Closing the Gap: Modeling Within-School Variance Heterogeneity in School Effect Studies

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Project 3.1 Methodologies for Assessing Student Progress Strand 2
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Abstract

Effective schools should be superior in both enhancing students' achievement levels and reducing the gap between high- and low-achieving students in the school. However, the focus has been placed mainly on schools' achievement levels in most school effect studies. In this article, we attend to the school-specific achievement dispersion as well as achievement level in determining effective schools. The achievement dispersion in a particular school can be captured by within-school variance in achievement (σ^2). Assuming heterogeneous within-school variance across schools in hierarchical modeling, we identified school factors related to high achievement level and a small gap between high- and low-achieving students. Schools with a high achievement level tended to be more homogeneous in achievement dispersion, but even among schools with the same achievement level, schools varied in their achievement dispersion, depending on classroom practices.

One of the fundamental questions that most school effect studies have continuously addressed is whether schools make a difference in student achievement, and if so, how much of the student achievement can be attributable to schools' effort. Regarding this question, most researchers have agreed that schools do have a measurable impact on student achievement, even though the source and the magnitude of the school effect are still heavily debated (Rumberger & Palardy, 2003).

Using the basic Hierarchical Model (HM), one can successfully show how much of the total variation in achievement comes from the student level (within-school variance, σ^2) and how much comes from the school level (between-school variance, τ). Many studies have found that between-school variance is much smaller than within-school variance. For example, using High School and Beyond (HS&B) data, Lee and Bryk (1989) found that about 19% of the total variation in student math achievement was attributable to school differences.

More complicated HM can be used to discover the source of these within- and between-school variances. Because school effect studies are usually focused on identifying effective schools after controlling for student background characteristics, or on finding out school practices that are effective in increasing student achievement, between-school variance (τ) plays an important role. Substantial τ is evidence of a school's contribution to student outcome, indicating the magnitude of variation among schools in their achievement levels (Raudenbush & Bryk, 2002). On the other hand, there has been little discussion on within-school variance (σ^2) in school effect studies.

We argue in this study that σ^2 can provide valuable information regarding effective schools because school effectiveness can be determined not only by student achievement levels, but also by the dispersion of student achievement in a particular school. Given that all the schools try to increase their students' achievement, it is clear that successful schools should have smaller variation in their student achievement levels. Additionally, these achievement levels themselves should be higher because smaller within-school variation indicates that the school has successfully directed all of its students to a certain level. In other words, effective schools should be superior in both increasing students' achievement levels and reducing the gap between high- and low-achieving students in the school. The former can be captured in common HM and has been addressed in many school effect studies. The latter—the dispersion of student achievement within a school—can be captured using within-school variance by assuming that σ^2 varies across schools with careful examination of variance heterogeneity in HM.

The purpose of this study was to illustrate how to detect variance heterogeneity and find a systematic relationship between within-school variance and school practices. If certain school practices are related to smaller within-school variance, this could provide important evidence that school practice can have an equalizing effect on student performance.

Data Description

Data from the Third International Mathematics and Science Study-Repeat (TIMSS-R) are used for this study. TIMSS-R is an international study of math and science achievement conducted by the International Association for the Evaluation of Educational Achievement (IEA) in 1999 (Eugenio & Julie, 2001). The target population was eighth-grade students, and 38 countries participated in the study. The dataset contains student, teacher, and school background data, as well as student math and science achievement scores. More information can be found at the TIMSS website, www.timss.org.

In the current study, because the purpose is not international comparison, data for a single country (Republic of Korea) and a single content area score (math achievement) were used. In TIMSS-R, this score is equated across countries using Item Response Theory and rescaled to have a mean of 500 and a standard deviation of 100. For the current study, from the larger TIMSS-R sample of 6,130 students from 150 Korean middle schools, 5,583 students in 143 schools who had complete data were used as our final sample. For the final sample, average achievement was 590.62, and the standard deviation was 77.60—almost 1 *SD* above the international average achievement level with smaller variation.

Earlier studies using the same dataset reported some student- and school-level variables affecting student achievement (Park, Park, & Kim, 2001; Yang & Kim, 2003). According to these studies, students' academic motivation and after-school time management were relatively powerful student-level predictors, and school average socioeconomic status (SES) level and school location were closely related to achievement at the school level. After preliminary screening based on this information, the variables for the current study were selected at each level (see Tables 1 and 2).

Table 1

Descriptive Statistics for Student-Level Variables

Name	Description	Category/Scale	Freq. (%)	Mean	*SD*
GENDER	Student gender	0: BOY	2872 (51.4)	0.49	.500
		1: GIRL	2711 (48.6)		
PED	Parents' highest education level	0: did not finish/did not go to primary	719 (12.9)	1.86	.942
			768 (13.8)		
		1: finished primary	2677 (47.9)		
		2: finished secondary/college	1419 (25.4)		
		3: finished university			
HOMERSC	Home educational resources index	0: low	283 (5.1)	1.10	.435
		1: medium	4471 (80.1)		
		2: high	829 (14.8)		
LESSON	Takes extra math lessons outside of school more than 1 hour/week	0: no	3288 (58.9)	0.41	.492
		1: yes	2295 (41.1)		
AOFREQ	Teacher explains rules at the beginning of new topic	1: never	229 (4.1)	3.32	.855
		2: sometimes	741 (13.3)		
		3: frequently	1602 (28.7)		
		4: always	3011 (53.9)		
STDUSEBOD	How often student uses board	1: never	932 (16.7)	2.38	.942
		2: sometimes	2511 (45.0)		
		3: frequently	1252 (22.4)		
		4: always	888 (15.9)		
MATATT	Positive attitude towards math	0: low	1487 (26.6)	0.82	.570
		1: medium	3594 (64.4)		
		2: high	502 (9.0)		
TIMEPLY	Spends 3 or more hours/day watching TV/video or playing with friends	0: no		0.63	.484
		1: yes	2083 (37.3)		
			3500 (62.7)		

Table 2

Descriptive Statistics for School-Level Variables

Name	Description	Category/Scale	Freq. (%)	Mean	*SD*
URBAN	Urban schools	0: no 1: yes	69 (48.3) 74 (51.7)	0.52	.501
SUBURBAN	Suburban schools	0: no 1: yes	89 (62.2) 54 (37.8)	0.38	.486
MPED	School mean PED	Continuous		1.85	.318
USEBOD	How often teacher uses board	Continuous (1: never ~ 4: always)		3.05	.167

Note. PED = parents' highest education level; USEBOD is entered for variance modeling.

For student background characteristics, student gender (GENDER), parents' highest education level (PED), and the home educational resources index (HOMERSC) were used. HOMERSC is a composite variable that IEA calculated based on students' responses regarding household possessions, such as computers, student's own desk, etc. For student's experience outside school, extra math lessons outside school (LESSON) and time spent on non-academic activities such as watching TV or playing with friends (TIMEPLY) were selected. Student responses to the frequency of teacher's advance organizer use (AOFREQ) and how often students used the board (STDUSEBOD) were selected to capture the impact of the classroom experience. Finally, student's positive attitude toward mathematics (MATATT) was selected to check the impact of student motivation.

