis, Part 11*. Montreal: McGill University.

Johnson, E. G., & Allen, N. L. (Eds.). (1992). *The NAEP 1990 technical report* (Report No. 21-TR-20). Washington, DC: National Center for Education Statistics.

Jones, P., Sabera, D., & Trosset, M. (1987, April). *Dimensionality assessment for dichotomously scored items using multidimensional scaling*. Paper presented at the annual meeting of the American Educational Research Association, Washington, DC.

Jöreskog, K. G. (1978). Structural analysis of covariance and correlation matrices. *Psychometrika*, *43*, 443-447.

Kaiser, H. F. (1970). A second generation little jiffy. *Psychometrika*, *35*, 401-415.

Kaiser, H. F., & Cerny, B. A. (1979). Factor analysis of the image correlation matrix. *Educational and Psychological Measurement*, *39*, 99-101.

Kirk, D. B. (1973). On the numerical approximation of the bivariate normal (tetrachoric) correlation coefficient. *Psychometrika*, *38*, 259-268.

Lawrence, I. M., & Dorans, N. J. (1987, April). *An assessment of the dimensionality of SAT-mathematics*. Paper presented at the annual meeting of the National Council on Measurement in Education, Washington, DC. (ERIC Document Reproduction No. ED 282 898)

Lord, F. M. (1958). Some relations between Guttman's principal components of scale analysis and other psychometric theory. *Psychometrika*, *23*, 291-296.

Lord, F. M. (1980). *Application of item response theory to practical testing problems*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Lord, F. M., & Novick, M. R. (1968). *Statistical theories of mental test scores*. Reading MA: Addison Wesley.

Loevinger, J. (1944). *A systematic approach to the construction of tests of ability*. Unpublished doctoral dissertation, Los Angeles, University of California.

Lumsden, J. (1957). A factorial approach to unidimensionality. *Australian Journal of Psychology*, *9*, 105-111.

Lumsden, J. (1959). *The construction of unidimensional tests*. Unpublished master's thesis, Perth, University of Western Australia.

Lumsden, J. (1961). the construction of unidimensional test. *Psychological Bulletin*, *58*, 122-131.

Marsh, H. W., Balla, J. R., & McDonald, R. P. (1988). Goodness of fit indices in confirmatory factor analysis: The effect of sample size. *Psychological Bulletin*, *103*, 391-410.

McDonald, R. P. (1967). Factor interaction in nonlinear factor analysis. *British Journal of Mathematical and Statistical Psychology, 20,* 82-99.

McDonald, R. P. (1970). The theoretical foundations of principal factor analysis, canonical factor analysis, and alpha factor analysis. *British Journal of Mathematical and Statistical Psychology*, *23*, 1-21.

McDonald, R. P. (1981). The dimensionality of tests and items. *British Journal of Mathematical and Statistical Psychology*, *34*, 100-117.

McDonald, R. P. (1982). Linear versus nonlinear models in item response theory. *Applied Psychological Measurement*, *6*, 379-396.

McDonald, R. P., & Ahlawat, K. S. (1974). Difficulty factors in binary data. *British Journal of Mathematical and Statistical Psychology, 27*, 82-99.

Mislevy, R. J. (1986). Recent developments in the factor analysis of categorical variables. *Journal of Educational Statistics*, *11*, 3-31.

Muthén, B. (1978). Contributions to factor analysis of dichotomous variables. *Psychometrika*, *43*, 551-560.

Muthén, B. (1981). Factor analysis of dichotomous variables: American attitudes toward abortion. In D. J. Jackson & E. F. Borgatta (Eds.), *Factor analysis and measurement in sociological research: A multidimensional perspective* (pp. 201-214). London: Sage.

Muthén, B. (1988). Some uses of structural equation modeling in validity studies: Extending IRT to external variables. In H. Wainer & H. Braun (Eds.), *Test validity* (pp. 213-238). Hillsdale, NJ: Lawrence Erlbaum.

Muthén, B. (1989a). Latent variable modeling in heterogeneous populations. *Psychometrika*, *54*, 557-585.

Muthén, B. (1989b). Using item-specific instructional information in achievement modeling. *Psychometrika*, *54*, 385-396.

Muthén, B. (1992). *Latent variable modeling of achievement development using national test data*. Los Angeles, University of California, Graduate School of Education, National Center for Research on Evaluation, Standards and Student Testing.

Muthén, B., & Christofferson, A. (1981). Simultaneous factor analysis of dichotomous variables in several groups. *Psychometrika*, *46*, 407-419.

Muthén, B., Kao, C., & Burstein, L. (1991). Instructionally sensitive psychometrics: Application of a new IRT-based detection technique to mathematics achievement test items. *Journal of Educational Measurement*, *28*, 1-22.

Nunnally, J. C. (1978). *Psychometric theory* (2nd ed.). New York: McGraw-Hill.

Olsson, U. (1979). On the robustness of factor analysis against crude classification of the observations. *Multivariate Behavioral Research*, *14*, 485-500.

Pedhazur, E. J. (1982). *Multiple regression in behavioral research: Explanation and prediction* (2nd ed.). New York: Holt, Rinehart, and Winston.

Reckase, M. D. (1979). Unifactor latent trait models applied to multifactor tests: Results and implications. *Journal of Educational Statistics*, *4*, 207-230.

Reckase, M. D. (1981). *The formation of homogeneous item sets when guessing is a factor in item responses* (Office of Naval Research Rep. 81-85). Columbia: University of Missouri, Department of Educational Psychology. [ERIC Document Reproduction No. ED20935]

Rock, D. A. (1991, October). *Subscale dimensionality*. Paper presented at the annual meeting of the NAEP Design and Analysis Committee, Washington, DC.

Rogers, H. J. (1984). *Fit statistics for latent trait models*. Unpublished master's thesis, Armidale, Australia, University of New England.

Stevens, J. (1992). *Applied multivariate statistics for the social sciences* (2nd ed.). Hillsdale, NJ: Lawrence Erlbaum Associates.

Stout, W. F. (1990). A new item response theory modeling approach with applications to unidimensionality assessment and ability estimation. *Psychometrika, 55*, 293-325.

Swinton , S. S., & Powers, D. E. (1980). *A factor analytic study of the restructured GRE Aptitude Test.* (GRE Board Professional Report GREB No. 77-6P). Princeton, NJ: Educational Testing Service.

Tucker, L. R., & Lewis, C. (1973). A reliability coefficient for maximum likelihood factor analysis. *Psychometrika*, *38*, 1-10.

van den Wollenberg, A. L. (1982a). A simple and effective method to test the dimensionality axiom of the Rasch model. *Applied Psychological Measurement*, *6*, 83-91.

Velicer, W. F., & Jackson, D. N. (1990). Component Analysis versus common factor analysis: some further observations. *Multivariate Behavioral Research, 25*, 97-114.

Yamamoto, K., & Jenkins, F. (1992). Data analysis for the mathematics assessment. In E. G. Johnson & N. L. Allen (Eds.), *The NAEP 1990 technical report* (Report No. 21-TR-20, pp. 243-274). Washington, DC: National Center for Education Statistics.

Zwick, R. (1985). *Assessment of the dimensionality of NAEP year 15 reading data* (ETS Res. Rep. No. 86-4). Princeton: Educational Testing Service.

Zwick, R. (1986). *Assessment of the dimensionality of NAEP year 15 reading data.* Princeton, NJ: Educational Testing Service. [ERIC Document Reproduction No. ED269420]

Zwick, R. (1987). Assessing the dimensionality of NAEP reading data. *Journal of Educational Measurement*, *24*, 293-308.

APPENDIX

Tables

Discriminant Analysis 1990 Data:	Tables A1–A27
Discriminant Analysis 1992 Data:	Tables B1–B18
Factor Analysis 1990 Data:	Tables TA1–TA9
	Tables 4A1–4A15
	Tables 8A1–8A12
	Tables 12A1–12A9
	Tables IA1–IA9
Factor Analysis 1992 Data:	Tables TB1–TB8
	Tables IB1–IB8

Discriminant Analysis 1990 Data: Tables A1–A27

Table A1
Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 4, Booklet 11, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Number	.92142	.92091	53.03	.0000
Measure	.77099	.94320	37.19	.0000
Geometry	.51570	.97376	16.64	.0000
Statistics	.78546	.94127	38.53	.0000
Algebra	.59410	.96463	22.64	.0000
Function 1	%Var = 98.12	Canon R = .30		
After Function 0	Λ =.9064	χ^2 = 121.18	df = 10	p < .0000

Table A2
Results of Discriminant Analysis on: "How Do You Feel About This Statement: I Am Good in Math"
(Grade 4, Booklet 11, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Number	.99283	.90241	65.16	.0000
Measure	.71850	.94634	34.16	.0000
Geometry	.51275	.97156	17.63	.0000
Statistics	.59753	.96144	24.16	.0000
Algebra	.56537	.96512	21.78	.0000
Function 1	%Var = 96.84	Canon R = .31		
After Function 0	Λ = .8979	χ^2 = 129.53	df = 10	p < .0000

Table A3
Results of Discriminant Analysis on: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 11, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Number	.88425	.95606	14.10	.0000
Measure	.76097	.96595	10.81	.0000
Geometry	.73046	.96910	9.78	.0000
Statistics	.66029	.97517	7.81	.0000
Algebra	.78369	.96571	10.89	.0000
Function 1	%Var = 83.52	Canon R = .23		
After Function 0	Λ = .9350	χ^2 = 82.34	df = 20	p < .0000

Table A4
Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 4, Booklet 12, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Number	.86356	.95144	31.47	.0000
Measure	.80552	.95729	27.51	.0000
Geometry	.66005	.96959	19.33	.0000
Statistics	.80546	.95706	27.66	.0000
Algebra	.77767	.95995	25.72	.0000
Function 1		%Var = 93.27	Canon R =.25	
After Function 0	Λ = .9314	χ^2 = 87.53	df = 10	$p < .0000$

Table A5
Results of Discriminant Analysis on: "How Do You Feel About This Statement: I Am Good in Math"
(Grade 4, Booklet 12, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Number	.88445	.94604	34.11	.0000
Measure	.88773	.94552	34.46	.0000
Geometry	.64649	.97017	18.39	.0000
Statistics	.59363	.97417	15.86	.0000
Algebra	.65519	.96935	18.91	.0000
Function 1		%Var = 95.50	Canon R = .26	
After Function 0	Λ = .9289	χ^2 = 87.99	df = 10	$p < .0000$

Table A6
Results of Discriminant Analysis on: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 12, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Number	.87713	.94187	18.89	.0000
Measure	.61246	.96835	10.00	.0000
Geometry	.74186	.95607	14.06	.0000
Statistics	.72039	.95887	13.13	.0000
Algebra	.78317	.95098	15.77	.0000
Function 1		%Var = 83.32	Canon R = .27	
After Function 0	Λ = .9116	χ^2 = 113.24	df = 20	$p < .0000$

Table A7
Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 4, Booklet 14, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Number	.91475	.93152	44.85	.0000
Measure	.83640	.94191	37.62	.0000
Geometry	.68310	.95917	25.96	.0000
Statistics	.73941	.95337	29.84	.0000
Algebra	.72215	.95568	28.29	.0000
Function 1		%Var = 93.36	Canon R = .28	
After Function 0	Λ = .9136	χ^2 = 110.07	df = 10	$p < .0000$

Table A8
Results of Discriminant Analysis on: "How Do You Feel About This Statement: I Am Good in Math"
(Grade 4, Booklet 14, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Number	.93878	.92254	49.41	.0000
Measure	.84450	.93631	40.03	.0000
Geometry	.51278	.97433	15.50	.0000
Statistics	.56274	.97033	18.00	.0000
Algebra	.57943	.96790	19.51	.0000
Function 1		%Var = 97.03	Canon R = .29	
After Function 0	Λ = .9104	χ^2 = 110.32	df = 10	$p < .0000$

Table A9
Results of Discriminant Analysis on: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 14, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Number	.90005	.97909	6.55	.0000
Measure	.86675	.98071	6.03	.0001
Geometry	.67952	.98431	4.89	.0006
Statistics	.59714	.98854	3.55	.0068
Algebra	.79814	.98325	5.23	.0004
Function 1		%Var = 66.81	Canon R = .16	
After Function 0	Λ = .9619	χ^2 = 47.52	df = 20	$p < .0005$

Table A10
Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 8, Booklet 8, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.95963	.91341	58.06	.0000
Measure	.78242	.94067	38.63	.0000
Geometry	.76156	.94277	37.18	.0000
Statistics	.84091	.93215	44.58	.0000
Algebra	.82414	.93466	42.82	.0000
Function 1	%Var = 97.47	Canon R = .31		
After Function 0	Λ = .9043	χ^2 = 123.05	df = 10	p < .0000

