Teacher Notebook:

Expresso

National Center for Research
on Evaluation, Standards, & Student Testing

UCLA | Graduate School of Education & Information Studies

National Center for Research on Evaluation, Standards, and Student Testing (CRESST)
Center for the Study of Evaluation (CSE)
Graduate School of Education & Information Studies
University of California, Los Angeles
300 Charles E. Young Drive North
GSE&IS Bldg., Box 951522
Los Angeles, CA 90095-1522
(310) 206-1532

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Overview of the Notebook

The goal of this notebook is to provide you with the necessary information to use this game with your students. In this notebook, you will find:

1) **Overview of the Game**: This section provides a brief introduction about the game including how to play and the math topics covered by the game.

2) **The Screen**: This page of the notebook shows an example of a typical screen and points out elements of the game that will help your students play such as game resources and screen features that may be important to know about as your students play the game.

3) **Suggested Topics for Classroom Instruction**: This list provides an overview of the math topics related to this game. Depending on your purpose for using the game (e.g., review, introduction, etc.), the math topics can be used as preview topics to the game or something to go over with students after they have played the game. You might teach the lesson to the whole class, a small group, or an individual student who is having difficulty.

4) **Common Errors Students Might Make in the Game**: This is a list of the most common errors students might make in this game. Review this list and watch carefully for these errors in the game. In many cases, the mistakes students make in the game provide hints to you about their deeper understanding and misconceptions about arithmetic operators and order of operations.

5) **Sample Lesson Plan**: This section provides a sample lesson that could be used as a guide for classroom instruction.

6) **FAQ**: This section provides you with frequently asked questions about this game.

7) **Standards Assessed**: This list provides an overview of the Common Core State Standards for Mathematics addressed in this game. Both middle school and elementary school standards may be listed.

Overview of the Game

Topics covered in the game: addition, subtraction, multiplication, division, and order of operations.

Expresso is a puzzle game about the manipulation of expressions. The player is presented with a limited set of elements and machines. Each element represents an expression and may contain any combination of positive or negative whole numbers and variables. The machines provided to the player represent the arithmetic operators. Each machine may require one or two elements to be placed on the machine's platforms before it can be operated. Players use the machines to manipulate the elements into the goal state and in a specific format. The goal state is represented as a mathematical expression, and the format of the goal state is restricted by the number of answer balls the player is required to fill.

Early levels of the game require the player to manipulate a small number of elements with only one or two machines to create simple expressions. As the game progresses, new mathematical concepts and techniques are introduced and practiced. In later levels, the goal states become more complex and require creative manipulations of the elements. This challenging problem-solving environment requires the player to explore the meaning of each operator and the rules governing the order of operations.

Levels 1-25: Order of operations with integers

Levels 26-43: Order of operations with integers and variables

The Screen

The Correct Answer button tells the player the required total value in all the elements. For a correct order to be accepted, all of the elements must be placed on the Answer Balls with no extra elements left over.

This shows the current total value in all the elements. When players carry out an operation, this number may change. Press this button to submit an answer.

This is the Answer Ball. When the player has finished transforming an element, the element can be dragged onto the answer ball.

Players must click on the center of each machine to carry out the operation.

These are the operators that players can use. If they are green, they are available to use. If they are gray, players cannot use that operator on this level. Click on an operator to view the machine.

This indicates the number of times the player can employ that operator.

The elements represent a combination of numbers and variables. The value of an element is indicated by green or black spheres or cubes orbiting the element.

Press this button to reset the level.

The Machines

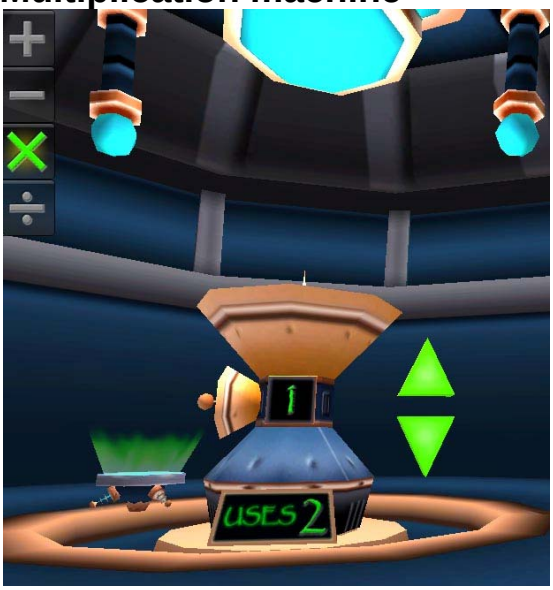
Addition machine

The Addition Machine has two platforms that can be filled with elements. When both platforms are occupied, tapping the center of the machine will combine the two elements into a single element with a value equal to the total value of the two input elements.