Some student-level variables are aggregated to the school level to measure contextual effect and school practice effect. The school mean of parents' education level (MPED) can be used to measure the contextual effect of SES, which will be discussed in the Results section. School location (rural, suburban or urban) is entered as a dummy variable to estimate the location contrast.

Math teacher's use of the board (USEBOD) was selected for an illustrative purpose in this study. This variable, when aggregated to the school level, describes an important classroom practice. If a teacher uses the board more frequently, students in that class will share the same instructional experience more often, and as a result, math achievement for those students will become more similar. Based on this assumption, USEBOD was entered to explain variance heterogeneity. If this variable has an equalizing effect, schools in which teachers use the board more frequently should have a smaller variation in student achievement. Controlling for school mean achievement level is also crucial in variance modeling because in effective schools, achievement should be both high and narrow in dispersion. In other words, we propose to show that even among schools with the same average achievement level, some schools have smaller variation then others and that this is related to the average frequency of the teachers' board use.

Methods and Models

A common practice in multilevel application is to assume that all errors at level 1 are drawn from an identical distribution, that is, $r_{ij} \sim N(0, \sigma^2)$. It is reported that the estimation of fixed effects and their standard errors does not change substantially when this assumption does not hold and σ^2 varies randomly (Kasim & Raudenbush, 1998). Because of the robustness of this assumption, school effect studies rarely pay attention to the possibility of heterogeneous variance. However, level 1 variance may differ across schools and can give valuable information regarding the equalizing effect of certain school practices.

However, one needs to specify the level 1 and level 2 models carefully before modeling the residual level 1 variance because variance heterogeneity can result from model misspecification. Bryk and Raudenbush (1988) pointed out that in randomized experiments, heterogeneous variance across groups can be viewed as an indicator that shows the possibility of treatment and aptitude interaction. Similarly in a multilevel situation, heterogeneity may be caused by model misspecification, either by omitting an important level 1 variable or by erroneously fixing a level 1 predictor slope (Raudenbush & Bryk, 2002). However, it should be pointed out also that modeling heterogeneous variance does not compensate for model misspecification. Heterogeneous variance only indicates the possibility of the misspecification of mean function, and finding a systematic relationship between level 1 variance and school characteristics does not reduce the bias in fixed effects estimates caused by the misspecification.

If we find heterogeneity in level 1 variances after establishing the final model, the next step is to model this residual variance to see whether there is a systematic pattern. Variance homogeneity can be tested by computing chi-square statistics for standardized dispersion (see Raudenbush & Bryk, 2002, pp. 263-265, for example). After checking the variance heterogeneity, the next step would be to examine the distribution of variances and set up a regression model to find a relationship with school characteristics. Our specific models and their development are discussed below.

First, we fit a fully unconditional model to decompose the total variance into student and school levels. The results showed that the grand mean math achievement was 590.22, between-school variance was 379.81, and within-school variance was 5652.50. These results indicate that only about 6.3% of the total variance is attributable to school differences and the remaining 93.7% of the total variance comes from individual differences among students within schools. Also, the test of homogeneous variance rejected the homogeneous variance assumption. The results are summarized in Table 3.

As noted above, variance heterogeneity could occur by omission of an important level 1 variable or by fixing the level 1 slope that is in fact varying across schools. To make sure this was not the case, we fit a series of HM as described below.

We entered all eight level 1 variables were entered in the model and random variation was allowed only for intercept (Model 1, Random intercept ANCOVA

Table 3

Results From Unconditional Model

Fixed effect	Estimate	*SE*	T-ratio	*p*-value
Grand mean	590.22	1.912	308.64	0.000

Variance components	Estimate	Chi-square	*p*-value
Between-school variance	379.81	507.70	0.000
Within-school variance	5652.50		
Homogeneity of level 1 var. test		177.63	0.02

model). The level 1 homogeneous variance test for this model still rejected the null hypothesis that level 1 residual errors are drawn from identical distribution. Following Raudenbush and Bryk (2002), we checked the variability of level 1 slopes across schools and found that LESSON, AOFREQ, and TIMEPLY effects varied significantly across schools. Therefore, in Model 2, we allowed random variation for intercept and the three slopes. Also in this model, school location and the average education level of parents (MPED) were entered to model the intercept (adjusted grand mean). The chi-square test for this model also rejected the homogeneous level 1 variance assumption. In Model 3, school location and MPED were entered for the three random slopes specified in Model 2, as well as for the intercept. This was the final mean structure model. The homogeneous variance assumption was again rejected in this model. Therefore, we moved to the heterogeneous variance model, keeping the mean structure as specified in Model 3. Results for Models 1 through 3 are summarized in Table 4. The statistics package HLM5 was used to fit the three models described above.

Table 4

Result Summary for Model 1 to Model 3 With Homogeneity of Level 1 Variance Test

	Model 1			Model 2			Model 3		
Fixed effects	Estimate	*SE*	T (*p*-value)	Estimate	*SE*	T (*p*-value)	Estimate	*SE*	T (*p*-value)
For adjusted grand mean, β_{0j}									
Adjusted grand mean, γ_{00}	590.46	1.49	396.24 (0.00)	590.41	1.31	450.69(0.00)	590.94	1.32	447.67 (0.00)
Urban schools, γ_{01}				19.73	5.40	3.66 (0.00)	18.44	5.5	3.34 (0.00)
Suburban schools, γ_{02}				16.46	5.41	3.04 (0.00)	14.69	5.49	2.67 (0.00)
School average parents ed. level, γ_{03}				11.84	4.21	2.81 (0.00)	14.92	4.46	3.33 (0.00)
Gender contrast, γ_{10}	–3.33	3.06	–1.08 (0.27)	–2.93	3.10	–0.95 (0.35)	–2.77	3.14	–0.88 (0.37)
Parent highest ed. slope, γ_{20}	6.92	1.22	5.68 (0.00)	6.24	1.23	5.09 (0.00)	6.12	1.23	4.94 (0.00)
Home resource slope, γ_{30}	32.86	2.64	12.44 (0.00)	32.02	2.64	12.11 (0.00)	32.16	2.63	12.21 (0.00)
For extra outside lesson slope, β_{4j}									
Adjusted mean effect, γ_{40}	21.73	2.30	9.44 (0.00)	20.62	2.34	8.81 (0.00)	21.22	2.34	9.06 (0.00)
Urban schools, γ_{41}							–10.56	9.66	–1.09 (0.27)
Suburban schools, γ_{42}							–4.68	9.78	–0.47 (0.63)
School average parents ed. level, γ_{43}							–15.21	7.21	–2.10 (0.03)
For "teacher explains rules at the beginning" slope, β_{5j}									
Adjusted mean effect, γ_{50}	14.94	1.33	11.22 (0.00)	14.75	1.32	11.17 (0.00)	14.53	1.3	11.17 (0.00)
Urban schools, γ_{51}							2.44	7.05	0.34 (0.72)
Suburban schools, γ_{52}							2.62	7.05	0.37 (0.71)
School average parents ed. level, γ_{53}							–10.25	4.98	–2.05 (0.03)
Students' board use slope, γ_{60}	6.48	1.12	5.78 (0.00)	6.61	1.11	5.95 (0.00)	6.63	1.09	6.05 (0.00)
Positive attitude toward math slope, γ_{70}	27.38	1.68	16.24 (0.00)	27.73	1.69	16.40 (0.00)	27.74	1.69	16.41 (0.00)
For "spend more than 3 hrs. playing/TV" slope, β_{8j}									
Adjusted mean effect, γ_{80}	–12.34	2.05	–5.99 (0.00)	–13.18	2.05	–6.42 (0.00)	–13.24	2.01	–6.57 (0.00)
Urban schools, γ_{81}							–13.92	7.49	–1.85 (0.06)
Suburban schools, γ_{82}							–14.27	7.49	–1.90 (0.05)
School average parents ed. level, γ_{83}							4.29	6.53	0.65 (0.51)