Table A11
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 8, Booklet 8, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.96642	.88661	37.60	.0000
Measure	.76513	.92550	23.67	.0000
Geometry	.57612	.95583	13.59	.0000
Statistics	.68958	.93821	19.36	.0000
Algebra	.82390	.91461	27.45	.0000
Function 1	%Var = 96.64	Canon R = .35		
After Function 0	Λ = .8754	χ^2 = 156.31	df = 20	p < .0000

Table A12
Results of Discriminant Analysis on: "What Kind of Math Class Are You Taking This Year"
(Grade 8, Booklet 8, 1990)

	Structural coeff.	Structural coeff.	Univariate test		
Variable	Func 1	Func 2	Wilks's lambda	F-ratio	Significance level
Numbers	.79019	.58101	.85323	49.67	.0000
Measure	.66102	.15475	.89499	33.88	.0000
Geometry	.67805	.22291	.89075	35.42	.0000
Statistics	.71981	.00059	.87804	40.11	.0000
Algebra	.98956	−.06623	.79470	74.60	.0000
Function 1	%Var = 90.08		Canon R = .46		
After Function 0	Λ = .7689	χ^2 = 303.28	df = 20	p < .0000	
Function 2	%Var = 7.26		Canon R = .14		
After Function 1	Λ = .9716	χ^2 = 33.23	df = 12	p < .0009	

Table A13
Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 8, Booklet 9, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.85788	.93530	42.78	.0000
Measure	.74839	.95003	32.53	.0000
Geometry	.74801	.95002	32.54	.0000
Statistics	.91205	.92753	48.33	.0000
Algebra	.79514	.94346	37.06	.0000
Function 1		%Var = 97.87	Canon R = .29	
After Function 0	Λ = .9123	χ^2 = 113.38	df = 10	p < .0000

Table A14
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 8, Booklet 9, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.92604	.92580	23.72	.0000
Measure	.72626	.95208	14.90	.0000
Geometry	.73849	.95065	15.37	.0000
Statistics	.77313	.94626	16.81	.0000
Algebra	.86510	.93432	20.81	.0000
Function 1		%Var = 89.25	Canon R = .29	
After Function 0	Λ = .9046	χ^2 = 118.61	df = 20	p < .0000

Table A15
Results of Discriminant Analysis on: "What Kind of Math Class Are You Taking This Year"
(Grade 8, Booklet 9, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.78329	.86801	43.68	.0000
Measure	.58020	.92198	24.31	.0000
Geometry	.64085	.90673	29.55	.0000
Statistics	.66936	.89735	32.86	.0000
Algebra	.96820	.81447	65.43	.0000
Function 1		%Var = 92.08	Canon R = .44	
After Function 0	Λ = .7883	χ^2 = 273.09	df = 20	p < .000

Table A16
Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 8, Booklet 10, 1990)

Variable	Structural coeff. Func 1	Univariate test Wilks's lambda	*F*-ratio	Significance level
Numbers	.84858	.94288	36.96	.0000
Measure	.86936	.93999	38.94	.0000
Geometry	.78795	.95029	31.91	.0000
Statistics	.93932	.93085	45.31	.0000
Algebra	.70604	.95974	25.59	.0000
Function 1		%Var = 98.58	Canon R = .28	
After Function 0	Λ = .9213	χ^2 = 99.85	df = 10	p < .0000

Table A17
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 8, Booklet 10, 1990)

Variable	Structural coeff. Func 1	Univariate test Wilks's lambda	*F*-ratio	Significance level
Numbers	.96113	.92868	22.69	.0000
Measure	.78409	.95039	15.42	.0000
Geometry	.82657	.94564	16.99	.0000
Statistics	.79663	.94945	15.73	.0000
Algebra	.76291	.95365	14.36	.0000
Function 1		%Var = 90.96	Canon R = .28	
After Function 0	Λ = .9158	χ^2 = 103.85	df = 20	p < .0000

Table A18
Results of Discriminant Analysis on: "What Kind Of Math Class Are You Taking This Year"
(Grade 8, Booklet 10, 1990)

Variable	Structural coeff. Func 1	Structural coeff. Func 2	Univariate test Wilks's lambda	*F*-ratio	Significance level
Numbers	.76769	.19552	.91338	27.36	.0000
Measure	.63357	.33094	.93743	19.26	.0000
Geometry	.74784	–.16270	.91767	25.88	.0000
Statistics	.90733	–.24769	.88381	37.93	.0000
Algebra	.87190	.41695	.89025	35.57	.0000
Function 1	%Var = 85.95		Canon R = .37		
After Function 0	Λ = .8415	χ^2 = 198.91	df = 20		p < .0000
Function 2	%Var = 8.96		Canon R = .13		
After Function 1	Λ = .9746	χ^2 = 26.61	df = 12		p < .0032

Table A19
Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 12, Booklet 8, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.77337	.96160	23.80	.0000
Measure	.59208	.97711	13.96	.0000
Geometry	.75161	.96327	22.73	.0000
Statistics	.81826	.95708	26.73	.0000
Algebra	.86699	.95198	30.07	.0000
Function 1	%Var = 96.38	Canon R = .25		
After Function 0	Λ = .9351	χ^2 = 79.85	df = 10	p < .0000

Table A20
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 12, Booklet 8, 1990)

	Structural coeff.	Structural coeff.	Univariate test		
Variable	Func 1	Func 2	Wilks's lambda	F-ratio	Significance level
Numbers	.92301	.29961	.87478	39.86	.0000
Measure	.86726	–.16915	.88820	35.05	.0000
Geometry	.70518	–.18771	.92023	24.14	.0000
Statistics	.55848	.43055	.94741	15.46	.0000
Algebra	.82897	.26179	.89695	32.00	.0000
Function 1	%Var = 88.01		Canon R = .38		
After Function 0	Λ = .8383		χ^2 = 196.36	df = 20	p < .0000
Function 2	%Var = 7.01		Canon R = .11		
After Function 1	Λ = .9777		χ^2 = 25.09	df = 12	p < .0144

Table A21
Results of Discriminant Analysis on: "In Math Class How Often Do Problems on Worksheet"
(Grade 12, Booklet 8, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.89435	.95960	12.48	.0000
Measure	.65637	.97446	7.77	.0000
Geometry	.71724	.97087	8.90	.0000
Statistics	.57640	.98211	5.40	.0003
Algebra	.92926	.95628	13.56	.0000
Function 1	%Var = 78.86	Canon R = .22		
After Function 0	Λ = .9370	χ^2 = 77.12	df = 20	p < .0000

Table A22
Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 12, Booklet 9, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.90034	.97291	16.61	.0000
Measure	.89537	.97662	14.28	.0000
Geometry	.85703	.97094	17.85	.0000
Statistics	.82678	.96803	19.70	.0000
Algebra	.76423	.96834	19.50	.0000
Function 1	%Var = 98.75	Canon R = .20		
After Function 0	Λ = .9604	χ^2 = 48.16	df = 10	p < .0000

Table A23
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 12, Booklet 9, 1990)

	Structural coeff.	Structural coeff.	Univariate test		
Variable	Func 1	Func 2	Wilks's lambda	F-ratio	Significance level
Numbers	.85173	.52293	.89467	32.35	.0000
Measure	.58027	.20856	.94881	14.82	.0000
Geometry	.80299	.00331	.90898	27.51	.0000
Statistics	.55085	.38588	.95127	14.07	.0000
Algebra	.93702	–.14325	.87949	37.65	.0000
Function 1	%Var = 87.55		Canon R = .37		
After Function 0	Λ = .8468		χ^2 = 182.53	df = 20	p < .0000
Function 2	%Var = 10.51		Canon R = .14		
After Function 1	Λ = .983		χ^2 = 24.06	df = 12	p < .0200

Table A24
Results of Discriminant Analysis on: "In Math Class How Often Do Problems on Worksheet"
(Grade 12, Booklet 9, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.91728	.95682	13.35	.0000
Measure	.90297	.96788	9.81	.0000
Geometry	.87612	.958346	14.44	.0000
Statistics	.81060	.96211	11.65	.0000
Algebra	.74155	.95511	13.90	.0000
Function 1	%Var = 82.86	Canon R = .23		
After Function 0	Λ = .9344	χ^2 = 80.19	df = 20	p < .0000

Table A25
Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 12, Booklet 10, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.92852	.95432	28.29	.0000
Measure	.86542	.96907	18.87	.0000
Geometry	.81223	.96222	23.20	.0000
Statistics	.78342	.94784	32.53	.0000
Algebra	.70617	.95964	24.86	.0000
Function 1	%Var = 98.50	Canon R = .24		
After Function 0	Λ = .9392	χ^2 = 74.07	df = 10	$p < .0000$

Table A26
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 12, Booklet 10, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.86963	.90685	27.99	.0000
Measure	.83835	.91249	26.13	.0000
Geometry	.83438	.91360	25.77	.0000
Statistics	.68814	.93642	18.50	.0000
Algebra	.89011	.90304	29.26	.0000
Function 1	%Var = 89.33	Canon R = .34		
After Function 0	Λ = .8671	χ^2 = 155.28	df = 20	$p < .0000$

Table A27
Results of Discriminant Analysis on: "In Math Class How Often Do Problems on Worksheet"
(Grade 12, Booklet 10, 1990)

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.76175	.96756	9.84	.0000
Measure	.86833	.95942	12.41	.0000
Geometry	.94488	.95223	14.72	.0000
Statistics	.74851	.96904	9.38	.0000
Algebra	.85332	.96024	12.15	.0000
Function 1	%Var = 88.54	Canon R = .23		
After Function 0	Λ = .9403	χ^2 = 72.22	df = 20	$p < .0000$

Discriminant Analysis 1992 Data: Tables B1–B18

Table B1
Results of Discriminant Analysis on: "Agree/Disagree: I Am Good at Math"
(Grade, 4, Booklet 15, 1992) $N = 349$

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.86099	.96002	7.204	.0009
Measure	.24375	.99536	.8058	.4476
Geometry	.65240	.97681	4.108	.0172
Statistics	.64654	.97637	4.186	.0160
Algebra	.17043	.99813	.3234	.7239
Function 1		%Var = 95.40	Canon $R = .230$	
After Function 0	$\Lambda = .945$	$\chi^2 = 19.57$	$df = 10$	$p < .03$

Table B2
Results of Discriminant Analysis on: "Agree/Disagree: I Like Math"
(Grade 4, Booklet 15, 1992) $N = 352$

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.59811	.97936	3.678	.0263
Measure	–.03866	.99991	.1534	.9848
Geometry	.73789	.96943	5.503	.0044
Statistics	.60857	.97912	3.722	.0252
Algebra	.30960	.99440	.9829	.3753
Function 1		%Var = 96.77	Canon $R = .23$	
After Function 0	$\Lambda = .944$	$\chi^2 = 20.09$	$df = 10$	$p < .03$

Table B3
Results of Discriminant Analysis on: "How Much Time Spent Each Day on Math Homework"
(Grade 4, Booklet 15, 1992) $N = 356$

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.94514	.90995	5.756	.0000
Measure	.41061	.97796	1.311	.2515
Geometry	.53189	.96784	1.933	.0748
Statistics	.81474	.92986	4.388	.0003
Algebra	.48986	.96683	1.996	.0656
Function 1		%Var = 80.10	Canon $R = .31$	
After Function 0	$\Lambda = .877$	$\chi^2 = 45.80$	$df = 30$	$p < .03$

Table B4
Results of Discriminant Analysis on: "Agree/Disagree: I Am Good at Math"
(Grade 4, Booklet 17, 1992) N = 348

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.86335	.93115	12.76	.0000
Measure	.72548	.95341	8.429	.0003
Geometry	.70891	.95627	7.889	.0004
Statistics	.84305	.93948	11.11	.0000
Algebra	.80478	.94330	10.37	.0000
Function 1		%Var = 77.00	Canon R = .29	
After Function 0	Λ = .89	χ^2 = 38.9	df = 10	p < .000

Table B5
Results of Discriminant Analysis on: "Agree/Disagree: I Like Math"
(Grade 4, Booklet 17, 1992) N =

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.62121	.96921	3.89	.0231
Measure	.22112	.99635	.969	.3831
Geometry	.74791	.96991	5.72	.0034
Statistics	.62821	.96832	3.93	.0211
Algebra	.37232	.99320	.989	.3921
Function 1		%Var =	Canon R =	
After Function 0	Λ = .947	χ^2 = 21.09	df = 10	p < .029

Table B6
Results of Discriminant Analysis on: "How Much Time Spent Each Day On Math Homework"
(Grade 4, Booklet 17, 1992) N = 358

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.79374	.94962	3.103	.0056
Measure	.59386	.96028	2.420	.0264
Geometry	.42830	.96161	2.335	.0317
Statistics	.60726	.96354	2.214	.0413
Algebra	.94701	.93733	3.911	.0009
Function 1		%Var = 50.74	Canon R = .26	
After Function 0	Λ = .867	χ^2 = 49.9	df = 30	p < .01

