

HUMAN COGNITION AND THE USE OF NEW TECHNOLOGIES

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Introduction

Computer technology is invading our nation's schools. However, the ultimate usefulness of this new technology may be viewed with either optimism or pessimism. In the optimistic view, computers will become aides for teachers, providing help in areas such as instruction, problem solving, and evaluation. In the pessimistic view, computers will become an expensive fad and eventually join their predecessors--teaching machines--collecting dust in the basements of schoolhouses across the nation.

This paper argues that the effective use of computer technology in schools requires an understanding of how humans learn and think. The fulfillment of the optimistic scenario of computers depends on their being used in a way that is consistent with what we know about the psychology of human cognitive processes. In order to avoid the pitfalls of the past, and thus to deny the fulfillment of the pessimistic scenario, we must not base the use of computer technology on psychological principles which are inappropriate.

The tremendous influx of computer technology into our nation's schools has been widely reported. In a recent report to school board members, Fortune (1983) points out that more than 100,000 microcomputers and terminals were installed in schools in 1982, and that there will be almost one million microcomputers in schools by 1985. Similarly, a recent report in News (1983) stated:

As of last spring, by one count, 29,000 schools provided... microcomputers and terminals for 4,711,000 school students. Another study released last fall found that 60 per cent of

the nation's school districts use computers for learning and that the number of elementary schools using them had increased 80 percent over the year before. In fact, computers are multiplying too fast to count; experts figure the statistics are obsolete when they are reported.

In California, the Apple Computer Foundation's "Kids Can't Wait" program is providing one computer system for every school in the state, and the state's "Investment in People" program is providing about \$10,000,000 for the improvement of education related to "high technology". Fortune (1983, p. 7) summarizes all of the new programs as follows: "One thing is clear: computers in the school are not just a passing fad."

The urgent need to prepare for the role of computers in schools has been widely recognized. For example, a recommendation from Technology in Science Education: The Next Ten Years (National Science Foundation, 1979) states that "there is an urgent, national need to create an educational system that fosters computer literacy in our society." The report points out that "American education is not only missing a great opportunity, it is failing to discharge a crucial responsibility" (Deringer & Molnar, 1982).

As another example, the President's Report on Science and Engineering Education in the 1980's and Beyond (National Science Foundation, 1980) cites the decline in national productivity and increase in foreign trade competition as rationale for preparing American students to become better educated in the use of computers. The French government has recognized the impending "computerization of society" and has committed France to a national policy of computer

education for all students (Nora & Minc, 1980). In addition, state departments of education in this country have begun to propose computer courses as part of the mandated graduation requirements (California State Department of Education, 1982).

A recent conference on National Goals for Computer Literacy in 1985 (Seidel, Anderson & Hunter, 1982) concluded by calling for "the presence of computers for instruction in all schools for all students" and "the availability of a critical mass of high-quality curricula and courseware." In particular, the conference supported the proposition that a computer should be in every classroom from kindergarten through eighth grade; in grades 8 through 12, computers should be available in a laboratory environment for every student."

The National Council of Teachers of Mathematics (1980) has issued similar recommendations in its report An Agenda for Action: Recommendations for Mathematics of the 1980's. One recommendation concerning computers stated: "Mathematics programs should take full advantage of the power of calculators and computers at all grade levels." More specifically, the report states, "All high school students should have work in computer literacy and hands-on use of computers."

Two Scenarios

The foregoing section demonstrates that computer technology has arrived in our schools. Let me try to describe two scenarios for the role of computers in improving our children's education: a pessimistic scenario and an optimistic scenario.

In order to fully appreciate the pessimistic scenario for the future, consider the past history of technology in the schools. In particular, recall the role of teaching machines in education, and the theory of learning and instruction which supported their use.

Teaching machines clattered onto the scene of American education about 25 years ago (Skinner, 1958). In his classic book The Technology of Teaching Skinner (1968, p.22) introduced an early version of a teaching machine:

The device is a box about the size of a small record player. On the top surface is a window through which a question or problem printed on paper tape may be seen. The child answers the question by moving one or more sliders upon which the digits 0 through 9 are printed. The answer appears in square holes punched in the paper upon which the question is printed. When the answer has been set, the child turns a knob. The operation is as simple as adjusting a television set. If the answer is right, the knob turns freely and can be made to ring a bell... If the answer is wrong, the knob will not turn. When the answer is right, a further turn of the knob engages a clutch which moves the next problem into place in the window.

Some more sophisticated versions of teaching machines involved answer keys instead of knobs, and even allowed the students to write an answer.

From the beginning, the technological development of teaching machines was closely tied to an underlying theory of human learning. The dominant force in psychology at the time was behaviorism. Hence, the principles of learning by reinforcement guided the use of teaching machines. In particular, the primary instructional materials for teaching machines were teaching programs--a series of simple questions, each requiring an overt response from the learner. For example, a program in high school physics began with the following items (Skinner, 1968, p. 45):

The important parts of a flashlight are the battery and the bulb. When we "turn on" a flashlight, we close a switch which connects the battery with the _____.