	Model 1			Model 2			Model 3		
Variance components	Estimate	Chi-sq	*p*-value	Estimate	Chi-sq	*p*-value	Estimate	Chi-sq	*p*-value
Within	4417.46			4312.19			4313.77		
Between (intercept, τ_{00})	205.83	397.40	0.000	135.33	284.60	0.000	133.57	283.13	0.000
Between (extra lesson slope, τ_{44})				198.45	184.12	0.009	169.02	174.50	0.019
Between (teacher explain rules slope, τ_{55})				65.51	196.90	0.002	61.75	189.59	0.003
Between (3 or more hrs playing slope, z_{88})				93.86	169.27	0.052	88.07	166.76	0.048
Homogeneity of level-1 var. test		174.37	0.033		175.30	0.030		183.37	0.010

In the heterogeneous variance model, level 1 variance is assumed to vary across schools. Therefore, we posed school-specific within-school residual variance, σ_j^2 for school *j*. The first step in our variance modeling was to check whether schools with higher achievement levels had smaller σ_j^2 or vice versa. For this we used a latent variable regression technique (Raudenbush & Bryk, 2002; Seltzer, Choi, & Thum, 2003), which essentially uses the unobserved latent variable (adjusted school mean, β_0 in this study) as a predictor for σ_j^2 (Model 4).

An effective school, according to our definition, is a school with high achievement and small variation in achievement among its students. Therefore, to determine school effectiveness it is crucial to examine school characteristics and practices that can reduce student achievement variation even after controlling for school mean achievement. Our final model (Model 5) illustrates this point. Both β_0 (average achievement level) and USEBOD were entered to predict σ_j . Therefore, a significant USEBOD effect will indicate that among schools with the same achievement level, schools in which teachers use the board more frequently have a smaller gap between high- and low-achieving students. Specification of the final models are shown in equations (1.1), (1.2), and (1.3).

Note that at the student level, PED, HOMERSC and LESSON are grand mean centered and other level 1 variables are group mean centered. These grand mean centered variables are related to either SES or academic input from outside the school and would be better controlled for in a school effects study because variation in student achievement due to these variables cannot be attributable to school practice. This is especially true if, for example, a school's average achievement is high because most of its students take extra lessons outside school; then it would be more reasonable to adjust for the effect of these outside lessons when we evaluate the school's performance. By virtue of this level 1 centering, β_{0j} now represents the average math achievement of school *j*, holding constant parents' education, home educational resources, and extra math lessons. β_{1j} through β_{8j} capture the effect of

Achievement Model

Within-school (level-1)

$$Y_{ij} = \beta_{0j} + \beta_{1j}(GENDERij) + \beta_{2j}(PED_{ij}) + \beta_{3j}(HOMERSC_{ij}) + \beta_{4j}(LESSON_{ij}) + \beta_{5j}(AOFREQ_{ij}) + \beta_{6j}(USEBOD_{ij}) + \beta_{7j}(MATATT_{ij}) + \beta_{8j}(TIMEPLY_{ij}) + r_{ij}, \quad (1.1)$$

$$r_{ij} \sim N(0, \sigma_j^2)$$

Between-school (level-2)

$$\begin{aligned}
\beta_{0j} &= \gamma_{00} + \gamma_{01}(URBAN_j) + \gamma_{02}(SUBURBAN_j) + \gamma_{03}(MPED_j) + u_{0j} \\
\beta_{1j} &= \gamma_{10} \\
\beta_{2j} &= \gamma_{20} \\
\beta_{3j} &= \gamma_{30} \\
\beta_{4j} &= \gamma_{40} + \gamma_{41}(URBAN_j) + \gamma_{42}(SUBURBAN_j) + \gamma_{43}(MPED_j) + u_{4j} \\
\beta_{5j} &= \gamma_{50} + \gamma_{51}(URBAN_j) + \gamma_{52}(SUBURBAN_j) + \gamma_{53}(MPED_j) + u_{5j} \\
\beta_{6j} &= \gamma_{60} \\
\beta_{7j} &= \gamma_{70} \\
\beta_{8j} &= \gamma_{80} + \gamma_{81}(URBAN_j) + \gamma_{82}(SUBURBAN_j) + \gamma_{83}(MPED_j) + u_{0j},
\end{aligned}$$

$$\begin{pmatrix} u_{0j} \\ u_{4j} \\ u_{5j} \\ u_{8j} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00} & \tau_{04} & \tau_{05} & \tau_{08} \\ & \tau_{44} & \tau_{45} & \tau_{48} \\ & & \tau_{55} & \tau_{58} \\ & & & \tau_{88} \end{pmatrix} \right) \quad (1.2)$$

Dispersion model

$$\sigma_j = d_0 + d_1(\beta_{0j} - \gamma_{00}) + d_2(TCHUSEBOD_j) + e_j, \qquad e_j \sim N(0, \delta^2) \quad (1.3)$$

corresponding variables, respectively—that is, the average increase/decrease of student achievement in school *j* when the value of the corresponding variable changes by one unit.

At the school level, some βs are allowed to vary across schools, and school location and average PED level (MPED) are entered as predictors—also note that all the school- level variables are grand mean centered. By this grand mean centering, γ_{00} now captures the adjusted grand mean achievement level. γ_{01} and γ_{02} indicate how much urban and suburban schools did better/worse than the grand mean. γ_{03} requires special attention for interpretation—this fixed effect captures the contextual effect of parents' education level. Because we already have adjusted for PED at the student level, γ_{03} captures, among students with similar parental education levels, how much extra advantage students receiving in schools with a one-unit-higher mean PED level.