Table B7
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 8, Booklet 1, 1992) $N = 384$

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.82099	.92690	7.472	.0000
Measure	.76098	.93639	6.437	.0001
Geometry	.43525	.97590	2.340	.0547
Statistics	.33349	.98181	1.755	.1372
Algebra	.75858	.92515	7.666	.0000
Function 1		%Var = 69.40	Canon R = .32	
After Function 0	Λ = .853	χ^2 = 60.10	df = 20	p < .000

Table B8
Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 8, Booklet 1, 1992) $N = 369$

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.77676	.95392	4.396	.0018
Measure	.90284	.93916	5.895	.0001
Geometry	.75494	.95653	4.135	.0027
Statistics	.65000	.96608	3.196	.0134
Algebra	.78563	.95140	4.648	.0011
Function 1		%Var = 86.53	Canon R = .27	
After Function 0	Λ = .916	χ^2 = 31.87	df = 20	p < .05

Table B9
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 8, Booklet 2, 1992) $N = 368$

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.92811	.91784	8.124	.0000
Measure	.87596	.92731	7.114	.0000
Geometry	.66933	.95676	4.101	.0029
Statistics	.80626	.93663	6.140	.0001
Algebra	.77434	.94082	5.708	.0002
Function 1		%Var = 78.95	Canon R = .30	
After Function 0	Λ = .885	χ^2 = 44.4	df = 20	p < .001

Table B10
Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 8, Booklet 2, 1992) N = 359

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.96364	.90222	9.591	.0000
Measure	.62237	.94669	4.984	.0006
Geometry	.56198	.95684	3.992	.0035
Statistics	.77975	.92498	7.178	.0000
Algebra	.69021	.94654	4.998	.0006
Function 1		%Var = 70.59	Canon R = .32	
After Function 0	Λ = .854	χ^2 = 55.59	df = 20	p < .000

Table B11
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 8, Booklet 15, 1992) N = 380

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.82615	.91284	8.952	.0000
Measure	.87615	.91248	8.992	.0000
Geometry	.61950	.95206	4.721	.0010
Statistics	.54393	.95981	3.926	.0039
Algebra	.76203	.93075	6.976	.0000
Function 1		%Var = 69.67	Canon R = .33	
After Function 0	Λ = .845	χ^2 = 63.07	df = 20	p < .000

Table B12
Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 8, Booklet 15, 1992) N = 369

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.68145	.93759	6.058	.0001
Measure	.90227	.89836	10.30	.0000
Geometry	.66814	.93692	6.127	.0001
Statistics	.85111	.90259	9.821	.0000
Algebra	.48114	.95681	4.107	.0029
Function 1		%Var = 74.11	Canon R = .35	
After Function 0	Λ = .838	χ^2 = 64.04	df = 20	p < .000

Table B13
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 12, Booklet 1, 1992) N = 368

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.55906	.92187	7.692	.0000
Measure	.74575	.87705	12.72	.0000
Geometry	.60306	.91342	8.602	.0000
Statistics	.63598	.90125	9.943	.0000
Algebra	.94818	.81630	20.42	.0000
Function 1	%Var = 82.78	Canon R = .44		
After Function 0	Λ = .763	χ^2 = 97.80	df = 20	p < .000

Table B14
Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 12, Booklet 1, 1992) N = 368

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.73453	.87300	13.20	.0000
Measure	.82546	.84670	16.43	.0000
Geometry	.58755	.91134	8.83	.0000
Statistics	.54652	.92429	7.43	.0000
Algebra	.89256	.82685	19.00	.0000
Function 1	%Var = 87.37	Canon R = .45		
After Function 0	Λ = .764	χ^2 = 97.40	df = 20	p = .000

Table B15
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 12, Booklet 15, 1992) N = 359

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	F-ratio	Significance level
Numbers	.65618	.94029	5.620	.0002
Measure	.64444	.93904	5.745	.0002
Geometry	.69825	.92939	6.745	.0000
Statistics	.40508	.97235	2.517	.0412
Algebra	.94793	.88545	11.45	.0000
Function 1	%Var = 78.30	Canon R = .35		
After Function 0	Λ = .84	χ^2 = 60.9	df = 20	p < .000

Table B16
Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 12, Booklet 15, 1992) $N = 359$

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.80529	.85825	14.62	.0000
Measure	.64338	.90418	9.379	.0000
Geometry	.82403	.85082	15.52	.0000
Statistics	.73157	.88180	11.86	.0000
Algebra	.93265	.82050	19.36	.0000
Function 1	%Var = 90.19	Canon R = .45		
After Function 0	Λ = .778	χ^2 = 88.40	df = 20	$p < .000$

Table B17
Results of Discriminant Analysis on: "Do You Agree: I Am Good in Math"
(Grade 12, Booklet 17, 1992) $N = 348$

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.87213	.88151	11.53	.0000
Measure	.84175	.87297	12.48	.0000
Geometry	.81331	.88781	10.84	.0000
Statistics	.65493	.91795	7.664	.0000
Algebra	.67827	.91720	7.741	.0000
Function 1	%Var = 84.30	Canon R = .40		
After Function 0	Λ = .813	χ^2 = 70.93	df = 20	$p < .000$

Table B18
Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 12, Booklet 17, 1992) $N = 347$

	Structural coeff.	Univariate test		
Variable	Func 1	Wilks's lambda	*F*-ratio	Significance level
Numbers	.90143	.90139	9.353	.0000
Measure	.63952	.94726	4.760	.0009
Geometry	.84905	.91162	8.289	.0000
Statistics	.74637	.93104	6.332	.0001
Algebra	.86036	.90981	8.476	.0000
Function 1	%Var = 90.94	Canon R = 34		
After Function 0	Λ = .87	χ^2 = 46.97	df = 20	$p < .001$

Factor Analysis 1990 Data: Tables TA1–TA9

Table TA1
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 4, Booklet 11, 1990) N = 1255

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	75.93	35	2.17	.980	.986	.989
Five Factor	44.78	25	1.79	.988	.991	.995
5+1 Factor	57.02	30	1.90	.985	.989	.993
Δ 5, 5+1χ^2 = 12.24		$\Delta\, df = 5$	$p < .03$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 4, Booklet 11, 1990) N = 1255

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.96	—			
Geometry	0.83	0.89	—		
Statistics	1.00	1.000	0.99	—	
Algebra	1.00	0.99	0.90	0.99	—
Factor loading	0.97	1.00	0.88	1.00	1.00

Table TA2
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 4, Booklet 12, 1990) N = 1250

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	51.02	35	1.46	.987	.995	.996
Five Factor	20.20	25	.81	.995	1.002	1.000
5+1 Factor	21.21	30	.71	.995	1.003	1.000
Δ 5, 5+1χ^2 = 1.01		$\Delta\, df = 5$	$p < .975$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 4, Booklet 12, 1990) N = 1250

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.90	—			
Geometry	0.87	0.88	—		
Statistics	0.96	0.96	0.92	—	
Algebra	0.95	0.96	0.89	1.00	—
Factor loading	0.95	0.96	0.91	1.00	1.00

Table TA3

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models

(Grade 4, Booklet 14, 1990) $N = 1242$

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	92.28	35	2.64	.979	.983	.987
Five Factor	47.27	25	1.89	.989	.991	.995
5+1 Factor	55.54	30	1.85	.987	.991	.994
Δ 5, 5+1χ^2 = 8.27	$\Delta\ df = 5$	$p < .100$				

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables

(Grade 4, Booklet 14, 1990) $N = 1242$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.90	—			
Geometry	0.92	0.93	—		
Statistics	1.00	0.96	0.96	—	
Algebra	0.92	0.88	0.89	1.00	—
Factor loading	0.96	0.94	0.96	1.00	0.96

Table TA4

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models

(Grade 8, Booklet 8, 1990) $N = 1234$

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	121.56	35	3.47	.983	.984	.988
Five Factor	35.15	25	1.41	.995	.997	.999
5+1 Factor	55.94	30	1.86	.992	.994	.996
Δ 5, 5+1χ^2 = 20.79	$\Delta\ df = 5$	$p < .001$				

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables

(Grade 8, Booklet 8, 1990) $N = 1234$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.90	—			
Geometry	0.90	0.92	—		
Statistics	0.93	0.91	0.90	—	
Algebra	0.97	0.90	0.96	0.93	—
Factor loading	0.97	0.94	0.95	0.96	0.99

Table TA5
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 8, Booklet 9, 1990) $N = 1234$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	126.39	35	3.61	.979	.980	.985
Five Factor	45.42	25	1.82	.992	.994	.997
5+1 Factor	68.79	30	2.29	.989	.990	.993
Δ 5, 5+1χ^2 = 23.37		Δ *df* = 5	$p < .001$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 8, Booklet 9, 1990) $N = 1234$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	1.00	—			
Geometry	0.88	0.98	—		
Statistics	0.95	1.00	0.95	—	
Algebra	0.92	0.93	0.90	0.93	—
Factor loading	0.97	1.00	0.94	0.99	0.95

Table TA6
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 8, Booklet 10, 1990) $N = 1230$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	171.44	35	4.90	.976	.975	.981
Five Factor	29.45	25	1.18	.996	.999	.999
5+1 Factor	50.60	30	1.69	.993	.996	.997
Δ 5, 5+1χ^2 = 21.15		Δ *df* = 5	$p < .001$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 8, Booklet 10, 1990) $N = 1230$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.93	—			
Geometry	0.85	0.88	—		
Statistics	0.95	0.92	0.91	—	
Algebra	0.94	0.91	0.91	0.97	—
Factor loading	0.96	0.95	0.91	0.99	0.98

Table TA7
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 8, 1990) $N = 1201$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	134.20	35	3.83	.978	.979	.984
Five Factor	39.00	25	1.56	.994	.996	.998
5+1 Factor	58.44	30	1.95	.990	.993	.995
Δ 5, 5+1χ^2 = 19.44	$\Delta\, df = 5$	$p < .005$				

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 8, 1990) $N = 1201$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.90	—			
Geometry	0.92	0.96	—		
Statistics	0.94	0.88	0.88	—	
Algebra	0.91	0.89	0.91	0.89	—
Factor loading	0.96	0.95	0.98	0.94	0.94

Table TA8
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 9, 1990) $N = 1201$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	185.72	35	5.31	.973	.972	.978
Five Factor	79.85	25	3.19	.989	.986	.992
5+1 Factor	118.99	30	3.97	.983	.981	.987
Δ 5, 5+1χ^2 = 39.14	$\Delta\, df = 5$	$p < .001$				

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 9, 1990) $N = 1201$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.96	—			
Geometry	0.94	0.94	—		
Statistics	0.93	0.98	0.91	—	
Algebra	0.93	0.89	0.97	0.88	—
Factor loading	0.97	0.97	0.99	0.94	0.95

Table TA9
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 10, 1990) $N = 1193$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	161.68	35	4.62	.976	.976	.981
Five Factor	52.02	25	2.08	.992	.993	.996
5+1 Factor	83.05	30	2.77	.988	.988	.992
Δ 5, 5+1χ^2 = 31.03		Δ *df* = 5	$p < .001$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 8, 1990) $N = 1193$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.93	—			
Geometry	0.89	0.99	—		
Statistics	0.92	0.90	0.88	—	
Algebra	0.91	0.95	0.96	0.89	—
Factor loading	0.94	0.99	0.98	0.92	0.97

Factor Analysis 1990 Data: Tables 4A1–4A15

Table 4A1
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "How Do You Feel About This Statement: I Am Good in Math"
Selected Response = "Undecided" [2]
(Grade 4, Booklet 11, 1990) N = 279

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	40.91	35	1.17	.934	.987	.990
Five Factor	20.93	25	.84	.966	1.013	1.000
5+1 Factor	36.49	30	1.22	.941	.983	.989
Δ 5, 5+1χ^2 = 15.56	Δ *df* = 5	$p < .01$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good in Math"
(Grade 4, Booklet 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.98	—			
Geometry	.68	.90	—		
Statistics	1.00	.84	.81	—	
Algebra	.91	.97	1.00	1.00	—
Factor loading	.95	1.00	.85	1.00	1.00

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Undecided, and [3] = Disagree. Lower half of the triangle reports for the selected response: "Undecided" [2] for the above background variable (question).