When we turn on a flashlight, an electric current flows through the fine wire in the _____ and causes it to grow hot.

When the hot wire glows brightly, we say that it gives off or sends out heat and _____.

For each item, the student fills in the missing word, and then uncovers the corresponding word or phrase. In the above example, the correct answers respectively are: bulb, bulb, and light. As you can see, the instructional materials are based on the idea that learners must make a response, and that the response must be immediately reinforced.

Skinner's arguments for bringing teaching machines into schools are remarkably similar to many current arguments for using computers in schools. For example, Skinner (1968, p.26) notes that new technology will aid rather than replace the teacher: "The changes proposed

should free her for the effective exercise of her (teaching)." Similarly, Skinner (1968, p. 27) addresses the issue of cost: "Can we afford to mechanize our schools? The answer is clearly Yes."

In spite of the early enthusiasm of Skinner and many others, teaching machines did not revolutionize education. This failure to "mechanize teaching" motivates the questions: Will the computers being introduced today soon join their teaching machine predecessors, collecting dust in schoolhouse basements? Will computers, like teaching machines, fail to live up to the claims that have been made for them, and instead become just another costly fad in education? Twenty-five years from now, will we look back on Papert's (1980, p. 13) observation that "very powerful kinds of learning" take place with computers in the same way we now smile at Skinner's (1968, p. 28) claim that "the equipment needed (for educational innovation) can easily be provided"?

Proponents of the pessimistic scenario may answer "yes" to these questions. In the pessimistic scenario, computers do not find a home in American schools. Yet, there are several factors which lessen the appeal of the pessimistic scenario. First, the computer technology of today is far more powerful than the teaching machine technology of 25 years ago. Computers are not constrained by having to provide a series of test items; instead; computers allow for storage of massive data bases, graphics and simulations, interactive communication, and so on. Second, the current state of psychology has changed dramatically over the past 25 years. The behaviorist theories of

learning, based largely on animal research, have been replaced by cognitive psychology. Cognitive psychology provides implications for the instructional use of computer technology that are very different from earlier behaviorist-inspired instructional materials.

In the optimistic scenario, modern theories of learning and cognition are used in the development of useful instructional materials for computers. For example, cognitive psychologists tend to view learning as the acquisition of knowledge rather than the acquisition of responses. Mayer (1981) has shown how the analytic theories of cognitive psychology have been applied to several kinds of knowledge:

semantic knowledge--factual knowledge about the world, such as rainfall patterns for South America.

procedural knowledge--knowledge about how to carry out some procedure, such as how to compute in long division.

strategic knowledge--knowledge about how to set goals and monitor progress towards solving a problem, such as how to plan the writing of a research paper.

One of the major accomplishments of cognitive psychology has been the development of techniques for facilitating each of these kinds of knowledge within specific domains (Mayer, 1981). These techniques have implications for how to design effective instructional uses of computers. In the remainder of this paper, examples are given of possible uses of computers to enhance acquisition of each type of knowledge.

The Computer as an Aid to Learning Semantic Knowledge

Semantic knowledge refers to a person's factual knowledge about the world. Examples include knowledge about geography, such as how climate and terrain are related to a region's major crops, or the determinants of the amount of rainfall in a region.

Recent research on the psychology of human learning and cognition suggests a different approach to instruction as compared to the behaviorist approach which dominated during the teaching machine revolution. These differences can be summarized as follows:

active understanding versus passive memorization--The cognitive approach views learning as an active process in which the learner searches for meaning in what is presented, rather than a passive process of performing and remembering what the instructor demands.

assimilative versus additive--The cognitive approach views learning as a process of connecting new information with existing knowledge structures, rather than adding isolated pieces of information to memory.

cognitive structures versus responses--The cognitive approach views the outcome of learning as a coherent body of knowledge (or "mental model") rather than a set of specific responses for specific stimuli.

If meaningful learning of semantic knowledge is an active process of assimilating and reorganizing information, then computers may be used in a way that encourages active exploration. For example, Collins and Stevens (1982) have developed an "intelligent tutor" that

uses an inquiry or Socratic method, and that can be used with existing computers. The system is based on the idea that learning about some new domain, such as geography or meteorology, involves the construction of a "mental model" which relates all of the variables in the system.

Based on the observations of good human tutors, Collins (1977) developed rules for how to engage in inquiry teaching. Some of the main rules for how to teach are summarized below:

1. Ask about a known case, such as "Do they grow rice in China?"
2. Ask for any factors, such as "Why can they grow rice in China?"
3. Ask for intermediate factors, such as "Why do monsoons make it possible to grow rice in China?"
4. Ask for prior factors, such as "What do you need to have enough water?"
5. Form a general rule for an insufficient factor, such as "Do you think any place with enough water can grow rice?"
6. Pick a counterexample for an insufficient factor, such as "Why don't they grow rice in Ireland?"
7. Form a general rule for an unnecessary factor, such as "Do you think it is necessary to have heavy rainfall in order to grow rice?"
8. Pick a counterexample for an unnecessary factor, such as "Why do they grow rice in Egypt when they don't have much rainfall?"