Because preliminary analysis found no variability in GENDER, PED, and HOMERSC effects across schools, these slopes are fixed at the school level. Therefore, γ_{10}, γ_{20}, and γ_{30} show overall gender difference (γ_{10}), PED effect (γ_{20}), and HOMERSC effect (γ_{30}), respectively. LESSON and AOFREQ slopes showed significant variability across schools, and these slopes are set to vary randomly across schools. γ_{40} captures the overall extra lessons effect. γ_{41} through γ_{43} show whether extra lessons are more effective in urban schools (γ_{41}), suburban schools (γ_{42}) or in schools with higher average SES levels (γ_{43}). γ_{50} through γ_{53} can be interpreted the same way as γ_{40} through γ_{43}. USEBOD and MATATT slopes are also fixed across schools. Therefore, γ_{60} and γ_{70} represent the overall USEBOD effect (γ_{60}) and MATATT effect (γ_{70}), respectively. γ_{80} is the overall achievement difference between students spending 3 or more hours playing/watching TV and students spending less than 3 hours in those non-academic activities. γ_{81} and γ_{82} show whether this difference is larger or smaller in urban schools (γ_{81}) and suburban schools (γ_{82}), and, if so, how much. Finally, γ_{83} shows whether the gap gets wider or narrower depending on school mean SES level.

As we specified the level 1 model (equation 1.1) such that each school has its own within-school residual variance (σ_j^2), σ_j^2 now captures the dispersion of student achievement in school j after explaining out the effect of student-level variables.

Before modeling the variance, we examined the distribution of σ_j^2.[1] Figure 1 shows the distribution of σ_j^2. The distribution seems positively skewed with one outlying school (school #142). Because variance can only take positive values, it is a common practice to log-transform the estimated variance to fit the model. This transformation reduces the skewedness and makes the transformed value able to take on a negative value. However, because log-transformation is a non-linear function, the interpretation becomes complex. Another option in this situation is a square root transformation. Even though this is also a non-linear transformation, the interpretation becomes straightforward, considering the fact that the square root of variance is the standard deviation.

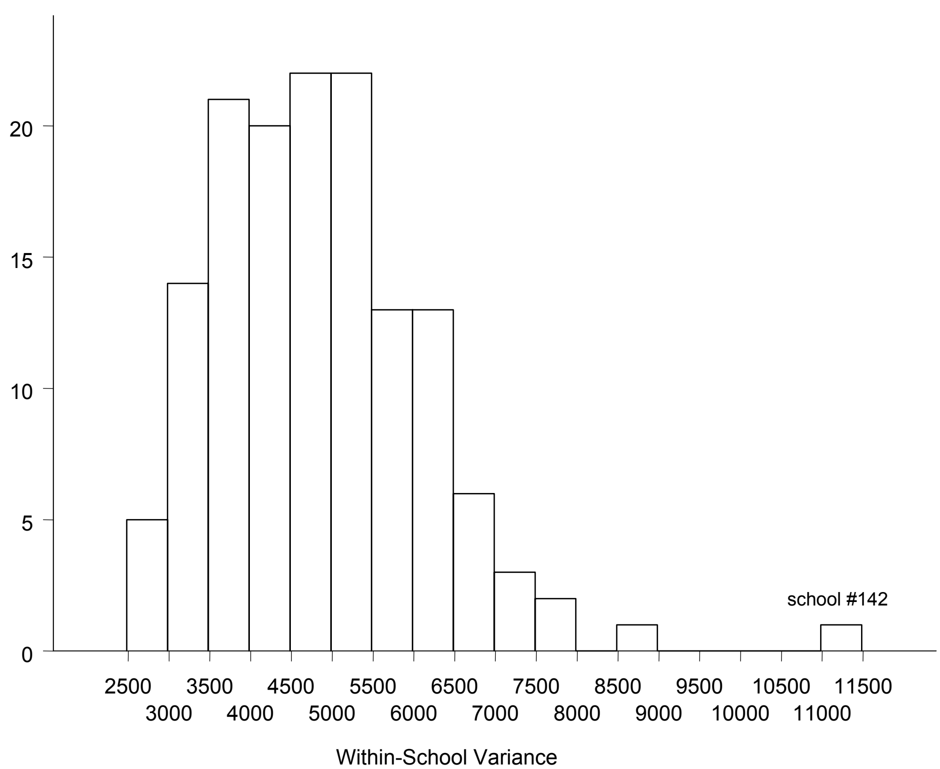

Figure 1. Distribution of posterior mean of σ_j^2.

[1] To examine the distribution of σ_j^2, we first fit the achievement model specified in equations (1.1) and (1.2) and obtained the estimate of σ_j^2, assuming that each school has its own level 1 variance.

Figure 2 shows the distribution after square root transformation. Note that the transformed data are less skewed. The skewedness of the original scale is 1.11, whereas the skewedness of the transformed scale is substantially reduced to 0.52. Therefore, we fit the variance regression model using the square root of the variance as the outcome. Note also that, for a sensitivity check, we fit the same model without school #142, which seems to have outlying variance. The result was not substantially different.[2]

Now, d_1 in equation (1.3) captures the relationship between the adjusted school mean (β_{0j}) and within-school residual standard deviation (σ_j). A negative d_1 estimate tells us that schools with high average achievement also have smaller variance. This

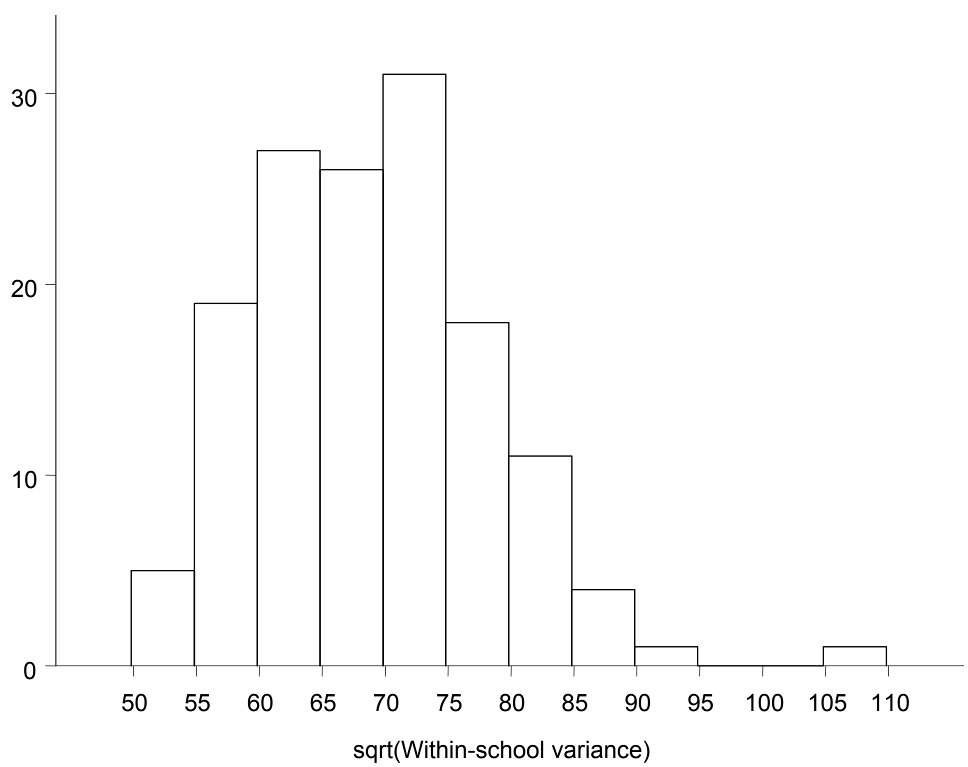

Figure 2. Distribution of posterior mean of σ_j .