Table 4A2
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "How Do You Feel About This Statement: I Am Good in Math"
Selected Response = "Agree" [1]
(Grade 4, Booklet 11, 1990) N = 763

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	60.66	35	1.73	.974	.986	.989
Five Factor	37.44	25	1.50	.984	.990	.995
5+1 Factor	47.52	30	1.58	.980	.989	.992
Δ 5, 5+1χ^2 = 10.08	Δ *df* = 5	p < .075				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "How Do You Feel About This Statement: I Am Good in Math"
Selected Response = "Disagree" [3]
(Grade 4, Booklet 11, 1990) N = 166

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	42.89	35	1.22	.886	.969	.976
Five Factor	38.13	25	1.20	.898	.928	.960
5+1 Factor	40.09	30	1.34	.893	.954	.969
Δ 5, 5+1χ^2 = 9.96	Δ *df* = 5	p < .080				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good in Math"
(Grade 4, Booklets 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	.94	.85	.95	.99	.95
Measurement	.97	—	.90	1.00	1.00	1.00
Geometry	.85	.87	—	.98	.90	.91
Statistics	.91	.77	.87	—	.90	1.00
Algebra	1.00	.91	.79	.88	—	1.00
Factor loading	1.00	.96	.86	.90	1.00	—

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Undecided, and [3] = Disagree. Upper half of the triangle reports for the selected response: "Strongly Agree" [1] and the lower half of the triangle reports for the selected response: "Disagree" [3] for the above background variable (question).

Table 4A3
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "How Do You Feel About This Statement: I Am Good in Math"
Selected Response = "Agree" [1]
(Grade 4, Booklet 12, 1990) N = 745

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	60.41	35	1.73	.976	.987	.990
Five Factor	31.28	25	1.25	.988	.996	.998
5+1 Factor	32.48	30	1.08	.987	.999	.999
Δ 5, 5+1χ^2 = 1.2		Δ df = 5	p < .900			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "How Do You Feel About This Statement: I Am Good in Math"
Selected Response = "Disagree" [3]
(Grade 4, Booklet 12, 1990) N = 155

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	39.35	35	1.12	.896	.983	.987
Five Factor	32.05	25	1.28	.916	.962	.979
5+1 Factor	35.43	30	1.18	.907	.976	.984
Δ 5, 5+1χ^2 = 3.38		Δ df = 5	p < .700			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good in Math
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.89	0.86	0.92	0.90	0.94
Measurement	0.97	—	0.87	0.94	0.93	0.95
Geometry	0.79	0.84	—	0.89	0.85	0.91
Statistics	1.00	0.97	0.77	—	0.98	0.99
Algebra	0.98	0.96	0.99	1.00	—	0.97
Factor loading	0.99	0.97	0.86	1.00	1.00	—

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Undecided, and [3] = Disagree. Upper half of the triangle reports for the selected response: "Strongly Agree" [1] and the lower half of the triangle reports for the selected response: "Disagree" [3] for the above background variable (question).

Table 4A4
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "How Do You Feel About This Statement: I Am Good in Math"
Selected Response = "Undecided" [2]
(Grade 4, Booklet 12, 1990) N = 299

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	22.22	35	.63	.966	1.027	1.000
Five Factor	13.21	25	.53	.980	1.035	1.000
5+1 Factor	17.20	30	.57	.974	1.032	1.000
Δ 5, 5+1χ^2 = 3.99	Δ *df* = 5	p < .550				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good in Math
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.87	—			
Geometry	0.97	0.86	—		
Statistics	0.99	0.91	0.94	—	
Algebra	1.00	0.86	0.77	0.91	—
Factor loading	1.00	0.89	0.94	0.99	0.99

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Undecided, and [3] = Disagree. Lower half of the triangle reports for the selected response: "Undecided" [2] for the above background variable (question).

Table 4A5

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "How Do You Feel About This Statement: I Am Good in Math"

Selected Response = "Agree" [1]

(Grade 4, Booklet 14, 1990) N = 732

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	60.54	35	1.73	.978	.988	.990
Five Factor	32.84	25	1.31	.988	.995	.997
5+1 Factor	39.75	30	1.32	.985	.995	.996
Δ 5, 5+1χ^2 = 6.91	Δ *df* = 5	p < .250				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "How Do You Feel About This Statement: I Am Good in Math

(Grade 4, Booklet 14, 1990)

Selected Response = "Disagree" [3] N = 161

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	26.13	35	.75	.927	1.036	.938
Five Factor	17.77	25	.71	.951	1.041	.972
5+1 Factor	20.56	30	.68	.943	1.045	.954
Δ 5, 5+1χ^2 = 2.79	Δ *df* = 5	p < .700				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good in Math

(Grade 4, Booklet 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.98	1.00	0.97	0.95	0.89
Measurement	0.98	—	1.00	0.90	1.00	0.91
Geometry	0.89	0.88	—	0.96	0.92	1.00
Statistics	0.90	0.82	1.00	—	0.95	0.94
Algebra	1.00	0.88	0.98	0.88	—	0.95
Factor loading	0.92	0.86	0.96	0.72	0.86	—

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Undecided, and [3] = Disagree. Upper half of the triangle reports for the selected response: "Strongly Agree" [1] and the lower half of the triangle reports for the selected response: "Disagree" [3] for the above background variable (question).

Table 4A6
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "How Do You Feel About This Statement: I Am Good in Math"
Selected Response = "Undecided" [2]
(Grade 4, Booklet 14, 1990) N = 287

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	57.67	35	1.65	.931	.963	.971
Five Factor	37.03	25	1.48	.956	.973	.985
5+1 Factor	48.02	30	1.60	.943	.966	.977
Δ 5, 5+1χ^2 = 10.99	Δ *df* = 5	$p < .055$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good in Math
(Grade 4, Booklets 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.88	—			
Geometry	1.00	0.99	—		
Statistics	1.00	0.85	0.82	—	
Algebra	0.86	0.76	0.85	1.00	—
Factor loading	0.99	0.91	1.00	1.00	0.89

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Undecided, and [3] = Disagree. Lower half of the triangle reports for the selected response: "Undecided" [2] for the above background variable (question).

Table 4A7
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types [1]
(Grade 4, Booklet 11, 1990) N = 395

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	36.77	35	1.05	.959	.997	.998
Five Factor	26.78	25	1.07	.970	.996	.998
5+1 Factor	32.36	30	1.08	.964	.996	.997
Δ 5, 5+1χ^2 = 5.58		Δ *df* = 5	$p < .400$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 4 Types [3]
(Grade 4, Booklet 11, 1990) N = 405

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	31.59	35	.90	.977	1.003	1.000
Five Factor	25.33	25	1.01	.981	1.000	1.000
5+1 Factor	26.28	30	.88	.981	1.004	1.000
Δ 5, 5+1χ^2 = .95		Δ *df* = 5	$p < .975$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Booklet 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.93	0.76	0.98	1.08	0.96
Measurement	0.98	—	0.90	0.99	0.99	0.98
Geometry	0.87	0.88	—	0.95	0.92	0.85
Statistics	0.99	1.00	0.94	—	1.00	1.00
Algebra	0.93	0.94	0.87	0.99	—	1.00
Factor loading	0.98	1.00	0.89	1.00	0.95	—

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, and [3] = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types [1] and the lower half of the triangle reports for the selected response: 4 Types [3] for the above background variable (question).

Table 4A8
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 3 Types [2]
(Grade 4, Booklet 11, 1990) $N = 438$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	71.02	35	2.03	.943	.962	.970
Five Factor	46.64	25	1.87	.963	.968	.982
5+1 Factor	58.85	30	1.96	.953	.964	.976
Δ 5, 5+1χ^2 = 12.21		$\Delta\, df = 5$	$p < .05$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Booklet 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.92	—			
Geometry	0.79	0.89	—		
Statistics	0.99	1.00	1.00	—	
Algebra	0.96	0.95	0.87	0.84	—
Factor loading	0.94	0.99	0.89	1.00	0.98

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, and [3] = 4 Types. Lower half of the triangle reports for the selected response: 3 Types [2] for the above background variable (question).

Table 4A9
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types [1]
(Grade 4, Booklet 12, 1990) $N = 408$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	32.94	35	.94	.970	1.003	1.000
Five Factor	20.79	25	.83	.981	1.007	1.000
5+1 Factor	25.02	30	.83	.977	1.007	1.000
Δ 5, 5+1χ^2 = 4.23		Δ *df* = 5	$p < .550$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 3 Types [2]
(Grade 4, Booklet 12, 1990) $N = 404$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	49.25	35	1.41	.968	.988	.990
Five Factor	30.52	25	1.22	.980	.993	.996
5+1 Factor	32.42	30	1.08	.979	.998	.998
Δ 5, 5+1χ^2 = 1.9		Δ *df* = 5	$p < .800$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.87	0.87	0.96	0.93	0.93
Measurement	0.90	—	0.92	0.91	1.00	0.95
Geometry	0.86	0.83	—	0.89	0.94	0.94
Statistics	0.92	0.92	0.92	—	1.00	1.00
Algebra	0.92	0.90	0.88	0.92	—	1.00
Factor loading	0.95	0.94	0.91	0.97	0.96	—

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, and [3] = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types [1] and the lower half of the triangle reports for the selected response: 4 Types [3] for the above variable (question).

Table 4A10
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 3 Types [2]
(Grade 4, Booklet 12, 1990) $N = 424$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	35.77	35	1.02	.968	.999	.999
Five Factor	24.39	25	.98	.978	1.001	1.000
5+1 Factor	27.45	30	.91	.975	1.004	1.000

Δ 5, 5+1χ^2 = 3.06 $\Delta\, df = 5$ $p < .800$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.90	—			
Geometry	0.84	0.88	—		
Statistics	0.98	1.00	0.87	—	
Algebra	0.91	0.91	0.78	1.00	—
Factor loading	0.95	0.98	0.87	1.00	0.95

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, and [3] = 4 Types. Lower half of the triangle reports for the selected response: 3 Types [2] for the above background variable (question).

Table 4A11
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Material"
Selected Response: 0-2 Types [1]
(Grade 4, Booklet 14, 1990) N = 409

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	49.25	35	1.41	.956	.983	.987
Five Factor	27.22	25	1.09	.976	.996	.998
5+1 Factor	30.24	30	1.01	.973	1.000	1.000
Δ 5, 5+1χ^2 = 3.02	Δ df = 5	p < .700				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 4 Types [3]
(Grade 4, Booklet 14, 1990) N = 383

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	44.55	35	1.27	.966	.990	.993
Five Factor	27.18	25	1.09	.979	.997	.998
5+1 Factor	31.81	30	1.06	.976	.998	.999
Δ 5, 5+1χ^2 = 4.63	Δ df = 5	p < .500				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Booklet 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.86	0.88	1.00	0.94	0.98
Measurement	0.94	—	0.86	0.88	0.79	0.88
Geometry	0.81	0.90	—	0.92	0.87	0.92
Statistics	1.00	1.00	0.83	—	0.95	1.00
Algebra	0.96	0.99	0.90	1.00	—	0.94
Factor loading	0.95	1.00	0.87	1.00	1.00	—

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, and [3] = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types [1] and the lower half of the triangle reports for the selected response: 4 Types [3] for the above background variable (question).

Table 4A12
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 3 Types [2]
(Grade 4, Booklet 14, 1990) $N = 431$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	63.14	35	1.80	.960	.976	.982
Five Factor	34.42	25	1.38	.978	.989	.994
5+1 Factor	40.99	30	1.37	.974	.989	.993
Δ 5, 5+1χ^2 = 6.57	Δ *df* = 5	$p < .275$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Book 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.86	—			
Geometry	1.00	0.97	—		
Statistics	0.97	0.92	1.00	—	
Algebra	0.89	0.85	0.90	1.00	—
Factor loading	0.96	0.91	1.00	1.00	0.93

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, and [3] = 4 Types. Lower half of the triangle reports for the selected response: 3 Types [2] for the above background variable (question).

Table 4A13
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
Selected Response: "Almost Every Day" [1]
(Grade 4, Booklet 11, 1990) $N = 337$

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	37.91	35	1.08	.953	.995	.996
Five Factor	27.16	25	1.09	.966	.995	.997
5+1 Factor	29.96	30	1.00	.963	1.000	1.000
Δ 5, 5+1χ^2 = 2.8		$\Delta\, df = 5$	$p < .750$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
Selected Response: "Never" [5]
(Grade 4, Booklet 11, 1990) $N = 382$

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	55.84	35	1.59	.948	.974	.980
Five Factor	31.22	25	1.25	.971	.989	.994
5+1 Factor	43.65	30	1.45	.959	.980	.987
Δ 5, 5+1χ^2 = 12.43		$\Delta\, df = 5$	$p < .03$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.88	0.84	0.98	0.98	0.93
Measurement	1.00	—	0.86	0.99	1.00	0.95
Geometry	0.72	0.79	—	1.00	1.00	0.97
Statistics	1.00	0.94	0.93	—	1.00	1.00
Algebra	0.98	1.00	0.70	0.83	—	1.00
Factor loading	1.00	1.00	0.77	0.99	0.97	—

Note. Range for this background variable question was: [1] = Almost Every Day, [2] = Several Times a Week, [3] = About Once a Week, [4] = Less Than Once a Week, and [5] = Never. Upper half of the triangle reports for the selected response: "Almost Every Day" [1] and the lower half of the triangle reports for the selected response: "Never" [5] for the above background variable (question).