Collins and Stevens (1982) have summarized the strategies that an intelligent tutor should use in teaching a student. Some strategies involve selecting a case, and then using counterexamples. An example of this strategy is demonstrated in the following dialogue (Collins & Stevens, 1982, p. 81):

Tutor: Why do they grow rice in Louisiana?

Student: It's a place where there is a lot of water. I think rice requires the ability to selectively flood fields.

Tutor: O.K. Do you think there's a lot of rice in, say, Washington and Oregon?

Collins' and Stevens' tutor requires a lot of specific knowledge (such as knowledge about geography), as well as procedures for asking questions and strategies for organizing the questions.

What is learned from a computerized tutor such as the one proposed by Collins and Stevens? A student may form a mental model of the factors involved in growing rice, such as summarized in Figure 1. As you can see, the student builds a coherent structure of factors and relations rather than a set of specific factual answers to specific questions. The mental model allows the student to generate answers to novel questions, and may be used in learning new information.

The use of computers as Socratic tutors represents an exciting possibility, especially in situations where the goal is to teach semantic knowledge. However, the main point in my example is that the way in which the computer is used is determined by the underlying theory of human learning and cognition that is currently available.

Thus, the success or failure of computer technology in teaching semantic knowledge depends as much on the educational implications of cognitive psychology as on the power of computer technology itself.

The Computer as an Aid to Learning Procedural Knowledge

Procedural knowledge refers to a person's knowledge about how to do something. Examples include knowledge about how to carry out long division or three-digit subtraction. The cognitive approach to procedural knowledge is based on analyzing any procedure into its parts. According to the cognitive approach, the description of procedural knowledge is based on what is learned rather than on how much is learned. Instead of focusing on the percentage of correct answers, the cognitive approach focuses on describing the procedure that the student is using to generate the answers.

Cognitive psychologists have been successful in analyzing many mathematical tasks into their constituent parts. For example, Groen and Parkman (1972) have described several different procedures that children might use to solve problems of the form $m + n$ (where the sum is less than 10). The models are based on the idea that the child uses counting as a way of finding answers to addition problems. Three possible procedures are:

counting-all--Set a counter to 0. Increment it m times and then increment it n times. For $3 + 5$, the child recites, "1,2,3...4,5,6,7,8."

counting-on--Set a counter to the first number (m); increment it n times. For $3 + 5$, the child states, "4,5,6,7,8."

min model (for counting-on)--Set a counter to the larger of m or n; increment the counter by the smaller of m or n. For $3 + 5$, the child states, "6,7,8."

Examples of these three procedures are given in Figure 2; the diamonds represent decisions and the rectangles represent operations. Fuson (1982) has observed a developmental progression in which children move from a counting-all procedure to a counting-on procedure, and eventually to a known-facts procedure in which the answers are memorized.

A slightly more complex computational task is three-digit subtraction, such as $697 - 354 = \dots$. Figure 3 shows a computational procedure which generates correct answers for three-digit subtraction problems. If a student possesses this knowledge, then the student will be able to generate correct answers for all three-digit subtraction problems. However, suppose that a student gives answers such as below:

521	819	712	481	655
<u>-418</u>	<u>-203</u>	<u>-531</u>	<u>-380</u>	<u>-160</u>
117	616	221	101	515

We could describe this student's performance by saying that he is right on 40% of the problems. However, a more useful approach is to try to describe the procedure that the student is following. For example, we could say that this student is using the procedure in Figure 3, but with small "bugs"; namely, at steps 2a, 2b, and 2c, the student subtracts the smaller number from the larger number regardless of which is on top.

Brown and Burton (1978) have argued that students' computational performance can be described by saying that they are using a procedure--perhaps with some bugs in it--and applying this procedure consistently to problems. In order to test this idea, Brown and Burton (1978) gave a set of 15 subtraction problems to 1,325 primary school children. Brown and Burton developed a computer program called BUGGY to analyze each student's procedural algorithm for three-digit subtraction. If the student's answers were all correct, BUGGY would categorize that subject as using the correct algorithm. If there were errors, BUGGY would attempt to find one bug that could account for all or most of the errors. If no single bug could account for the errors, then all possible combinations were tried, until BUGGY found combinations that best accounted for the errors. Figure 4 lists some of the most common bugs, such as "borrowing from zero" or subtracting smaller from larger". The BUGGY program was able to describe the performance of about half of the students by providing a list of each student's "bugs". Thus, Brown's and Burton's work provides a means for pinpointing specific bugs in students' computational procedures.