Subtraction machine

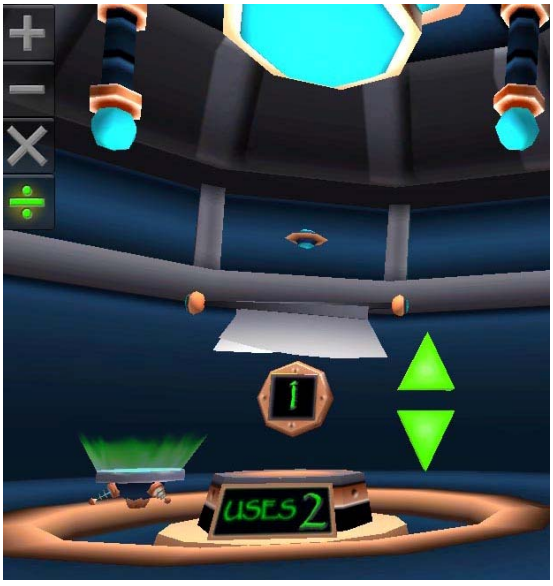
The Subtraction Machine has two platforms that can be filled with elements. When one or both platforms are occupied, tapping the center of the machine will result in a single element with a value equal to the value of the left element minus the value of the right element. Tapping the center of the machine while leaving the left platform empty subtracts the right element from zero and makes the element the opposite of its original value.

Multiplication machine



The Multiplication Machine has one platform that can be filled with an element. It also has an up-and-down arrow. These arrows control the "Parameter" value displayed at the machine's center. When the platform is occupied, tapping the center of the machine will create a number of copies (groups) of the input element. The number of copies (groups) is determined by the Parameter value.

Division machine



The Division Machine has one platform that can be filled with an element. It also has an up-and-down arrow. These arrows control the "Parameter" value displayed at the machine's center. When the platform is occupied, tapping the top of the machine will transform the input element into an element with a value equal to the initial value divided by the Parameter. This represents partitive division in that the Parameter specifies how many groups an element will be evenly broken into. The machine will not work if the answer to the division (quotient) is not a factor of the dividend.

Element values

Each small bright green sphere orbiting an element represents a positive one.

Each small black sphere orbiting an element represents a negative one.

A bright green sphere that grows larger and smaller while orbiting an element represents a positive *x* variable.

A black sphere that grows larger and smaller while orbiting an element represents a negative *x* variable.

A bright green cube that grows larger and smaller while orbiting an element represents a positive *y* variable.

A black cube that grows larger and smaller while orbiting an element represents a negative *y* variable.

Suggested Topics for Classroom Instruction

Below are the topics associated with this game. If students are having a particular type of trouble with a game, you might want to address that topic with the whole class or individual students.

Addition

Addition allows us to combine things together. If the things are identical, the result of adding can be expressed as a single term. For example, two ones plus five ones is seven ones, but 2 + *x* can't be expressed as a single term. While these ideas can be expressed as "combine like terms" we feel it is important for students to understand the reason behind such a statement and relate this idea to the way addition is used throughout the domain of mathematics (e.g., miles and miles per hour cannot be combined into a single term unless one or both are "converted" into equivalent quantities with like units). Addition is commutative so the order two things are added does not matter. Addition is associative so when three or more numbers are added, the sum is the same regardless of the grouping of the addends. For example, (2 + 3) + 4 = 2 + (3 + 4); in both expressions, the result is 9.

Subtraction

Subtraction allows us to take one quantity away from another, or to find the difference between two quantities. The subtraction of two quantities can be rewritten as an equivalent addition using the additive identity and additive inverse properties. In the game, a quantity can be made negative by subtracting the quantity **from** zero. Subtraction can be used to undo an addition but is not commutative. Consequently, the order things are subtracted is important.

Multiplication

Multiplication can be thought of as representing a certain number of groups with the same number of things in each group. While this can be represented as repeated addition when the multiplier is an integer, the repeated addition model can have shortcomings when the multiplier is a fraction. Among these shortcomings is that a repeated addition model can lead to the belief that multiplication always makes a quantity larger. Students should realize that, even in the case of integer multiplication, multiplication of a negative number by a positive multiplier can make the result (product) smaller than either the multiplier or multiplicand. Multiplication is commutative so the order things are multiplied will not affect the result.