Table 4A14
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
Selected Response: "Almost Every Day" [1]
(Grade 4, Booklet 12, 1990) $N = 318$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	51.02	35	1.46	.949	.978	.983
Five Factor	17.83	25	.71	.982	1.014	1.000
5+1 Factor	21.51	30	.72	.978	1.013	1.000
Δ 5, 5+1χ^2 = 3.68		Δ *df* = 5	$p < .600$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
Selected Response: "Never" [5]
(Grade 4, Booklet 12, 1990) $N = 377$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	24.87	35	.71	.978	1.012	1.000
Five Factor	16.34	25	.65	.985	1.015	1.000
5+1 Factor	18.34	30	.61	.983	1.016	1.000
Δ 5, 5+1χ^2 = 2.00		Δ *df* = 5	$p < .875$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.77	0.78	0.97	0.85	0.89
Measurement	0.95	—	0.77	0.88	0.88	0.87
Geometry	0.84	0.82	—	0.92	0.92	0.90
Statistics	1.00	1.00	0.83	—	1.00	1.00
Algebra	1.00	0.92	0.83	0.99	—	0.99
Factor loading	1.00	0.97	0.84	1.00	1.00	—

Note. Range for this background variable question was: [1] = Almost Every Day, [2] = Several Times a Week, [3] = About Once a Week, [4] = Less Than Once a Week, and [5] = Never. Upper half of the triangle reports for the selected response: "Almost Every Day" [1] and the lower half of the triangle reports for the selected response: "Never" [5] for the above background variable (question).

Table 4A15
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: “In Math Class How Often Do You Work With Rulers, Blocks, Shapes”
Selected Response: “Almost Every Day” [1] and “Several Times a Week” [2]
(Grade 4, Booklet 14, 1990) N = 329

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	67.37	35	1.92	.944	.964	.972
Five Factor	55.58	25	2.22	.954	.953	.974
5+1 Factor	62.55	30	2.08	.948	.958	.972
Δ 5, 5+1χ^2 = 6.97	Δ *df* = 5	p < .250				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: “In Math Class How Often Do You Work With Rulers, Blocks, Shapes”
Selected Response: “Never” [5]
(Grade 4, Booklet 14, 1990) N = 368

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	55.68	35	1.59	.950	.975	.981
Five Factor	19.85	25	.79	.982	1.009	1.000
5+1 Factor	25.59	30	.85	.977	1.006	1.000
Δ 5, 5+1χ^2 = 5.74	Δ *df* = 5	p < .300				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: “In Math Class How Often Do You Work With Rulers, Blocks, Shapes”
(Grade 4, Booklet 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.91	0.93	0.89	1.00	0.96
Measurement	0.82	—	0.95	0.98	0.94	0.96
Geometry	0.89	0.84	—	1.00	0.92	0.97
Statistics	1.00	0.85	0.83	—	1.00	1.00
Algebra	0.88	0.76	0.80	0.96	—	1.00
Factor loading	0.98	0.86	0.91	1.00	0.90	—

Note. Range for this background variable question was: [1] = Almost Every Day, [2] = Several Times a Week, [3] = About Once a Week, [4] = Less Than Once a Week, and [5] = Never. Upper half of the triangle reports for the selected response: “Almost Every Day” [1] and “Several Times a Week” [2], and the lower half of the triangle reports for the selected response: “Never” [5] for the above background variable (question).

Factor Analysis 1990 Data: Tables 8A1–8A12

Table 8A1
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Do You Agree: I Am Good in Math"
Selected Response: "Strongly Agree" [1]
(Grade 8, Booklet 8, 1990) $N = 710$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	92.26	35	2.64	.977	.981	0.985
Five Factor	30.61	25	1.22	.992	.997	0.999
5+1 Factor	44.52	30	1.48	.989	.994	0.996
Δ 5, 5+1χ^2 = 13.91	Δ *df* = 5	$p < .020$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Do You Agree: I Am Good in Math
Selected Response: "Disagree" [4]
(Grade 8, Booklet 8, 1990) $N = 217$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	28.11	35	.80	.977	1.088	1.000
Five Factor	9.33	25	.37	.992	1.024	1.000
5+1 Factor	13.56	30	.45	.989	1.021	1.000
Δ 5, 5+1χ^2 = 4.23	Δ *df* = 5	$p < .575$				

Correlations Between and Factor Loadings of the five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree: I Am Good in Math
(Grade 8, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.88	0.92	0.92	0.96	0.96
Measurement	0.88	—	0.91	0.90	0.89	0.92
Geometry	0.92	0.91	—	0.90	0.99	0.97
Statistics	0.92	0.90	0.90	—	0.91	0.95
Algebra	0.96	0.88	0.98	0.91	—	0.99
Factor loading	0.96	0.92	0.97	0.94	0.99	—

Note: Range for this background variable question was: [1] = Strongly Agree, [2] = Agree, [3] = Undecided, [4] = Disagree, and [5] = Strongly Disagree. Upper half of the triangle reports for the selected response: "Strongly Agree" [1] and the lower half of the triangle reports for the selected response: "Disagree" [4] for the above background variable (question).

Table 8A2
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types [1]
(Grade 8, Booklet 8, 1990) N = 268

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	59.47	35	1.70	.954	.973	.979
Five Factor	34.18	25	1.37	.972	.986	.992
5+1 Factor	36.51	30	1.22	.970	.992	.994
Δ 5, 5+1χ^2 = 2.33		Δ *df* = 5	$p < .700$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 4 Types [3]
(Grade 8, Booklet 8, 1990) N = 594

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	80.97	35	2.31	.974	.981	.985
Five Factor	38.28	25	1.53	.988	.992	.996
5+1 Factor	47.22	30	1.57	.985	.992	.994
Δ 5, 5+1χ^2 = 8.94		Δ *df* = 5	$p < .750$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 8, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.88	0.89	0.89	0.88	0.95
Measurement	0.89	—	0.86	0.88	0.81	0.91
Geometry	0.91	0.94	—	0.90	0.91	0.95
Statistics	0.92	0.87	0.89	—	0.88	0.94
Algebra	0.96	0.90	0.97	0.91	—	.92
Factor loading	.96	0.93	0.97	0.94	0.98	—

Note: Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, [3] = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types [1] and the lower half of the triangle reports for the selected response: 4 Types [3] for the above background variable (question).

Table 8A3
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
Selected Response: "Algebra" [4]
(Grade 8, Booklet 8, 1990) N = 183

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	66.78	35	1.91	.951	.969	.976
Five Factor	35.91	25	1.44	.974	.985	.992
5+1 Factor	46.16	30	1.54	.966	.982	.988
Δ 5, 5+1χ^2 = 10.25	Δ *df* = 5	$p < .075$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
Selected Response: "Eighth Grade Math" [2]
(Grade 8, Booklet 8, 1990)

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	57.46	35	1.63	.973	.986	.989
Five Factor	31.71	25	1.27	.985	.994	.997
5+1 Factor	34.03	30	1.13	.984	.997	.998
Δ 5, 5+1χ^2 = 2.32	Δ *df* = 5	$p < .800$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "What Kind of Math Class Are You Taking This Year"
(Grade 8, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.88	0.88	0.94	1.00	0.98
Measurement	0.89	—	0.92	0.89	0.91	0.92
Geometry	0.87	0.89	—	0.89	0.95	0.93
Statistics	0.89	0.90	0.87	—	0.95	0.96
Algebra	0.98	0.96	0.99	0.95	—	1.00
Factor loading	0.94	0.95	0.94	0.94	1.00	—

Note: Range for this background variable question was: [1] = No Math This Year, [2] = Eighth Grade Math, [3] = Pre-Algebra, [4] = Algebra, and [5] = Other. Upper half of the triangle reports for the selected response: "Algebra" [4] and the lower half of the triangle reports for the selected response: "Eighth-Grade Math" [2] for the above background variable (question).

Table 8A4
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
Selected Response: "Pre-Algebra" [3]
(Grade 8, Booklet 8, 1990) N = 235

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	70.05	35	2.00	.935	.956	.966
Five Factor	38.58	25	1.54	.964	.976	.987
5+1 Factor	49.78	30	1.66	.954	.971	.981
Δ 5, 5+1χ^2 = 11.2	Δ *df* = 5	$p < .050$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "What Kind of Math Class Are You Taking This Year"
(Grade 8, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.83	—			
Geometry	0.91	0.93	—		
Statistics	0.92	0.84	0.87	—	
Algebra	0.95	0.75	0.86	0.83	—
Factor loading	0.99	0.87	0.95	0.92	0.93

Note. Range for this background variable question was: [1] = No Math This Year, [2] = Eighth-Grade Math, [3] = Pre-Algebra, [4] = Algebra, and [5] = Other. Lower half of the triangle reports for the selected response: "Pre-Algebra" [3] for the above background variable (question).

Table 8A5
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Do You Agree: I Am Good in Math"
Selected Response: "Strongly Agree" [1]
(Grade 8, Booklet 9, 1990) N = 746

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	112.25	35	3.20	.969	.972	.978
Five Factor	48.98	25	1.96	.987	.990	.994
5+1 Factor	60.85	30	2.03	.983	.987	.991
Δ 5, 5+1χ^2 = 11.87		Δ *df* = 5	$p < .05$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Do You Agree: I Am Good in Math"
Selected Response: "Agree" [2]
(Grade 8, Booklet 9, 1990) N = 189

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	48.48	35	1.39	.933	.974	.980
Five Factor	32.70	25	1.31	.955	.980	.989
5+1 Factor	38.64	30	1.29	.946	.981	.987
Δ 5, 5+1χ^2 = 5.95		Δ *df* = 5	$p < .450$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree: I Am Good in Math"
(Grade 8, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.94	0.88	0.91	0.99	1.00
Measurement	0.94	—	0.95	0.89	1.00	0.87
Geometry	1.00	0.84	—	0.94	0.93	0.90
Statistics	0.92	0.89	0.98	—	1.00	0.92
Algebra	1.00	0.86	1.00	0.83	—	0.96
Factor loading	0.92	1.00	0.95	0.99	1.00	—

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Agree, [3] = Undecided, [4] = Disagree, and [5] = Strongly Disagree. Upper half of the triangle reports for the selected response: "Strongly Agree" [1] and the lower half of the triangle reports for the selected response: "Agree" [4] for the above background variable (question).

Table 8A6
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types [1]
(Grade 8, Booklet 9, 1990) N = 263

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	49.23	35	1.41	.958	.984	.987
Five Factor	35.73	25	1.43	.968	.983	.990
5+1 Factor	43.04	30	1.43	.963	.983	.988
Δ 5, 5+1χ^2 = 7.35	Δ *df* = 5	p < .200				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 3 Types [2]
(Grade 8, Booklet 9, 1990) N = 606

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	95.93	35	2.74	.965	.971	.780
Five Factor	48.82	25	1.95	.982	.984	.991
5+1 Factor	61.05	30	2.04	.978	.983	.989
Δ 5, 5+1χ^2 = 12.23	Δ *df* = 5	p < .05				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: Home Environment-Reading Materials
(Grade 8, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.95	0.99	1.00	1.00	1.00
Measurement	0.95	—	1.00	0.91	1.00	0.89
Geometry	1.00	0.88	—	0.96	0.91	0.96
Statistics	0.95	0.95	0.93	—	1.00	0.94
Algebra	1.00	0.85	1.00	0.86	—	0.94
Factor loading	0.92	0.92	0.96	1.00	1.00	—

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, [3] = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types [1] and the lower half of the triangle reports for the selected response: 3 Types [2] for the above background variable (question).

Table 8A7
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
Selected Response: "Pre-Algebra" [3]
(Grade 8, Booklet 9, 1990) $N = 242$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	48.74	35	1.39	.959	.985	.988
Five Factor	36.19	25	1.45	.970	.982	.990
5+1 Factor	39.49	30	1.32	.967	.988	.992
Δ 5, 5+1χ^2 = 3.3	Δ *df* = 5	$p < .650$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
Selected Response: "Algebra" [4]
(Grade 8, Booklet 9, 1990) $N = 176$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	58.50	35	1.67	.94	.97	.98
Five Factor	30.87	25	1.23	.97	.99	.99
5+1 Factor	39.89	30	1.33	.96	.98	.99
Δ 5, 5+1χ^2 = 9.02	Δ *df* = 5	$p < .150$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
(Grade 8, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	1.00	0.92	1.00	0.96	1.00
Measurement	1.00	—	0.92	1.00	0.95	1.00
Geometry	0.86	0.98	—	0.99	0.90	0.94
Statistics	0.90	0.97	0.91	—	0.92	1.00
Algebra	0.89	0.90	0.91	0.94	—	0.95
Factor loading	0.94	1.00	0.94	0.97	0.95	—

Note. Range for this background variable question was: [1] = No Math This Year, [2] = Eighth-Grade Math, [3] = Pre-Algebra, [4] = Algebra, and [5] = Other. Upper half of the triangle reports for the selected response: "Pre-Algebra" [3] and the lower half of the triangle reports for the selected response: "Algebra" [4] for the above background variable (question).