The BUGGY program provides an example of how computer technology can be used to improve the teaching of procedural knowledge. The BUGGY program provides the teacher with a detailed diagnosis of errors in "what is learned" so that the student can be given instruction aimed specifically at remediating the bugs. Again, my point is that the use of computers in teaching of procedural knowledge can be closely guided by existing theories in cognitive psychology.

The Computer as an Aid to Learning Strategic Knowledge

Strategic knowledge refers to knowledge concerning how to set goals, select procedures for achieving goals, and monitor progress toward goals. Examples include knowledge of how to plan the writing of a research paper or how to produce a computer program that accomplishes some task. Research in cognitive psychology emphasizes the role of process rather than product in creative problem solving. For example, consider the following assignments: "Write an essay on whether children should be allowed to choose their own courses in school" or "Write a BASIC program that will take a list of names as input and give an alphabetized list as output." Instruction could focus on the final product, such as a holistic rating of the final essay or whether the BASIC program runs properly, or could focus on the processes that a person went through in generating the final product, including setting of goals, etc.

Research on the process of writing (Hayes & Flower, 1980) has identified the following processes in writing: planning, in which the author searches memory for ideas and uses these ideas to establish a plan for generating text; translating, the actual production of text; and reviewing, the improvement of the written text. According to these researchers, writing may be viewed as a problem-solving process in which goals are set and monitored.

How can the computer become involved as an aid in writing? One current area is to use the word processing power of computers to stimulate interest in writing and to free children from some of the low level aspects of writing (such as correct spelling, punctuation and penmanship). For example, Scardamalia, Bereiter and Geolman (1982) propose that since the information processing capacity of young writers is limited, and since the mechanical and syntactic aspects of writing are not automatic, emphasis on correctly formed sentences results in poorer overall writing quality. The low level aspects of writing interfere with higher level planning. Evidence for this assertion includes the finding that when children are allowed to dictate their essays (which presumably frees them from some of the low level aspects of writing) they produce longer and higher quality essays as compared to writing.

Currently available word processing systems make revision much easier and free the writer from some aspects of production (such as penmanship and spelling). However, word processors of the future will be even more helpful in stimulating high quality writing. For example, the "Writer's Workbench" (Macdonald, Frase, Gingrich, & Keenan, 1982) is an intelligent computer coach. It consists of a collection of programs which analyze written prose and make suggestions for revisions. The Writer's Workbench is actually in use at Bell Laboratories, with over 1,000 users. You can type your text into the computer, using a standard word processing system. Then, once you have finished your first draft, you can ask the programs from the writer's workbench to suggest revisions in your manuscript.

The writer's workbench consists of three major parts: a proofreader, a style analyzer, and an on-line English reference guide. The proofreader consists of the following programs:

spelling--lists all words that may be misspelled, and allows the user to specify any new words (such as jargon, proper names, and acronyms) to the list of acceptable words.

punctuation--lists cases where punctuation may be needed or where existing punctuation may be incorrect.

double words--lists all cases in which a word is repeated.

faulty phrasing--lists phrases which may not be coherent.

split infinitives--lists all instances of split infinitives.

An example of the output of the proofreading program is shown in Figure 5. As can be seen, the program points out possible errors as well as making suggestions for how to correct the errors.

The style analyzer consists of the following programs:

style--provides readability indices, measures of average word length and average sentence length, the percentage of verbs in the passive voice, the percentage of nouns that are nominalizations, the number of sentences that begin with expletives, and other such information.

prose--compares the style statistics listed above with some standard measures; if the text's measures are outside of the standards, the program prints an explanation of why the text may be hard to read and prints suggestions for how to correct the problem.

find--locates individual sentences that contain passive verbs, expletives, nominalizations, "to be" verb forms, and other potential problem sentences.

The on-line reference programs include information on the correct use of 300 commonly misused words and phrases, a computerized dictionary, and general information about the writer's workbench. Additional programs rate the words in the text for abstractness-concreteness, rate the paragraph organization, and detect possible instances of sexist language.

Other writer's helper systems include JOURNALISM, a proofreader that comments on the organization and style of news stories (Bishop, 1975), and CRES, a proofreader that identifies uncommon words, long

sentences, and difficult phrases in NAVY documents (Kincaid, Aagard, O'Hara, & Cottrell, 1981).

Intelligent computer coaches for writing may help writers to develop more productive writing strategies. For example, in early drafts more attention can be devoted to the organization and goals of the document, since proofreaders will detect lower level errors. In addition, writers are encouraged to engage in more extensive revision cycles, allowing for refinement of writing strategies. Unfortunately, there is very little empirical information concerning the effectiveness of writing coaches, but Macdonald et al. (1982) report that writers tend to like the programs.

Goldstein (1980) has developed a computer coach to teach general problem-solving strategies. For example, a student is asked to play a computer game that requires the use of strategic thinking. Throughout the game, the computer coach makes suggestions or observations about the strategy that the student is using. Goldstein (1980, p. 53) states that "the coach's function is to intervene occasionally in student-generated situations to discuss appropriate skills that might improve the student's play." Thus, an ultimate use of computers may be to expand the power of human strategic thinking. However, as Hayes and Flower (1980) and Goldstein (1980) have pointed out, successful computer coaches must be based on useful theories of human thinking (such as Newell & Simon, 1972). Again, the usefulness of a computer coach is tied to the underlying theory of cognitive processing.