Division

Division has two meanings. In partitive division, the divisor tells one how many groups to divide some quantity (dividend) into; in measurement division, the divisor tells one how many things to put in each group and the result (quotient) represents how many whole groups will result. In part, these two meanings result from the fact that division can undo multiplication and the fact that multiplication is commutative (three groups of four produce the same result as four groups of three, so when reversing the process it can be difficult to tell which order was originally used). Unlike multiplication, division is not commutative so the order things are divided is important. Because of limitations in how the results of division are represented, *Expresso* only represents partitive division. Moreover, because of the desire to integrate previous student understanding, only a single identical group remains after division. For example, when students divide the quantity $3x + 6$ by 3, three groups of $x + 2$ are shown (partitive division); however, because students expect that the division $(3x + 6) / 3$ should produce $x + 2$, the other two groups of $x + 2$ disappear from the screen after a few seconds.

Order of Operations

Students should see that the order they choose to perform operations in the game is important. For example, adding $1 + x$ and then multiplying by 2 will result in two groups of $1 + x$ (a total of $2 + 2x$), but multiplying 2 times x and then adding one will result in two groups of x plus one more (a total of $2x + 1$). The order of the four operations—multiply or divide before adding or subtracting, but add (subtract) or multiply (divide) in order from left to right—can be reinforced as a gaming strategy.

The Importance of WHY!

Students often have trouble connecting the symbolism of expressions (school) to their use in the world. While the game is one means to help them do so, it is vital that they begin to understand why the mathematical operators behave like they do and what the symbols in an expression represent. An appropriate question to ask in instruction and when helping students play the game is “Why?” For example, why do you get a certain result after a specific multiplication?

Common Errors
Students Might Make in the Game

Multiplication makes larger

Because of an early exposure to integer multiplication, many students believe multiplication makes larger. Have students who hold this misconception multiply negative numbers by a positive multiplier or multiply a positive number by a negative multiplier. Make sure students understand that the result is smaller (i.e., more negative) than either the multiplier or the multiplicand.

Division makes smaller

Because of an early exposure to integer division, many students believe division makes smaller. Have students who hold this misconception divide a negative integer by a positive divisor. Make sure students understand that the result is larger (i.e., less negative) than the dividend. Make sure students understand that the result is larger than the dividend.

Positive and negative numbers are the same

In *Expresso*, positive integer values are represented by green dots and negative numbers are represented by black dots. Positive and negative variables have a similar distinction. These quantities are opposites. Opposites can be combined into a single sum.

The order of operations is unimportant

The order that students use operators is an important part of what the game is trying to teach. Students can experiment to determine which order will allow them to reach a certain goal. It is important, however, that teachers make sure that students relate this strategy to the way expressions are represented symbolically. This notion becomes very important given that there are limits on how many times each operation can be used on various levels.

Using available operations in order from top to bottom

Some students believe that they must use all the additions first, followed by all the subtractions, etc. Generally, they abandon this misconception after a single admonition to consider using other operators before addition.

The large green orbs represent a single number

The large green orbs can be thought of as parentheses. Although the orb can represent individual numbers or variables, they can also represent quantities like $x + 1$. Help students to see this by multiplying such a quantity by 2 or 3 and explaining the resulting product.

How to find the value of the *x* and/or *y* variables

As the name suggests, the values of *x* and *y* vary, that is, they can have any value. In *Expresso*, *x* and *y* can represent any real number and their value cannot be specifically determined.

Sample Lesson Plan

When using *Expresso* with your students, the focus of your lesson could vary depending on if you wanted to use the game for review or initial instruction. Below is a sample lesson plan intended for students who have previously learned about arithmetic operators and order of operations and are now using *Expresso* as practice of the concepts.

The lessons below can be done on consecutive days or spaced further apart as necessary. Lessons can also be combined depending on how fast students play each level and/or how proficient students are with each topic.

Lesson 1: Quick review of addition, subtraction, multiplication, division, and order of operations (list from Math Topics section); introduce game to students at the front of class (play first few levels together).

Lesson 2: Students play levels 1-25 on individual computers. At the end of the class, debrief student difficulties and successes.

Lesson 3: Quick review student difficulties and successes from the day before; have students play levels 26-43. After students have completed all levels, review the topics of the game and make explicit the connection between the mathematics from the game and the mathematics learned in the classroom.

Expresso FAQs

Can students skip levels?

No. Students are only allowed to play the level after the last completed level. However, students can return to previously completed levels if they want to replay a level.

Can I help students while they play the game?

Yes.

We have found that students seem to learn best when they make sense of the math in the game on their own in order to succeed at each level.

That said, you are probably in the best position to determine how to help. If students are getting frustrated or unmotivated to play, it may be best to help them through one or more levels until they better understand the goal, mechanics, and math of the game.

Standards Addressed in the Game

Common Core State Standards

Mathematics (2010)

Grade 1

Operations and Algebraic Thinking

1.OA.A: Represent and solve problems involving addition and subtraction.

Grade 3

Operations and Algebraic Thinking

3.OA.A: Represent and solve problems involving multiplication and division.

Grade 6

Expressions and Equations

6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.3: Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*.