Table 8A8
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
Selected Response: "Eighth-Grade Math" [2]
(Grade 8, Booklet 9, 1990) N = 558

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	65.55	35	1.87	.967	.980	.984
Five Factor	38.35	25	1.53	.981	.988	.993
5+1 Factor	48.62	30	1.62	.976	.986	.990
Δ 5, 5+1χ^2 = 10.27		Δ *df* = 5	$p < .075$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "What Kind of Math Class Are You Taking This Year"
(Grade 8, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.98	—			
Geometry	0.83	0.96	—		
Statistics	0.90	1.00	0.95	—	
Algebra	0.92	0.93	0.89	0.88	—
Factor loading	0.93	1.00	0.94	0.99	0.94

Note. Range for this background variable question was: [1] = No Math This Year, [2] = Eighth-Grade Math, [3] = Pre-Algebra, [4] = Algebra, and [5] = Other. Lower half of the triangle reports for the selected response: "Eighth-Grade Math" [2] for the above background variable (question).

Table 8A9
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Do You Agree: I Am Good in Math"
Selected Response: "Strongly Agree" [1] and "Agree" [2]
(Grade 8, Booklet 9, 1990) $N = 742$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	116.37	35	3.32	.973	.976	.981
Five Factor	37.41	25	1.50	.991	.995	.997
5+1 Factor	47.98	30	1.60	.989	.994	.996
Δ 5, 5+1χ^2 = 10.57		$\Delta\ df = 5$	$p < .075$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Do You Agree: I Am Good in Math"
Selected Response: "Disagree" [4] and "Strongly Disagree" [5]
(Grade 8, Booklet 9, 1990) $N = 196$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	81.55	35	2.33	.914	.934	.948
Five Factor	32.99	25	1.32	.965	.984	.991
5+1 Factor	47.08	30	1.57	.950	.972	.981
Δ 5, 5+1χ^2 = 14.09		$\Delta\ df = 5$	$p < .025$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree: I Am Good in Math"
(Grade 8, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.92	0.85	0.94	0.96	0.95
Measurement	0.87	—	0.92	0.93	0.95	0.97
Geometry	0.78	0.74	—	0.90	0.94	0.92
Statistics	0.97	0.77	0.90	—	1.00	0.98
Algebra	0.87	0.71	0.83	0.89	—	1.00
Factor loading	0.97	0.84	0.86	0.99	0.89	—

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Agree, [3] = Undecided, [4] = Disagree, and [5] = Strongly Disagree. Upper half of the triangle reports for the selected response: "Strongly Agree" [1] and "Agree" [2], and the lower half of the triangle reports for the selected response: "Disagree" [4] and "Strongly Disagree" [5] for the above background variable (question).

Table 8A10
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types [1]
(Grade 8, Booklet 10, 1990) N = 285

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	75.85	35	2.17	.949	.964	.972
Five Factor	33.42	25	1.34	.978	.990	.994
5+1 Factor	44.86	30	1.49	.970	.985	.990
Δ 5, 5+1χ^2 = 11.44	Δ *df* = 5	p < .050				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 4 Types [3]
(Grade 8, Booklet 10, 1990) N = 576

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	125.98	35	3.60	.960	.963	.971
Five Factor	26.41	25	1.06	.992	.999	1.000
5+1 Factor	38.00	30	1.27	.988	.996	.997
Δ 5, 5+1χ^2 = 11.59	Δ *df* = 5	p < .050				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: Home Environment-Reading Materials
(Grade 8, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.89	0.81	0.96	0.94	0.95
Measurement	0.92	—	0.85	0.86	0.88	0.91
Geometry	0.80	0.85	—	0.91	0.93	0.90
Statistics	0.94	0.95	0.91	—	1.00	1.00
Algebra	0.90	0.86	0.86	0.96	—	1.00
Factor loading	0.94	0.96	0.89	1.00	0.94	—

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, [3] = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types [1], and the lower half of the triangle reports for the selected response: 4 Types [3] for the above background variable (question).

Table 8A11

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
Selected Response: "Algebra" [4]
(Grade 8, Booklet 10, 1990) N = 165

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	79.31	35	2.27	.935	.952	.962
Five Factor	50.53	25	2.02	.959	.961	.978
5+1 Factor	57.58	30	1.92	.953	.965	.977
Δ 5, 5+1χ^2 = 7.05		Δ *df* = 5	$p < .200$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
Selected Response: "Eighth-Grade Math" [2]
(Grade 8, Booklet 10, 1990) N = 561

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	95.66	35	2.73	.962	.968	.975
Five Factor	23.03	25	.92	.991	1.001	1.000
5+1 Factor	31.77	30	1.06	.987	.999	.999
Δ 5, 5+1χ^2 = 8.74		Δ *df* = 5	$p < .150$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
(Grade 8, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.91	0.91	0.92	0.87	0.92
Measurement	0.92	—	0.98	0.97	0.87	0.97
Geometry	0.78	0.83	—	0.99	0.92	0.99
Statistics	0.94	0.90	0.86	—	1.00	1.00
Algebra	0.91	0.92	0.84	0.96	—	0.95
Factor loading	0.95	0.95	0.85	0.98	0.97	—

Note. Range for this background variable question was: [1] = No Math This Year, [2] = Eighth-Grade Math, [3] = Pre-Algebra, [4] = Algebra, and [5] = Other. Upper half of the triangle reports for the selected response: "Algebra" [4] and the lower half of the triangle reports for the selected response: "Eighth-Grade Math" [2] for the above background variable (question).

Table 8A12
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
Selected Response: "Pre-Algebra" [3]
(Grade 8, Booklet 10, 1990) N = 248

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	55.09	35	1.57	.954	.977	.982
Five Factor	25.53	25	1.02	.979	.999	1.000
5+1 Factor	33.16	30	1.11	.972	.996	.997
Δ 5, 5+1χ^2 = 7.63		Δ *df* = 5	$p < .150$			

Correlations Between Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "What Kind of Math Class Are You Taking This Year"
(Grade 8, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.90	—			
Geometry	0.81	0.88	—		
Statistics	0.93	0.90	0.90	—	
Algebra	0.96	0.87	0.90	1.00	—
Factor loading	0.94	0.94	0.90	0.99	1.00

Note. Range for this background variable question was: [1] = No Math This Year, [2] = Eighth-Grade Math, [3] = Pre-Algebra, [4] = Algebra, and [5] = Other. Upper half of the triangle reports for the selected response: "Pre-Algebra" [3] for the above background variable (question).

Factor Analysis 1990 Data: Tables 12A1–12A9

Table 12A1
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Do You Agree/Disagree: I Am Good in Math"
Selected Response: "Strongly Agree" [1]
(Grade 12, Booklet 8, 1990) N = 651

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	72.69	35	2.08	.978	.985	.988
Five Factor	21.12	25	0.84	.994	1.002	1.000
5+1 Factor	31.65	30	1.06	.990	.999	.999
Δ 5, 5+1χ^2 = 10.53		Δ *df* = 5	$p < .075$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Do You Agree/Disagree: I Am Good in Math"
Selected Response: "Agree" [2]
(Grade 12, Booklet 8, 1990) N = 254

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	28.29	35	0.81	.978	1.007	1.000
Five Factor	8.22	25	0.33	.994	1.024	1.000
5+1 Factor	12.32	30	0.41	.990	1.021	1.000
Δ 5, 5+1χ^2 = 4.1		Δ *df* = 5	$p < .550$			

Correlations Between and Factor Loadings of the five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree/Disagree: I Am Good in Math"
(Grade 12, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.90	0.92	0.94	0.91	0.96
Measurement	0.90	—	0.96	0.88	0.89	0.95
Geometry	0.90	0.96	—	0.88	0.91	0.98
Statistics	0.94	0.88	0.88	—	0.89	0.94
Algebra	0.91	0.89	0.91	0.89	—	0.94
Factor loading	0.96	0.95	0.98	0.94	0.94	—

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Agree, [3] = Undecided, [4] = Disagree, [5] = Strongly Disagree. Upper half of the triangle reports for the selected response: "Strongly Agree" [1] and the lower half of the triangle reports for the selected response: "Agree" [2] for the above background variable (question).

Table 12A2
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types [1]
(Grade 12, Booklet 8, 1990) $N = 176$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	52.68	35	1.51	.937	.971	.978
Five Factor	38.28	25	1.53	.955	.970	.983
5+1 Factor	46.53	30	1.55	.945	.969	.979
Δ 5, 5+1χ^2 = 8.25	Δ *df* = 5	$p < .150$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 3 Types [2]
Grade 12, Booklet 8, 1990) $N = 718$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	88.94	35	2.54	.975	.980	.985
Five Factor	36.00	25	1.44	.990	.994	.997
5+1 Factor	38.65	30	1.29	.989	.996	.998
Δ 5, 5+1χ^2 = 2.65	Δ *df* = 5	$p < .750$				

Correlations Between and Factor Loadings of the five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 12, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.91	0.95	1.00	0.92	1.00
Measurement	0.89	—	0.99	0.87	0.88	0.95
Geometry	0.90	0.93	—	0.83	0.89	0.97
Statistics	0.88	0.87	0.87	—	0.93	1.00
Algebra	0.90	0.92	0.90	0.90	—	0.93
Factor loading	0.94	0.96	0.96	0.92	0.96	—

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, [3] = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types [1] and the lower half of the triangle reports for the selected response: 3 Types [2] for the above background variable (question).

Table 12A3

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do Problems on Worksheet"

Selected Response: "Almost Ever Day" [1]

(Grade 12, Booklet 8, 1990) $N = 347$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	69.13	35	1.98	.960	.974	.980
Five Factor	39.09	25	1.56	.978	.985	.992
5+1 Factor	46.85	30	1.56	.973	.985	.990
Δ 5, 5+1χ^2 = 7.76	Δ *df* = 5	$p < .200$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do Problems on Worksheet"

Selected Response: "Several Times a Week" [2]

(Grade 12, Booklet 8, 1990) $N = 292$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	54.32	35	1.55	.960	.981	.985
Five Factor	29.76	25	1.19	.978	.994	.996
5+1 Factor	32.31	30	1.08	.976	.997	.998
Δ 5, 5+1χ^2 = 2.55	Δ *df* = 5	$p < .900$				

Correlations Between and Factor Loadings of the five Subscale Latent Variables Based on the Levels of: "In Math Class How Often Do Problems on Worksheet"

(Grade 12, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.91	0.95	0.95	0.89	0.97
Measurement	0.88	—	0.96	0.86	0.85	0.94
Geometry	0.88	0.98	—	0.87	0.93	0.99
Statistics	0.91	0.95	0.93	—	0.91	0.95
Algebra	0.89	0.93	0.91	0.94	—	0.93
Factor loading	0.92	0.98	0.97	0.98	0.95	—

Note. Range for this background variable question was: [1] = Almost Every Day, [2] = Several Times a Week, [3] = About Once a Week, [4] = Less Than Once a Week, and [5] = Never. Upper half of the triangle reports for the selected response: "Almost Every Day" [1] and the lower half of the triangle reports for the selected response: "Several Times a Week" [2] for the above background variable (question).

Table 12A4
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: “Do You Agree: I Am Good in Math”
Selected Response: “Strongly Agree” [1]
(Grade 12, Booklet 9, 1990) $N = 601$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	113.92	35	3.25	.969	.972	.978
Five Factor	60.63	25	2.43	.983	.998	.990
5+1 Factor	76.83	30	2.56	.979	.980	.987
Δ 5, 5+1χ^2 = 16.2		Δ *df* = 5	$p < .01$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: “Do You Agree: I Am Good in Math”
Selected Response: “Agree” [2]
(Grade 12, Booklet 9, 1990) $N = 250$)

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	43.30	35	1.24	.960	.990	.992
Five Factor	30.29	25	1.21	.972	.991	.995
5+1 Factor	37.12	30	1.24	.966	.990	.993
Δ 5, 5+1χ^2 = 6.83		Δ *df* = 5	$p < .250$			

Correlations Between and Factor Loadings of the five Subscale Latent Variables Based on the Levels of Item Correlations With: “Do You Agree: I Am Good in Math”
(Grade 12, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.98	0.94	0.92	0.92	0.96
Measurement	0.94	—	0.96	1.00	0.93	1.00
Geometry	0.93	0.96	—	0.93	0.97	0.99
Statistics	0.94	1.00	0.89	—	0.88	0.95
Algebra	0.94	0.91	0.98	0.90	—	0.95
Factor loading	0.97	0.99	0.97	0.95	0.96	—

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Agree, [3] = Undecided, [4] = Disagree, [5] = Strongly Disagree. Upper half of the triangle reports for the selected response: “Strongly Agree” [1] and the lower half of the triangle reports for the selected response: “Agree” [2] for the above background variable (question).