Conclusion

We began with a pessimistic and an optimistic scenario for the role of computers in education. This paper then briefly explored examples of how computers can be used to help learners acquire semantic, procedural, and strategic knowledge. The major theme of this paper has been that the effective use of computer technology in schools is tied to the educational value of current theories of human learning and cognition. Another way to state this theme is to say that the future of computer technology in schools depends on both the technological power of computers and the pedagogic usefulness of cognitive psychology.

A quarter of a century ago, American education was introduced to the technological innovation of teaching machines supported by a behaviorist psychology of learning. Today, schools are again being asked to participate in a technological revolution; however, the technological innovation involves computers, and the dominant psychology of learning is cognitive psychology. The realization of the optimistic scenario depends on our ability to extract what is useful from the cognitive psychology of human learning and cognition and to creatively apply the information to the development of computer-based instructional materials. Blindly using computers, without making use of what we now know about human learning and cognition, is likely to result in the realization of the pessimistic scenario.

References

- Bishop, R.L. (1975). The JOURNALISM programs: Help for the weary writer. Creative Computing, 1, 28-30.
- Brown, J.S., & Burton, R.R. (1978). Diagnostic models for procedural bugs in basic mathematical skills. Cognitive Science, 2, 155-192.
- California State Department of Education. (1982). Proposed graduation requirements. Sacramento, California.
- Collins, A. (1977). Processes in acquiring knowledge. In R.C. Anderson, R.J. Spiro, & W.E. Montague (Eds.), Schooling and the acquisition of knowledge. Hillsdale, NJ: Erlbaum.
- Collins, A., & Stevens, A.L. (1982). Goals and strategies of inquiry teachers. In R. Glaser (Ed.), Advances in instructional psychology: Vol. 2. Hillsdale, NJ: Erlbaum.
- Deringer, D.K., & Molnar, A. (1982). Key components for a national computer literacy program. In R.J. Seidel, R.E. Anderson, & B. Hunter (Eds.), Computer literacy. New York: Academic Press.
- Fortune, R.F. (1983). The computer: Its impact on the classroom. California School Boards, 42, (3), 5-6.
- Fuson, K.C. (1982). An analysis of the counting-on procedure in addition. In T.P. Carpenter, J.M. Moser, & T.A. Romberg (Eds.), Addition and subtraction: A cognitive perspective. Hillsdale, NJ: Erlbaum.
- Goldstein, I. (1980). Developing a computational representation for problem solving skills. In D.T. Tuma & F. Reid (Eds.), Problem solving and education. Hillsdale, NJ: Erlbaum.
- Groen, G., & Parkman, J.M. (1972). A chronometric analysis of simple addition. Psychological Review, 79, 329-343.
- Hayes, J.R. & Flower, L. (1980). Identifying the organization of writing processes. In L.W. Gregg & E.R. Steinberg (Eds.), Cognitive processes in writing. Hillsdale, NJ: Erlbaum.
- Kincaid, J.P., Aagard, J.A., O'Hara, J.W., & Cottrell, L.K. (1981). Computer readability editing system. IEEE Transactions on Profession Communication, PC-24, 38-41.
- Macdonald, N.H., Frase, L.T., Gingrich, P.S., & Keenan, S.A. (1982). The writer's workbench: Computer aide for text analysis. Educational Psychologist, 17, 172-179.

- Mayer, R.E. (1981). The promise of cognitive psychology. San Francisco: Freeman.
- National Council of Teachers of Mathematics. (1980). An agenda for action: Recommendations for mathematics in the 1980's. Reston, Virginia.
- National Science Foundation. (1979, 1980). Science and engineering education for the 1980's and beyond. Washington, DC.
- Newell, A., & Simon, H.A. (1972). Human problem solving. Englewood Cliffs, NJ: Prentice-Hall.
- Nora, S., & Minc, A. (1980). The computer of society: A report to the President of France. Cambridge, MA: MIT Press.
- Papert, S. (1980). Mindstorms. New York: Basic Books.
- Scardamalia, M., Bereiter, C., & Goelman, H. (1982). The role of production factors in writing ability. In M. Nystrand (Ed.), What writers know. New York: Academic Press.
- Seidel, R.J., Anderson, R.E., & Hunter, B. (1982). Computer literacy. New York: Academic Press.
- Skinner, B.F. (1968). The technology of teaching. New York: Appleton-Century-Crofts.
- Skinner, B.F. (1958). Teaching machines. Science, 128, 969-977.