[2] The result for Model 5 without school #142 is as follows: d_0 = 65.37 ($p > 0 : 1.00$), d_1 = –1.83 ($p > 0 : 0.26$) and d_2 = –0.15 ($p > 0 : 0.01$).

could possibly occur due to a ceiling effect or other successful instructional factors. Note also that β_{0j} is centered around its grand mean (γ_{00}) so that the intercept (d_0) can represent the average within-school variation of 143 schools. d_2 in equation (1.3) shows whether teachers' frequent use of the board can reduce σ_j, even after controlling for school mean achievement. USEBOD is also grand mean centered. Results for this variance model (Models 4 and 5) are summarized in Table 5. Note that Model 4 and Model 5 are analyzed using a fully Bayesian approach via Gibbs Sampler implemented in WinBUGS (Spiegelhalter, Thomas, Bets, & Lunn, 2003).

Results

The results for Models 1 through 4 are preliminary analyses showing the step-by- step procedure. Therefore, we will discuss only the results for the final model (Model 5). General fixed effects in the mean structure model (achievement model) will be discussed first; then, more importantly, the result for the variance model (dispersion model) will be discussed.

Achievement Model

After adjusting for the effect of parents' education level, home educational resources, and extra outside school math lessons, the grand mean estimate is 591 (γ_{00}). The mean for urban schools was 18.03 points above average (γ_{01}). The mean for suburban schools was 14.21 points above average (605.21). The contextual effect of the aggregate parent education level was 15.33 (γ_{03}). Because the standard deviation of MPED is .318 (see Table 2), if we compare two students with the same parental education level in two schools differing by 1 *SD* MPED level, we would expect the student in the school in the 1 *SD* higher MPED level to show 4.87 points (i.e., 15.33*.318=4.87) higher achievement than the other student in the other school.

We found no gender difference in math achievement (γ_{10}). Parents' education level did make a difference in student math achievement (γ_{20}). Note that the possible difference in math achievement between students in the lowest parents' education level and the highest is 18.54 (i.e., 6.183*3=18.54). However, as mentioned above,

Table 5

Result Summary for Heterogeneous Variance Modeling

	Model 4			Model 5		
	Mean (*SE*)	95% interval	Prob. >0	Mean (*SE*)	95% interval	Prob. >0
Mean model						
For adjusted grand mean, β_{0j}						
Adjusted grand mean, γ_{00}	591.0 (1.36)	588.4, 593.7	1.000	591.0 (1.36)	588.3, 593.7	1.000
Urban schools, γ_{01}	18.08 (5.31)	7.55, 28.38	1.000	18.26 (5.35)	7.69, 28.65	1.000
Suburban schools, γ_{02}	14.27 (5.27)	3.80, 24.45	0.996	14.55 (5.27)	4.18, 24.78	0.997
School average parents ed. level, γ_{03}	15.45 (4.92)	5.87, 25.12	0.999	15.28 (4.93)	5.63, 24.96	0.999
Gender contrast, γ_{10}	–2.60 (3.13)	–8.74, 3.55	0.203	–2.44 (3.12)	–8.57, 3.68	0.217
Parent highest ed. slope, γ_{20}	6.15 (1.20)	3.79, 8.49	1.000	6.16 (1.20)	3.82, 8.51	1.000
Home resource slope, γ_{30}	31.84 (2.61)	26.71, 36.95	1.000	31.89 (2.61)	26.78, 37.00	1.000
For extra outside lesson slope, β_{4j}						
Adjusted average effect, γ_{40}	21.15 (2.35)	16.51, 25.75	1.000	21.01 (2.34)	16.42, 25.60	1.000
Urban schools, γ_{41}	–10.99 (9.52)	–30.02, 7.53	0.121	10.86 (9.63)	–30.03, 7.99	0.128
Suburban schools, γ_{42}	–4.99 (9.42)	–23.54, 13.46	0.299	–4.81 (9.58)	–23.60, 14.10	0.305
School average parents ed. level, γ_{43}	–15.33 (8.05)	–31.01, 0.49	0.029	14.94 (7.99)	–30.69, 0.85	0.031
For "Adv. Org. frequency" slope, β_{5j}						
Adjusted average effect, γ_{50}	14.61 (1.33)	12.01, 17.24	1.000	14.63 (1.33)	12.02, 17.25	1.000
Urban schools, γ_{51}	2.32 (5.22)	–7.71, 12.63	0.669	2.73 (5.08)	–7.19, 12.67	0.706
Suburban schools, γ_{52}	2.27 (5.13)	–7.50, 12.38	0.666	2.74 (5.03)	–7.15, 12.61	0.705
School average parents ed. level, γ_{53}	–10.53 (4.63)	–19.59, –1.49	0.011	10.68 (4.61)	–19.76, –1.64	0.011
Students' board use slope, γ_{60}	6.63 (1.04)	4.61, 8.67	1.000	6.65 (1.04)	4.61, 8.69	1.000
Positive attitude toward math slope, γ_{70}	27.72 (1.63)	24.52, 30.92	1.000	27.74 (1.63)	24.55, 30.94	1.000
For "play time" slope, β_{8j}						
Adjusted average effect, γ_{80}	–13.41 (2.16)	–17.65, –9.19	0.000	13.43 (2.19)	–17.74, –9.14	0.000
Urban schools, γ_{81}	–13.71 (8.54)	–30.11, 3.70	0.056	13.95 (8.39)	–30.63, 2.35	0.047
Suburban schools, γ_{82}	–13.70 (8.46)	–30.10, 3.30	0.054	13.76 (8.30)	–30.26, 2.24	0.048
School average parents ed. level, γ_{83}	3.81 (7.51)	–10.83, 18.58	0.693	4.15 (7.42)	–10.19, 18.90	0.712
Variance model for σ_j						
Average within-school residual *SD*, d_0	65.62 (0.74)	64.18, 67.10	1.000	65.62 (0.73)	64.21, 67.09	1.000
School mean achievement slope, d_1	–0.14 (0.06)	–0.27, –0.02	0.012	–0.13 (0.06)	–0.25, –0.01	0.019
Teachers' board use slope, d_2				–8.57 (4.31)	–16.92, –0.05	0.024
Random effects variance matrix (**T**)	$\begin{pmatrix} 140.3 & -57.73 & -7.20 & -14.73 \\ & 2087 & -12.43 & 35.30 \\ & & 65.32 & -11.06 \\ & & & 133.2 \end{pmatrix}$			$\begin{pmatrix} 139.8 & -54.38 & -8.68 & -13.23 \\ & 2025 & -11.25 & 35.72 \\ & & 67.69 & -10.82 \\ & & & 1327 \end{pmatrix}$		

depending on the school's average PED level, this gap can get wider or narrower. Home resources had a strong effect on math achievement (the effect estimate is 31.81, γ_{30}). Because this variable is coded 0 to 2, the expected difference between students with low and high home resources is 63.62 (i.e., 31.81*2=63.62). However, note that most of the students (80%, Table 1) had a medium home resources level.