Table 12A5
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types [1]
(Grade 12, Booklet 9, 1990) $N = 176$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	78.89	35	2.25	.926	.945	.957
Five Factor	37.09	25	1.48	.965	.979	.988
5+1 Factor	48.84	30	1.63	.954	.972	.982
Δ 5, 5+1χ^2 = 11.75	Δ *df* = .5	$p < .050$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 3 Types [2]
(Grade 12, Booklet 9, 1990) $N = 693$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	100.15	35	2.86	.974	.978	.983
Five Factor	59.79	25	2.39	.984	.983	.991
5+1 Factor	76.59	30	2.55	.980	.981	.988
Δ 5, 5+1χ^2 = 16.8	Δ *df* = 5	$p < .005$				

Correlations Between and Factor Loadings of the five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 12, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	1.00	0.90	0.91	0.86	0.96
Measurement	0.97	—	0.90	1.00	0.88	1.00
Geometry	0.96	0.97	—	0.85	0.93	0.95
Statistics	0.95	1.00	0.94	—	0.84	0.93
Algebra	0.94	0.90	0.97	0.89	—	0.91
Factor loading	0.98	0.98	1.00	0.96	0.96	—

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, [3] = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types [1],and the lower half of the triangle reports for the selected response: 3 Types [2] for the above background variable (question).

Table 12A6

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "How Often Do Problems on Worksheet"

Selected Response: "Almost Every Day "[1]

(Grade 12, Booklet 9, 1990) $N = 386$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	91.63	35	2.62	.958	.966	.974
Five Factor	49.59	25	1.98	.977	.979	.989
5+1 Factor	58.36	30	1.95	.973	.980	.987
Δ 5, 5+1χ^2 = 8.77		Δ *df* = 5	$p < .150$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do Problems on Worksheet"

Selected Response: "Several Times a Week" [2]

(Grade 12, Booklet 9, 1990) $N = 299$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	68.59	35	2.00	.959	.973	.979
Five Factor	34.86	25	1.39	.979	.989	.994
5+1 Factor	42.26	30	1.41	.975	.989	.992
Δ 5, 5+1χ^2 = 7.4		Δ *df* = 5	$p < .200$			

Correlations Between and Factor Loadings of the five Subscale Latent Variables Based on the Levels of Item Correlations With: "In Math Class How Often Do Problems on Worksheet"

(Grade 12, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.94	0.94	0.90	0.88	0.95
Measurement	0.97	—	0.95	1.00	0.90	1.00
Geometry	0.94	0.91	—	0.92	0.94	0.98
Statistics	0.91	0.91	0.91	—	0.87	0.95
Algebra	0.91	0.85	0.95	0.85	—	0.93
Factor loading	0.97	0.95	0.98	0.92	0.94	—

Note. Range for this background variable question was: [1] = Almost Every Day, [2] = Several Times a Week, [3] = About Once a Week, [4] = Less Than Once a Week, and [5] = Never. Upper half of the triangle reports for the selected response: "Almost Every Day" [1] and the lower half of the triangle reports for the selected response: "Several Times a Week" [2] for the above background variable (question).

Table 12A7
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Do You Agree: I Am Good in Math"
Selected Response: "Strongly Agree" [1]
(Grade 12, Booklet 10, 1990) N = 629

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	93.27	35	2.66	.976	.981	.985
Five Factor	35.55	25	1.42	.991	.995	.997
5+1 Factor	47.94	30	1.50	.988	.993	.995
Δ 5, 5+1χ^2 = 12.39	Δ *df* = 5	$p < .05$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Do You Agree: I Am Good in Math"
Selected Response: "Agree" [2]
(Grade 12, Booklet 10, 1990) N = 273

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	43.30	35	1.24	.960	.990	.992
Five Factor	30.29	25	1.21	.972	.991	.995
5+1 Factor	37.12	30	1.24	.966	.990	.993
Δ 5, 5+1χ^2 = 15.64	Δ *df* = 5	$p < .01$				

Correlations Between and Factor Loadings of the five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree: I Am Good in Math"
(Grade 12, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.92	0.91	0.91	0.91	0.94
Measurement	0.82	—	0.99	0.91	0.98	0.99
Geometry	0.65	0.90	—	0.88	0.97	0.98
Statistics	0.87	0.86	0.81	—	0.90	0.92
Algebra	0.79	0.77	0.87	0.84	—	0.98
Factor loading	0.87	0.93	0.88	0.94	0.90	—

Note. Range for this background variable question was: [1] = Strongly Agree, [2] = Agree, [3] = Undecided, [4] = Disagree, [5] = Strongly Disagree. Upper half of the triangle reports for the selected response: "Strongly Agree" [1] and the lower half of the triangle reports for the selected response: "Agree" [2] for the above background variable (question).

Table 12A8

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types [1]
(Grade 12, Booklet 10, 1990) $N = 182$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	50.95	35	1.46	.932	.971	.977
Five Factor	18.11	25	0.72	.976	1.018	1.000
5+1 Factor	26.26	30	0.88	.965	1.008	1.000
Δ 5, 5+1χ^2 = 8.15		Δ *df* = 5	$p < .150$			

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
Selected Response: 3 Types
(Grade 12, Booklet 10, 1990) $N = 680$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	83.32	35	2.38	.978	.984	.987
Five Factor	28.43	25	1.14	.993	.998	.999
5+1 Factor	48.78	30	1.63	.987	.993	.995
Δ 5, 5+1χ^2 = 20.35		Δ *df* = 5	$p < .005$			

Correlations Between and Factor Loadings of the five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 12, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.75	0.77	0.85	0.83	0.87
Measurement	0.96	—	0.92	0.79	0.87	0.92
Geometry	0.89	1.00	—	0.78	0.88	0.92
Statistics	0.93	0.90	0.88	—	0.81	0.88
Algebra	0.93	0.97	0.97	0.89	—	0.94
Factor loading	0.96	1.00	0.98	0.92	0.98	—

Note. Range for this background variable question was: [1] = 0-2 Types, [2] = 3 Types, [3] = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types [1] and the lower half of the triangle reports for the selected response: 3 Types [2] for the above background variable (question).

Table 12A9
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do Problems on Worksheet"
(Grade 12, Booklet 10, 1990) $N = 366$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	70.18	35	2.01	.963	.975	.981
Five Factor	40.39	25	1.62	.979	.985	.992
5+1 Factor	55.51	30	1.85	.971	.979	.986
Δ 5, 5+1χ^2 = 15.12	Δ *df* = 5	$p < .010$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do Problems on Worksheet"
(Grade 12, Booklet 10, 1990) $N = 306$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	60.42	35	1.73	.963	.979	.984
Five Factor	32.74	25	1.31	.980	.991	.995
5+1 Factor	43.28	30	1.44	.974	.987	.992
Δ 5, 5+1χ^2 = 10.54	Δ *df* = 5	$p < .100$				

Correlations Between and Factor Loadings of the five Subscale Latent Variables Based on the Levels of Item Correlations With: "In Math Class How Often Do Problems on Worksheet"
(Grade 12, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor loading
Numbers	—	0.93	0.90	0.95	0.91	0.95
Measurement	0.90	—	0.97	0.90	0.98	0.98
Geometry	0.84	1.00	—	0.88	1.00	0.98
Statistics	0.89	0.90	0.88	—	0.89	0.93
Algebra	0.91	0.94	0.94	0.92	—	0.99
Factor loading	0.92	.99	0.97	0.93	0.98	—

Note. Range for this background variable question was: [1] = Almost Every Day, [2] = Several Times a Week, [3] = About Once a Week, [4] = Less Than Once a Week, and [5] = Never. Upper half of the triangle reports for the selected response: "Almost Every Day" [1], and the lower half of the triangle reports for the selected response: "Several Times a Week" [2] to the above background variable (question).

Factor Analysis 1990 Data: Tables IA1–IA9

Table IA1
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlations With: "What Kind of Math Class Are You Taking This Year"
(Grade 4, Booklet 11, 1990) $N = 1255$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	65.75	35	1.88	.917	.947	.959
Five Factor	29.10	25	1.16	.963	.990	.995
5+1 Factor	35.57	30	1.19	.955	.989	.993
Δ 5, 5+1χ^2 = 6.47		Δ *df* = 5	$p < .275$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "What Kind of Math Class Are You Taking This Year"
(Grade 4, Booklet 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.571	—			
Geometry	.725	.625	—		
Statistics	.727	.668	1.000	—	
Algebra	1.000	.742	1.000	1.000	—
Factor loading	.919	.671	1.000	1.000	1.000

Table IA2
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "In Math Class How Often Do You Take Math Tests"
(Grade 4, Booklet 12, 1990) $N = 1240$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	59.35	35	1.70	.956	.976	.981
Five Factor	49.82	25	1.99	.963	.966	.981
5+1 Factor	55.42	30	1.85	.959	.971	.980
Δ 5, 5+1χ^2 = 5.67		Δ *df* = 5	$p < .350$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "In Math Class How Often Do You Take Math Tests"
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	1.000	—			
Geometry	.891	.778	—		
Statistics	.895	.966	.736	—	
Algebra	.723	.937	.738	1.000	—
Factor loading	1.000	.976	.779	1.000	1.000

Table IA3
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "In Math Class How Often Do You Take Math Tests"
(Grade 4, Booklet 14, 1990) $N = 1220$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	73.95	35	2.11	.940	.958	.967
Five Factor	66.89	25	2.68	.945	.936	.965
5+1 Factor	68.82	30	2.29	.944	.951	.967
Δ 5, 5+1χ^2 = 1.93	$\Delta\, df = 5$	$p < .900$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "In Math Class How Often Do You Take Math Tests"
(Grade 4, Booklet 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.614	—			
Geometry	.678	.982	—		
Statistics	.707	1.000	.962	—	
Algebra	.745	1.000	1.000	1.000	—
Factor loading	.699	1.000	.999	1.000	1.000

Table IA4
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "What Kind of Math Class Are You Taking This Year," "Parents' Education Level"
(Grade 8, Booklet 8, 1990) $N = 1234$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	57.84	35	1.65	.941	.968	.975
Five Factor	29.22	25	1.17	.970	.992	.995
5+1 Factor	34.44	30	1.15	.965	.993	.995
Δ 5, 5+1χ^2 = 5.22	$\Delta\, df = 5$	$p < .500$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "What Kind of Math Class Are You Taking This Year," "Parents' Education Level"
(Grade 8, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.597	—			
Geometry	.601	.740	—		
Statistics	.584	.817	.836	—	
Algebra	.672	.744	.939	.790	—
Factor loading	.681	.827	.960	.866	.942

Table IA5

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "What Kind of Math Class Are You Taking This Year," Agree/Disagree: "I Am Good in Math"

(Grade 8, Booklet 9, 1990) $N = 1244$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	92.81	35	2.65	.897	.913	.932
Five Factor	66.27	25	2.65	.926	.913	.952
5+1 Factor	69.77	30	2.33	.922	.930	.953
Δ 5, 5+1χ^2 = 3.5		Δ *df* = 5	$p < .750$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "What Kind of Math Class Are You Taking This Year," Agree/Disagree: "I Am Good in Math"

(Grade 8, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	1.000	—			
Geometry	.968	.964	—		
Statistics	.834	.971	1.000	—	
Algebra	.778	.837	.733	.710	—
Factor loading	1.000	1.000	.994	.931	.777

Table IA6

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "What Kind of Math Class Are You Taking This Year," "Father's Education Level," Agree/Disagree: "I Am Good in Math"

(Grade 8, Booklet 10, 1990) $N = 1230$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	129.25	35	3.69	.917	.920	.938
Five Factor	84.56	25	3.38	.946	.929	.966
5+1 Factor	97.53	30	3.25	.938	.933	.955
Δ 5, 5+1χ^2 = 12.97		Δ *df* = 5	$p < .025$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "What Kind of Math Class Are Taking This Year," "Father's Education Level," Agree/Disagree: "I Am Good in Math"

(Grade 8, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.993	—			
Geometry	.760	1.000	—		
Statistics	.767	.838	.702	—	
Algebra	.863	1.000	.738	.755	—
Factor loading	.923	1.000	.924	.806	1.000

Table IA7

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Algebra and Calculus Course Taking," Agree/Disagree: "I Am Good in Math"

(Grade 12, Booklet 8, 1990) $N = 1197$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	89.68	35	2.56	.856	.878	.905
Five Factor	67.07	25	2.68	.892	.869	.927
5+1 Factor	85.21	30	2.84	.863	.857	.905
Δ 5, 5+1χ^2 = 18.14		Δ *df* = 5	$p < .003$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "Algebra and Calculus Course Taking," Agree/Disagree: "I Am Good in Math"