FIGURE 1. Factors Influencing the Growing of Rice

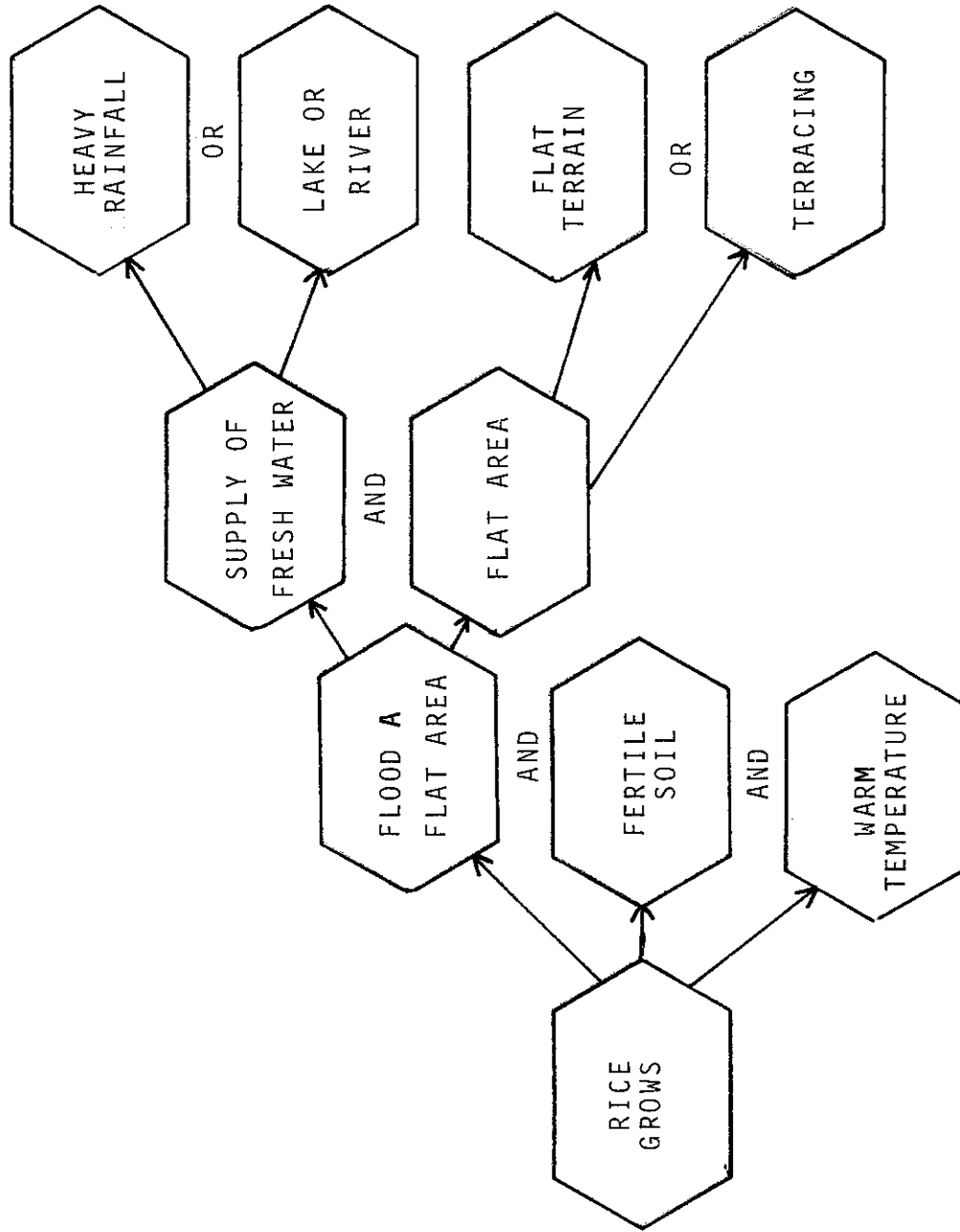
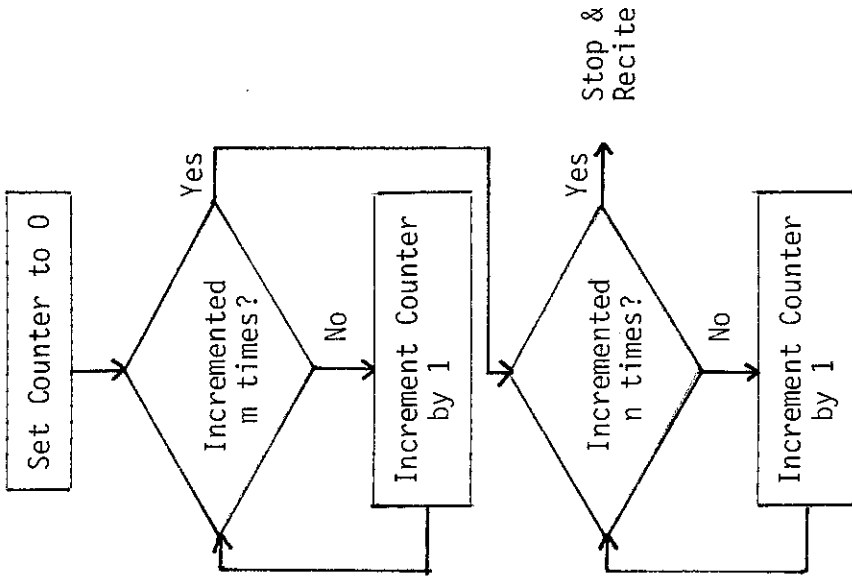