Students who took extra math lessons outside school more than 1 hour per week scored about 21 points better on average (γ_{40}). However, students in high MPED level schools got less benefit from extra lessons (γ_{43}). Students' frequent exposure to teacher's advance organizer (AO) did increase students' achievement (γ_{50}). Also, in high MPED level schools, this AO effect was smaller than average (γ_{53}). For example, the average AO effect was 14.60, and the AO effect for schools at 2 *SD* above the average MPED level was 7.77 (14.60–(2*.318*10.74)=7.77). The reason for extra lessons and AO being less effective in high SES schools requires further investigation. However, one possible explanation might be that in high SES schools, students could have various other educational resources and different environments (e.g., peer/family pressure and better classroom instruction) not specified in this study that contribute to student achievement, compared to low SES schools, in which students have fewer options to take extra lessons, for example.

Students' more frequent board use was positively associated with math achievement (γ_{60}). Also, students reporting a high positive attitude toward math showed higher math achievement (γ_{70}). These effects did not vary significantly across schools.

γ_{80} shows that students who spend more than 3 hours doing non-academic activities after school scored 13.46 points less on average. Interestingly, this gap gets wider in both urban (γ_{81}) and suburban schools (γ_{82}). In urban schools, the gap became 27.49 (–13.46 – 14.03 = –27.49), and in suburban schools, the gap is 27.46 (–13.46 – 14.0 = –27.46). In general, the gap between the two activity groups was smaller in rural schools than in nonrural schools.

Dispersion Model

Variance model results (d_0 to d_2 in Model 5; see Table 5) tell us that average σ_j was 65.61(d_0). School mean achievement was significantly related to smaller σ_j (d_1 = –.13). d_2, the effect of USEBOD, was –8.57 with prob. > 0 equal to 0.024. This shows that 97.6% of the posterior distribution of d_2 falls below zero—strong evidence of a negative relationship. Therefore, even after adjusting for school mean, using the board frequently in classroom instruction seems to reduce the achievement gap within schools. Table 2 shows that teachers already used the board frequently in the classroom (mean = 3.05, *SD* = .167). We expect a 1.43 point (8.57*.167 = 1.43) decrease in σ_j when USEBOD increases by 1 *SD*. If we compare two schools with a 2 *SD* difference in USEBOD and the same achievement level, the school with higher USEBOD will have about 11.2 points smaller 95% interval.[3] This interval can alternatively be interpreted as the gap between upper and lower 2.5% achievement level in a school. Therefore, the gap between the upper and lower 2.5% students will also be smaller by 11.2 points in schools with 2 *SD* above the USEBOD level. This variance model result is summarized in Figure 3. Each bar in Figure 3 represents the predicted 95% achievement range in a school.

As noted before, this can be interpreted as the achievement gap between the highest and lowest 2.5% of students in a school. For Figure 3, we chose three achievement levels (2 *SD* below average, average, and 2 *SD* above average), and within each achievement level, we selected three USEBOD levels (2 *SD* below average, average, and 2 *SD* above average). This figure clearly shows that high-achievement schools have a smaller gap, and among schools with the same achievement level, high USEBOD schools have an even smaller gap.

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[3] The 95% interval, which captures the middle 95% of the predicted achievement distribution in a school, can be calculated as $_{0j} \pm 1.96^* \sigma_j$. This interval becomes smaller in schools with high achievement or with higher USEBOD level because σ_j becomes smaller in these schools.

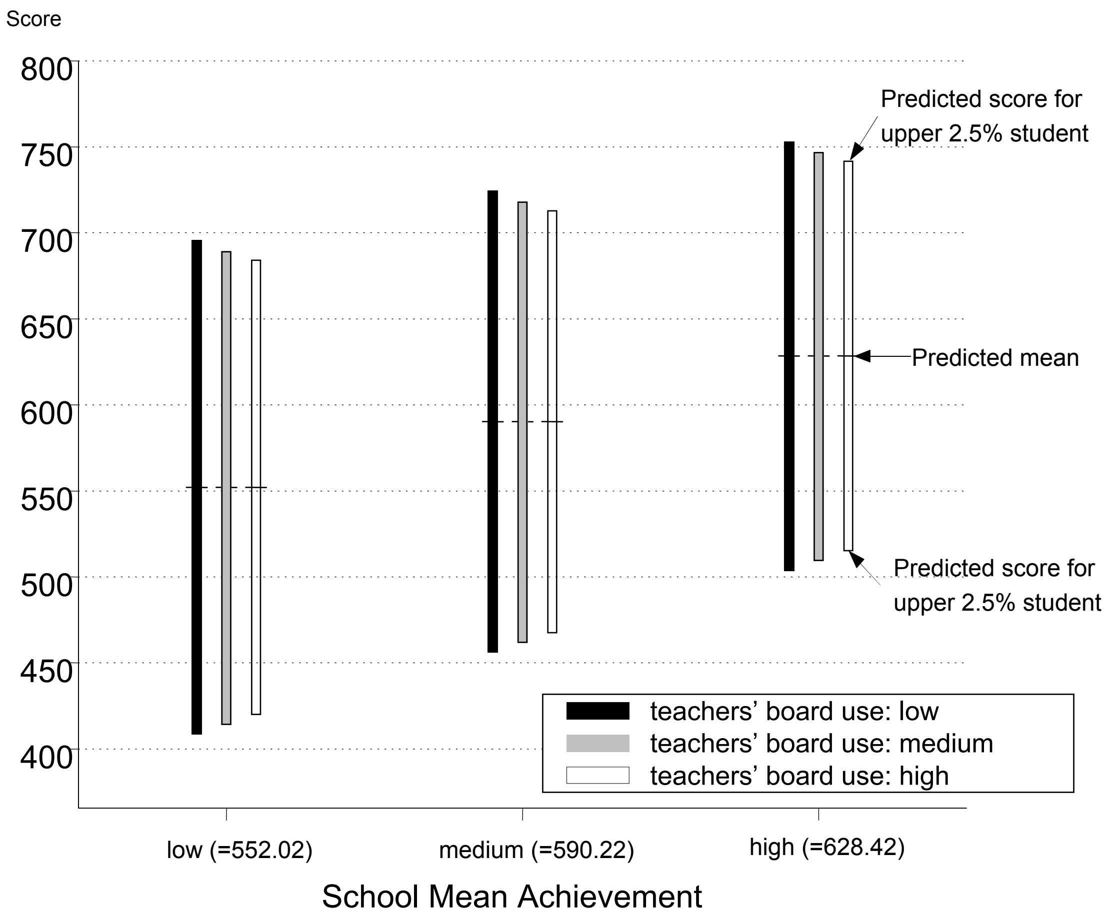

Figure 3. Comparing achievement gap between high- and low-achieving students in schools with different achievement levels and USEBOD levels.