(Grade 12, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.695	—			
Geometry	.880	.895	—		
Statistics	.982	1.000	.935	—	
Algebra	1.000	.506	.601	1.000	—
Factor loading	.975	.884	.928	1.000	.833

Table IA8

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Algebra and Calculus Course Taking," "Geometry-Trigonometry Course Taking," "Prior Knowledge Before the Test"

(Grade 12, Booklet 9, 1990) $N = 1176$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	104.83	35	3.00	.899	.909	.930
Five Factor	63.55	25	2.54	.939	.930	.961
5+1 Factor	71.21	30	2.37	.931	.938	.958
Δ 5, 5+1χ^2 = 7.66		Δ *df* = 5	$p < .200$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "Algebra and Calculus Course," "Geometry-Trigonometry Course Taking," "Prior Knowledge Before the Test"

(Grade 12, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.916	—			
Geometry	.772	.869	—		
Statistics	.966	1.000	.930	—	
Algebra	.501	.543	.683	6.000	—
Factor loading	.909	1.000	.907	1.000	.601

Table IA9
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Algebra and Calculus Course Taking," "Geometry-Trigonometry Course Taking"
(Grade 12, Booklet 10, 1990) $N = 1190$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	108.22	35	3.09	.887	.897	.920
Five Factor	77.22	25	3.09	.919	.897	.943
5+1 Factor	91.44	30	1.05	.905	.899	.933
Δ 5, 5+1χ^2 = 3.05	Δ *df* = 5	$p < .700$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "Algebra and Calculus Course Taking," "Geometry-Trigonometry Course Taking"
(Grade 12, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.955	—			
Geometry	.636	.793	—		
Statistics	1.000	.997	.901	—	
Algebra	.944	1.000	.996	.930	—
Factor loading	.868	1.000	.840	1.000	1.000

Factor Analysis 1992 Data: Tables TB1–TB8

Table TB1
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 4, Booklet 15, 1992) N = 361

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	50.60	35	1.45	.955	.981	.985
Five Factor	44.29	25	1.77	.960	.968	.982
5+1 Factor	49.96	30	1.67	.955	.972	.981
Δ 5, 5+1χ^2 = 5.67		Δ *df* = 5	p < .500			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 4, Booklet 15, 1992) N = 361

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.94	—			
Geometry	0.94	0.92	—		
Statistics	1.00	0.94	1.00	—	
Algebra	0.89	0.95	1.00	0.97	—
Factor loading	0.99	0.95	1.00	1.00	1.00

Table TB2
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 4, Booklet 17, 1992) N = 367

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	46.12	35	1.32	.965	.989	.991
Five Factor	33.99	25	1.36	.974	.987	.993
5+1 Factor	42.11	30	1.40	.968	.986	.991
Δ 5, 5+1χ^2 = 8.12		Δ *df* = 5	p < .250			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 4, Booklet 17, 1992) N = 367

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.91	—			
Geometry	.89	.88	—		
Statistics	1.00	.88	1.00	—	
Algebra	.97	.94	.90	0.95	—
Factor loading	0.99	0.93	0.95	1.00	0.98

Table TB3
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 8, Booklet 1, 1992) $N = 395$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	112.95	35	3.30	.934	.940	.953
Five Factor	30.40	25	1.20	.982	.994	.997
5+1 Factor	31.52	30	1.10	.982	.999	.999
Δ 5, 5+1χ^2 = 1.12		$\Delta\, df = 5$	$p < .950$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 8, Booklet 1, 1992) $N = 395$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.93	—			
Geometry	0.97	1.00	—		
Statistics	0.78	0.78	0.82	—	
Algebra	0.95	0.97	0.97	0.81	—
Factor loading	0.96	0.98	1.00	0.81	0.99

Table TB4
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 8, Booklet 2, 1992) $N = 382$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	40.32	35	1.15	.980	.997	.997
Five Factor	15.18	25	0.61	.993	1.009	1.000
5+1 Factor	19.56	30	0.65	.990	1.008	1.000
Δ 5, 5+1χ^2 = 4.38		$\Delta\, df = 5$	$p < .500$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 8, Booklet 2, 1992) $N = 382$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.92	—			
Geometry	0.90	0.88	—		
Statistics	0.99	0.99	1.00	—	
Algebra	0.95	0.89	0.91	0.99	—
Factor loading	0.97	0.94	0.94	1.00	0.96

Table TB5

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models

(Grade 8, Booklet 15, 1992) $N = 389$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	39.86	35	1.14	.980	.997	.997
Five Factor	24.24	25	0.97	.988	1.001	1.000
5+1 Factor	28.60	30	0.95	.985	1.001	1.000
Δ 5, 5+1χ^2 = 4.36		Δ *df* = 5	$p < .500$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables

(Grade 8, Booklet 15, 1992) $N = 389$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.95	—			
Geometry	0.94	0.97	—		
Statistics	0.94	0.97	0.96	—	
Algebra	0.89	0.89	0.96	0.96	—
Factor loading	0.95	0.98	0.99	0.99	0.95

Table TB6

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models

(Grade 12, Booklet 1, 1992) $N = 270$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	57.38	35	1.64	.957	.978	.983
Five Factor	33.97	25	1.36	.975	.988	.993
5+1 Factor	36.60	30	1.22	.973	.992	.995
Δ 5, 5+1χ^2 = 2.63	Δ *df* = 5		$p < .750$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables

(Grade 12, Booklet 1, 1992) $N = 370$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.91	—			
Geometry	0.81	0.87	—		
Statistics	0.79	0.87	0.86	—	
Algebra	0.87	0.92	0.88	0.82	—
Factor loading	0.91	0.97	0.92	0.89	0.95

Table TB7
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 15, 1992) $N = 360$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	88.91	35	2.5	.949	.960	.969
Five Factor	55.13	25	2.2	.969	.968	.982
5+1 Factor	64.89	30	2.2	.963	.969	.980
Δ 5, 5+1χ^2 = 9.76		Δ *df* = 5	$p < .100$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 15, 1992) $N = 360$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.95	—			
Geometry	0.96	0.89	—		
Statistics	0.95	0.92	0.91	—	
Algebra	0.93	0.91	0.91	0.80	—
Factor loading	1.00	0.96	0.96	0.93	0.92

Table TB8
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 17, 1992) $N = 350$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	100.98	35	2.89	.945	.953	.963
Five Factor	35.08	25	1.40	.981	.990	.994
5+1 Factor	41.83	30	1.39	.977	.990	.993
Δ 5, 5+1χ^2 = 6.75		Δ *df* = 5	$p < .250$			

Note. NFI = normed fit index; NNFI = non-normed index; CFI = comparative fit index.

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 17, 1992) $N = 350$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	0.95	—			
Geometry	0.88	0.90	—		
Statistics	0.90	0.82	0.80	—	
Algebra	1.00	0.93	0.95	0.87	—
Factor loading	1.00	0.95	0.92	0.88	1.00

Factor Analysis 1992 Data: Tables IB1–IB8

Table IB1

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "I Like Math," Agree/Disagree: "I Am Good in Math"

(Grade 4, Book 15, 1992) $N = 361$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	27.28	35	0.78	.940	1.024	1.000
Five Factor	20.57	25	0.82	.955	1.019	1.000
5+1 Factor	26.80	30	0.89	.941	1.012	1.000
Δ 5, 5+1χ^2 = 6.23		$\Delta df = 5$	$p < .300$			

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "I Like Math," Agree/Disagree: "I Am Good in Math"

(Grade 4, Book 15, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.986	—			
Geometry	.807	.754	—		
Statistics	1.000	1.000	.790	—	
Algebra	.730	.978	.956	1.000	—
Loading on second order factor	1.000	1.000	.834	1.000	1.000

Table IB2

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "I Like Math," "How Much Time Spent Each Day on Math Homework"

(Grade 4, Book 17, 1992) $N = 367$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	33.47	25	1.34	.956	.978	.988
Five Factor	37.87	35	1.08	.950	.995	.996
5+1 Factor	36.97	30	1.23	.951	.985	.990
Δ 5, 5+1χ^2 = 3.5		$\Delta df = 5$	$p < .600$			

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "I Like Math," "How Much Time Spent Each Day on Math Homework"

(Grade 4, Book 17, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.820	—			
Geometry	.917	.937	—		
Statistics	1.000	.915	1.000	—	
Algebra	.962	.963	.927	.957	—
Loading on second order factor	.994	.939	1.000	1.000	.977

Table IB3

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: “Do You Agree: I Am Good in Math”

(Grade 8, Book 1, 1992) $N = 395$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	85.39	35	2.44	.815	.845	.879
Five Factor	34.35	25	1.37	.926	.960	.978
5+1 Factor	38.40	30	1.28	.917	.970	.980
Δ 5, 5+1χ^2 = 4.05		$\Delta\, df = 5$	$p < .550$			

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: “Do You Agree: I Am Good in Math”

(Grade 8, Book 1, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.816	—			
Geometry	.574	.890	—		
Statistics	.526	.532	.599	—	
Algebra	.635	.953	.831	.653	—
Loading on second order factor	.726	.983	.871	.671	.950

Table IB4

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: Agree/Disagree: “Math Is Mostly Memorizing Facts,” “Do You Agree: I Am Good in Math”

(Grade 8, Book 2, 1992) $N = 382$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	44.62	35	1.27	.884	.964	.972
Five Factor	42.38	25	1.70	.890	.908	.949
5 +1 Factor	43.86	30	1.46	.886	.939	.959
Δ 5, 5+1χ^2 = 1.48		$\Delta\, df = 5$	$p < .975$			

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: Agree/Disagree: “Math Is Mostly Memorizing Facts,” “Do You Agree: I Am Good in Math”

(Grade 8, Book 2, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.871	—			
Geometry	1.000	.888	—		
Statistics	.916	1.000	1.000	—	
Algebra	.790	.847	.933	.878	—
Loading on second order factor	.970	.979	1.000	1.000	.885

Table IB5

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Do You Agree: I Am Good in Math"

(Grade 8, Book 15, 1992) $N = 394$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	78.80	35	2.25	.834	.869	.898
Five Factor	30.43	25	1.23	.936	.977	.987
5+1 Factor	37.24	30	1.24	.921	.975	.983
Δ 5, 5+1χ^2 = 6.81		$\Delta\, df = 5$	$p < .250$			

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Do You Agree: I Am Good in Math"

(Grade 8, Book 15, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.682	—			
Geometry	.893	.560	—		
Statistics	.806	.692	.709	—	
Algebra	.483	.468	.713	.701	—
Loading on second order factor	.903	.721	.845	.931	.702

Table IB6

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Do You Agree: I Am Good in Math," Agree/Disagree: "Math Is Mostly Memorizing Facts"

(Grade 12, Book 1, 1992) $N = 370$

	χ^2	*df*	χ^2/df	NFI	NNFI	CFI
One Factor	35.14	35	1.00	.849	.999	.999
Five Factor	21.06	25	0.84	.910	1.038	1.000
5+1 Factor	25.77	30	0.86	.889	.034	1.000
Δ 5, 5+1χ^2 = 4.75		$\Delta\, df = 5$	$p < .500$			

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: Do You Agree: "I Am Good in Math," Agree/Disagree: "Math Is Mostly Memorizing Facts"

(Grade 12, Book 1, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.510	—			
Geometry	.313	.746	—		
Statistics	.346	1.000	.765	—	
Algebra	.557	.764	.893	.866	—
Loading on second order factor	.450	1.000	.848	.972	.969

Table IB7

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "How Much Time Spent Each Day on Math Homework"

(Grade 12, Book 15, 1992) $N = 360$

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	37.42	35	1.07	.900	.991	.993
Five Factor	33.78	25	1.35	.910	.952	.973
5+1 Factor	36.96	30	1.23	.901	.968	.979
Δ 5, 5+1χ^2 = 3.18		$\Delta\, df = 5$	$p < .700$			

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "How Much Time Spent Each Day on Math Homework"

(Grade 12, Book 15, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	1.000	—			
Geometry	.929	.964	—		
Statistics	.878	.925	1.000	—	
Algebra	.512	.291	.335	.323	—
Loading on second order factor	.978	1.000	1.000	1.000	.468

Table IB8

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Do You Agree: I Am Good in Math"

(Grade 12, Book 17, 1992) $N = 35$

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	64.80	35	1.85	.885	.926	.943
Five Factor	38.24	25	1.53	.932	.954	.974
5+1 Factor	50.47	30	1.68	.911	.941	.961
Δ 5, 5+1χ^2 = 12.23		$\Delta\, df = 5$	$p < .030$			

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Do You Agree: I Am Good in Math "

(Grade 12, Book 17, 1992) $N = 35$

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.652	—			
Geometry	.649	.881	—		
Statistics	.735	.820	.909	—	
Algebra	.639	.638	1.000	.592	—
Loading on second order factor	.713	.865	1.000	.892	.886