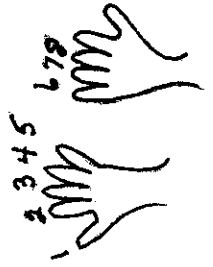
FIGURE 2. Three Counting Models of Simple Addition

Counting-All Model



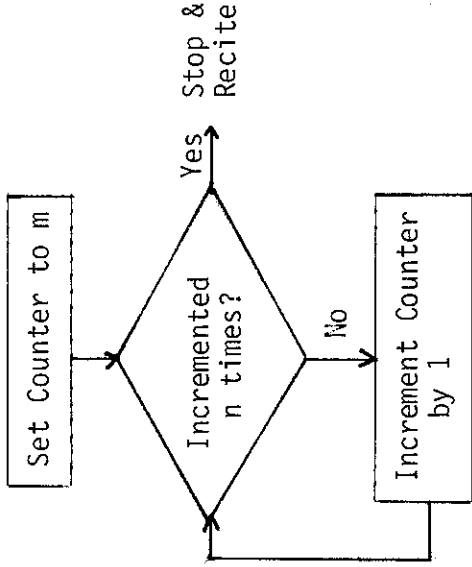
Example:

$$3 + 5 = \underline{\quad}$$



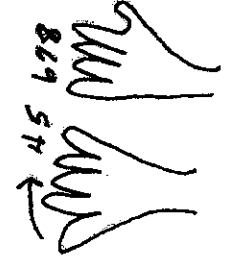
"1, 2, 3, 4, 5, 6, 7, 8"

Counting-On Model (Standard)



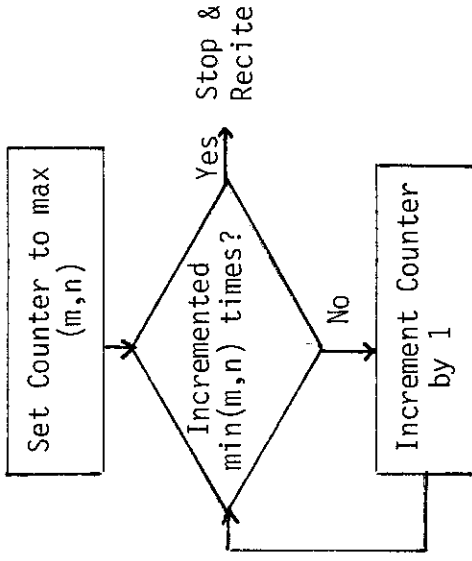
Example:

$$3 + 5 = \underline{\quad}$$



"4, 5, 6, 7, 8"

Counting-On Model (Min)



Example:

$$3 + 5 = \underline{\quad}$$



"6, 7, 8"