Summary and Implications

Our results can be summarized as follows:

1. Student background characteristics, such as family SES level and home educational resources, do affect student achievement, and the magnitude of these effects is constant across schools, regardless of school characteristics.

2. Students' experiences outside school, such as extra lessons and amount of playtime, are significantly associated with student achievement level. However, the magnitude of these effects varies depending on which school a student attends. In rural schools, after-school playtime does not affect

student achievement, in contrast to nonrural schools. Also, the effect of extra after-school lessons is magnified in low SES schools.

3. Students' classroom experience, such as teacher's advance organizer use and students' board use, is positively related to student achievement. This is an especially important point regarding school effects because classroom experience is under the control of the individual school. This result shows that schools' practice, not the context, can increase student achievement level.
4. School context and background characteristics also affect student achievement in school. Rural schools showed substantially lower achievement levels than nonrural schools. Also, students in high SES schools achieved more compared to students with the same SES level in lower SES schools.
5. Regarding the achievement dispersion within school, schools with high achievement levels tend to be more homogeneous in achievement distribution. However, even among schools with the same achievement level, schools did vary in their achievement dispersion, depending on classroom practice.

In this study, we tried to answer some important questions in school effect studies, such as: What elements make a good school? and What kind of school is effective in closing the gap between high- and low-achieving students? In this regard, we argue that effective schools not only increase student achievement on average, but also reduce the gap between student achievement levels. Looking at within-school variation is especially promising in studying the gap. We chose the three schools in Figure 4, based on our results, to illustrate this point. Note that solid reference lines represent the estimated upper 2.5% achievement level, the grand mean, and the lower 2.5% achievement level in the population, respectively. First,

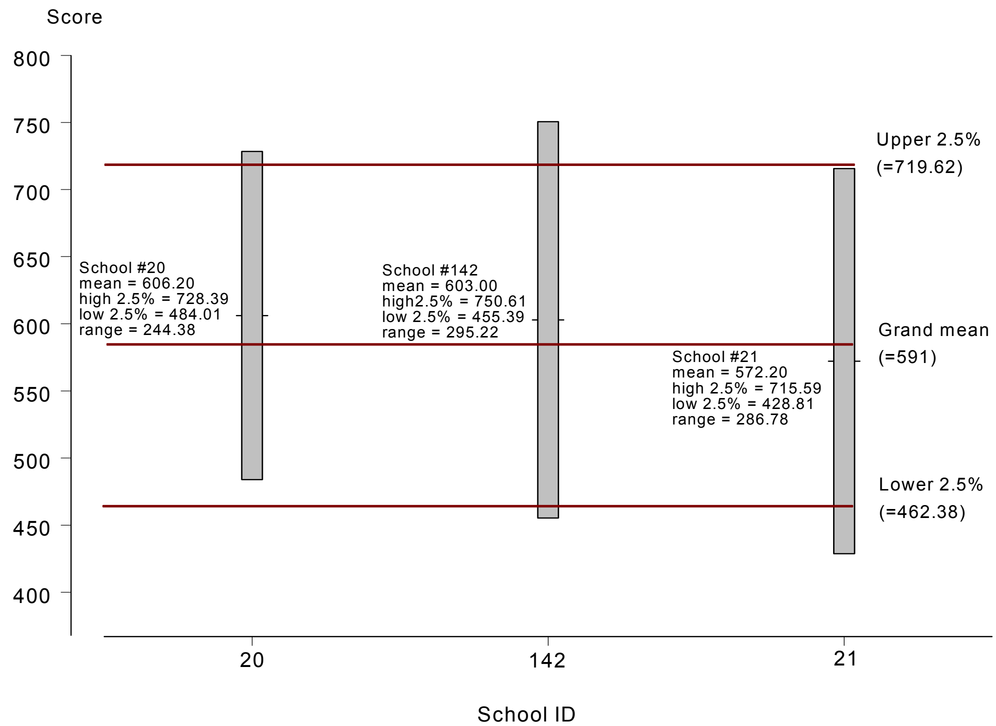

Figure 4. Contrasting three type of schools: high achievement and small gap (#20); high achievement and large gap (#142); and low achievement and large gap (#21).

schools #20 and #142 have similar mean achievement levels (606 and 603). However, if we compare the predicted gap between the upper and lower 2.5% of students in the two schools, we see that the gap is about 50 points smaller in school #20. Therefore, in terms of closing the gap, school #20 is more effective than school #142. School #21 is an example of a less effective school in dispersion as well as achievement, that is, low achievement and large gap. (See Appendix for estimated means and 95% intervals for all 143 sample schools.)

As exemplified above, the advantage of variance modeling is that we can actually calculate the gap between any two achievement percentile scores within a school (for example, 25% and 75%), and this can be used as a school indicator along with school performance level. This study also has an important implication regarding the No Child Left Behind (NCLB) Act (2002) in that closing the gap is one of NCLB's main concerns. In particular, we can study the school characteristics or practices that reduce or magnify the gap and use the information for school reform to direct as many students as possible towards the achievement goal. Also, for evaluation purposes, we can utilize more information from large-scale assessment data to identify effective schools.