FIGURE 3. A Process Model for Three Column Subtraction

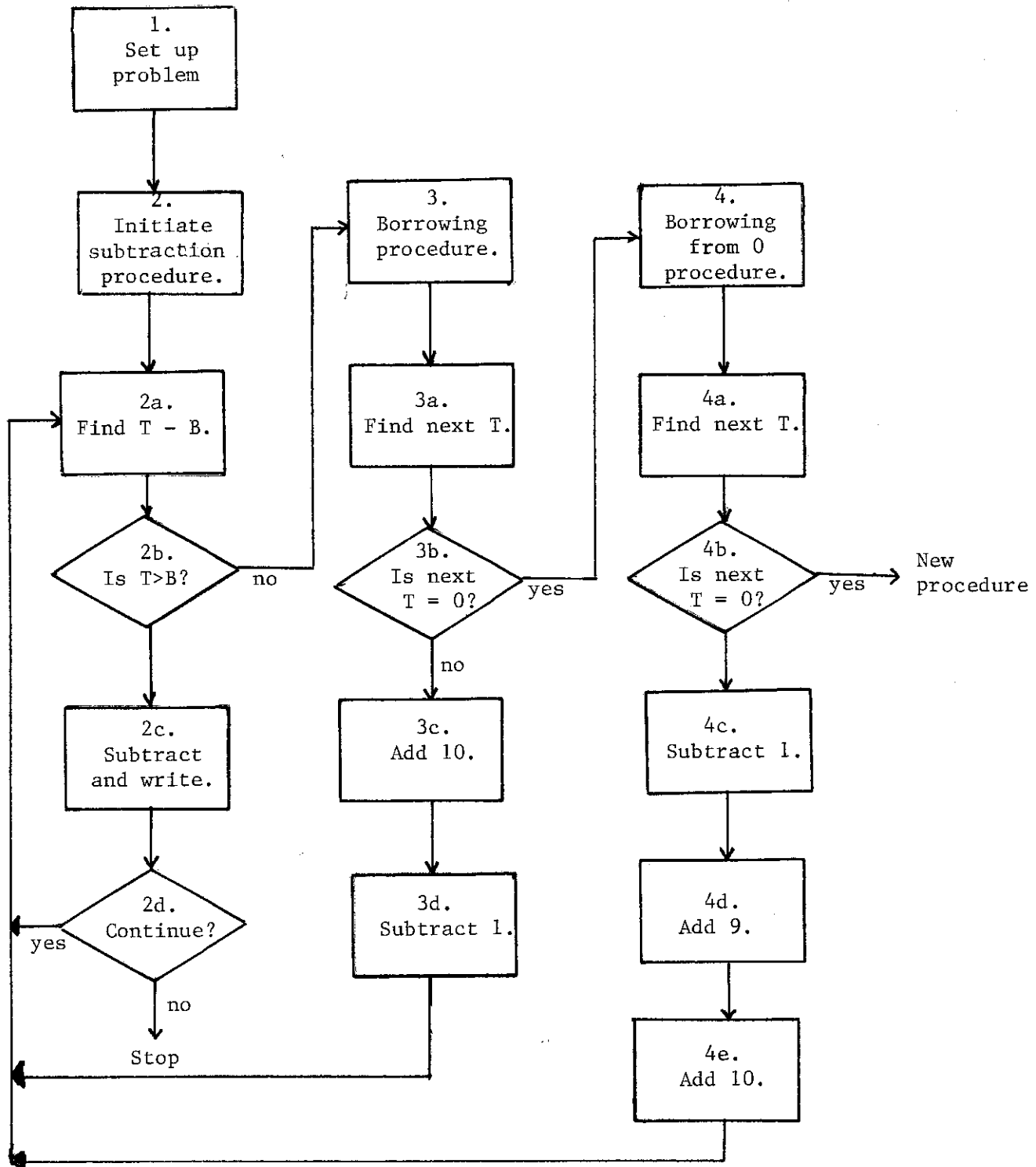


FIGURE 4. Some Common Subtraction Bugs

<u>Number of Occurrences in 1325 Students</u>	<u>Name</u>	<u>Example</u>	<u>Description</u>
57	Borrow from zero	$\begin{array}{r} 103 \\ - 45 \\ \hline 158 \end{array}$	When borrowing from a column whose top digit is 0, the student writes 9, but does not continue borrowing from the column to the left of zero.
54	Smaller from larger	$\begin{array}{r} 253 \\ -118 \\ \hline \end{array}$	The student subtracts the smaller digit in each column from the larger, regardless of which one is on top.
10	Diff 0-N=N	$\begin{array}{r} 140 \\ - 21 \\ \hline \end{array}$	Whenever the top digit in a column is 0, the student writes the bottom digit as the answer.
34	Diff 0-N=N <u>and</u> move over zero	$\begin{array}{r} 304 \\ - 75 \\ \hline \end{array}$	Whenever the top digit in a column is 0, the student writes the bottom digit as the answer. When the student needs to borrow from a column whose top digit is zero, he skips that column and borrows from the next one.

FIGURE 5. Output From A Proofreader Program

INPUT: Our report, "The Basic Fundamentals of Computatonal Complexity," is enclosed. Please send any recomended changes at your earliest convenience. thanks.

PROOFRSPELLING.....

OUTPUT: Possible spelling errors in examplefile are:

Computatonal recomended

If any of these words are spelled correctly, later type
spelladd word1 word2...wordn
to have them added to your spelldict file.

.....PUNCTUATION.....

The punctuation in examplefile is first described.

2 double quotes and 0 single quotes
0 apostrophes
0 left parentheses and 0 right ones

The program next prints any sentence that it thinks is incorrectly punctuated and follows it by its correction.

line 1

OLD: Our report, "The Basic Fundamentals of Computatonal Complexity".

NEW: Our report, "The Basic Fundamentals of Computatonal Complexity."

line 3

OLD: earliest convenience. thanks.

NEW: earliest convenience. Thanks.

For more information about punctuation rules, type:

puncrules

.....DOUBLE WORDS.....

For file examplefile:

No double words found

.....WORD CHOICE.....

Sentences with possibly wordy or misused phrases are listed next, followed by suggested revisions.

beginning line 1 examplefile

Our report, "The "[Basic Fundamentals]" of Computatonal Complexity", is enclosed.

beginning line 2 examplefile

Please send any recomended changes "[at your earliest convenience]".

file examplefile: number of lines 3, number of phrases found 2

.....Table of Substitutions.....

PHRASE SUBSTITUTION

at your earliest convenience: use "soon" for "at your earliest convenience"

basic fundamentals: use "fundamentals" for "basic fundamentals"

.....SPLIT INFINITIVES.....

For file examplefile:

No split infinitives found