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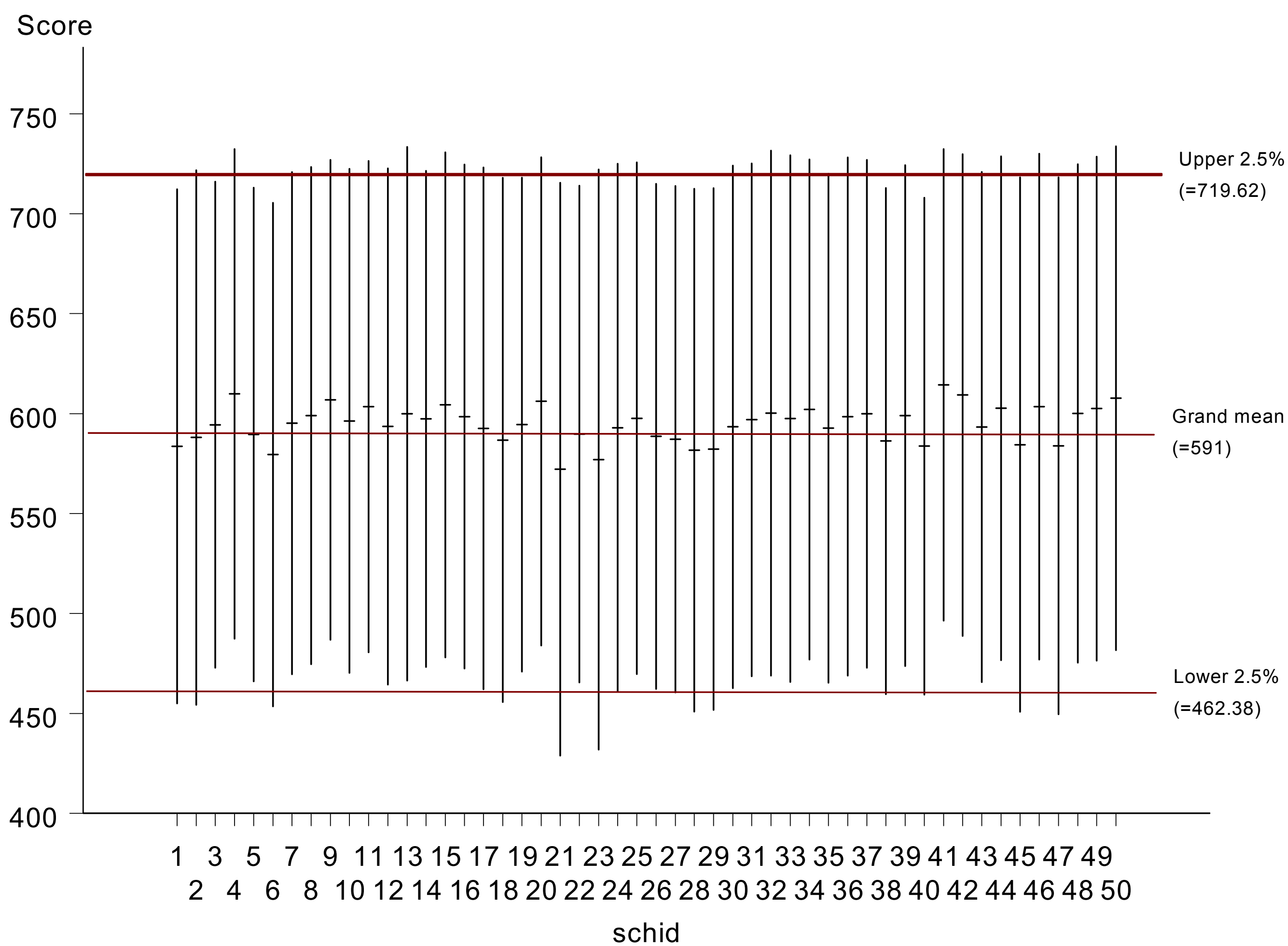
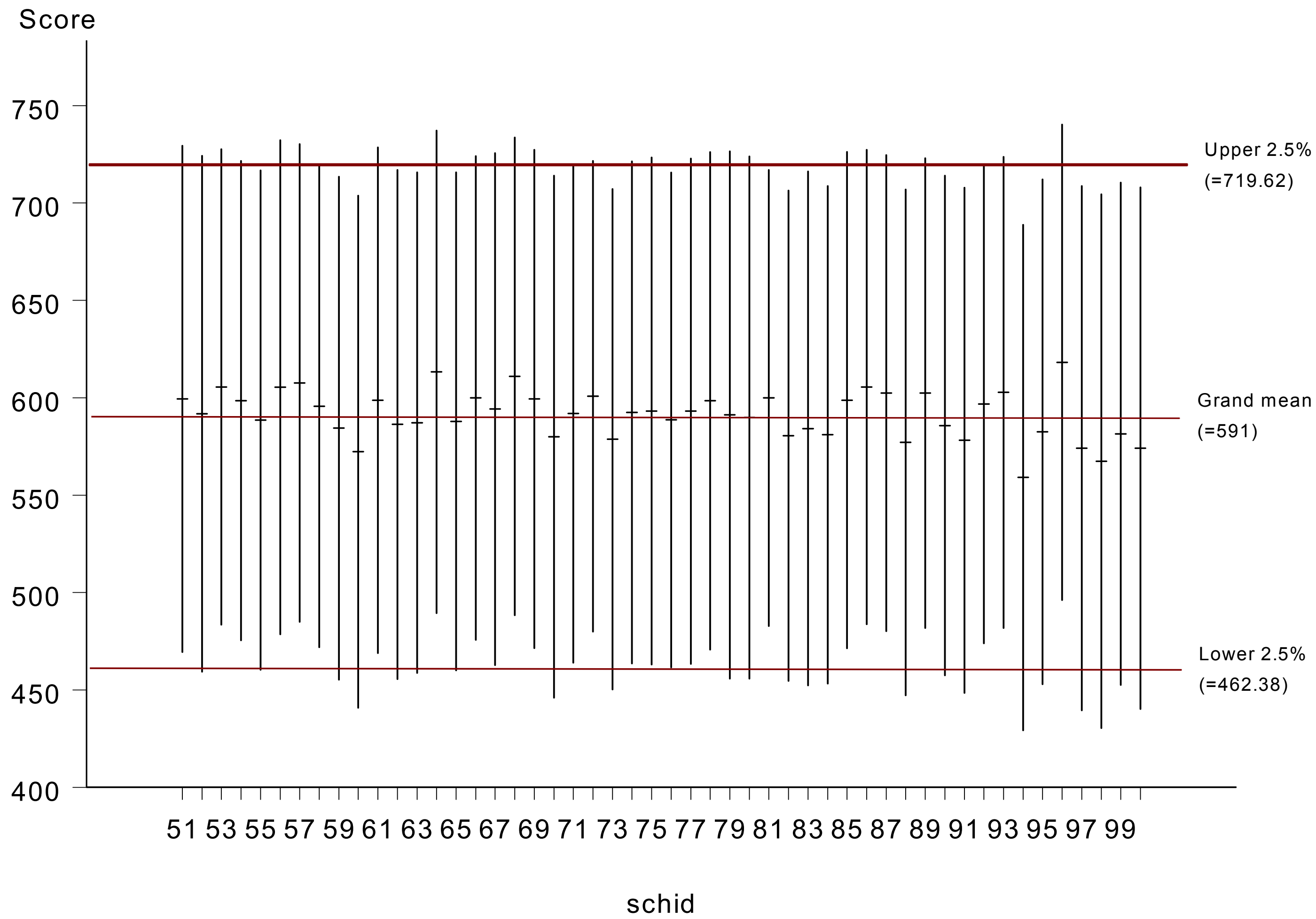
APPENDIX

Estimated mean and 95% document interval for 143 schools



Estimated mean and 95% interval for 143 schools (continued)



Estimated mean and 95% interval for 143 schools (continued)