

**CRESST REPORT 793**

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**THE DEVELOPMENT  
AND IMPACT OF  
POWERSOURCE<sup>®</sup>: YEAR 3**

MAY, 2011



**The National Center for Research on Evaluation, Standards, and Student Testing**

Graduate School of Education & Information Sciences  
UCLA | University of California, Los Angeles



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## EXECUTIVE SUMMARY

### POWERSOURCE<sup>®</sup> Background and Rationale

The POWERSOURCE<sup>®</sup> intervention is intended as a generalizable and powerful formative assessment-based strategy that can be integrated with any ongoing mathematics curriculum to improve teachers' knowledge and practice and, in turn, student learning. Combining theory and research in cognition, assessment, and learning with design elements to support the transformation of practice within existing constraints, POWERSOURCE<sup>®</sup> includes both a system of learning-based assessments and an infrastructure to support teachers' use of those assessments to improve student learning. The current study focuses on middle school mathematics, starting in 6<sup>th</sup> grade, and on helping to assure that students possess key understandings they need for success in Algebra I. Our primary research objectives are based on our hypotheses that as a result of POWERSOURCE<sup>®</sup>, teachers will become more proficient in their subject matter knowledge, more skilled in their formative use of assessment, and better focus their instruction on key ideas; in turn, teachers will be more effective in helping students to improve their understanding—as shown by measures of student learning.

A striking innovation in POWERSOURCE<sup>®</sup> is its targeting of the big ideas—fundamental concepts and principles—and their interrelationships that underlie and define a field of knowledge (rather treating specific concepts and topics in isolation, as do traditionally developed tests). The POWERSOURCE<sup>®</sup> intervention targets big ideas and related skills in four domains underlying success in Algebra I: a) rational number equivalence (RNE), b) properties of arithmetic (PA), c) principles for solving linear equations (SE) and d) application of core principles in these domains to other critical areas of mathematics, such as geometry and probability (RA). These domains were chosen because of their importance to later mastery of algebra and their significant place in mathematics standards across 6<sup>th</sup> through the 8<sup>th</sup> grade.

In each domain we designed a series of short POWERSOURCE<sup>®</sup> assessments comprised of multiple item types, which are called *Checks for Understanding*, to help teachers assess their students' understanding of basic mathematical principles as well as to connect their instruction and provide feedback to support deeper understanding. A set of instructional resources and targeted professional development activities were also developed for each of these domains. POWERSOURCE<sup>®</sup> materials are designed to complement existing curricula.

## **Updated Results from 2006-07 POWERSOURCE<sup>®</sup> Field Test**

During the 2006-07 school year we conducted two types of inter-related studies building on the prior project years' work: a) continuing investigation of item quality of the *Checks for Understanding*, supplementing data we obtained on items in the previous year; and b) experimental tests (using random assignment) of the 6<sup>th</sup> grade POWERSOURCE<sup>®</sup> materials in four districts in two states. Analyses of these two types of data were conducted during the 2007-08 project year.

### **Item Quality Analyses of 2006-07 Field Test Data**

Several kinds of item-level analyses were carried out: confirmatory factor analyses, reliability analyses, and Item Response Theory (IRT) analyses. Our typical scheme for analyzing each set of *Checks for Understanding* was to first calculate reliability coefficients (Cronbach's alpha) for the items comprising the set. Second, as another check of item quality, we conducted a principal component analysis and a confirmatory factor analysis for each test form to check whether the items exhibited the factor structure we expected (e.g., whether the computation items loaded on the same factor). Third, IRT analyses based on Rasch models, were conducted in order to obtain item parameters (difficulties) and item characteristic and information curves so that we could use them to select items for future testing.

Results were generally supportive of the statistical quality of the measures. Although a few items appeared not to fit well by Rasch models, this finding does not invalidate the measure. It simply indicates that beyond the strong overall achievement measured by each domain's test forms, there are also some minor dimensions of achievement that impact the individual item scores of individual students. That the overall dimensions (or principal components) measured by each subject assessment are very strong is demonstrated by both 1) strong Cronbach's alpha internal consistency reliabilities (a measure of measurement precision of the overall dimension derived outside the IRT model) and 2) the positive results from the confirmatory factor analysis and principal component analysis.

### **POWERSOURCE<sup>®</sup> Field Test 2006-07: Experimental Comparison Findings**

As noted in the previous annual report, sixty-six 6<sup>th</sup> grade teachers were recruited from middle-schools in Arizona and California to participate in the 2006-07 field test. Within each district, teachers were randomly assigned to experimental (POWERSOURCE<sup>®</sup>) and comparison groups. Experimental group teachers in all cases participated in initial summer professional development and after school follow-up sessions; they also used project materials, such as the *Checks for Understanding* as well as instructional supports (including

teacher instructional handbooks). Comparison group experiences varied slightly depending on district need and configuration, with all comparison group teachers having no access to the POWERSOURCE<sup>®</sup> instructional supports. All teachers gave eight *Checks for Understandings* throughout the school year—two for each of the four POWERSOURCE<sup>®</sup> modules.

Significant differences were found between POWERSOURCE<sup>®</sup> and control students' performance on project-developed measures for all districts and domains. The effect sizes ranged from .45 to 3.30 across the four districts and the five content areas, with a median effect size of 0.84 (mean effect size is 1.05). The median effect sizes by content area are 1.12, 0.80, 0.81, and 2.04, respectively for each of the units. The median effect sizes by district ranged from 0.54 to 1.01. These findings pertain to POWERSOURCE<sup>®</sup> tasks, which are broadly representative of fundamental pre-algebra content.

In terms of next steps, we are in the process of collecting and analyzing both prior and concurrent state standards test data in mathematics for both the POWERSOURCE<sup>®</sup> and control groups. The prior year's data would serve to document the initial comparability of the two groups and could be used as a covariate for analysis; the concurrent year data would serve as a partial transfer measure.

As part of the 2006-07 field test, based on an activity that asked teachers to rank a series of student assessment responses, we also examined changes in teacher knowledge across professional development participation. This activity was integrated into the POWERSOURCE<sup>®</sup> professional development meetings at both the beginning and end of the 2006-07 school year. We analyzed results from this measure to provide some basic statistical quality information about the measures and also to assess pre- and post- professional development differences in teacher knowledge.

In summary, we found the teacher knowledge ranking task to be a short, relatively non-invasive tool for assessing teacher knowledge that produces relatively reliable results as well as a scale that allows for cumulative interpretations. Among 2006-07 teachers who took both the pre- and the post- ranking assessment, composite scores demonstrate statistically significant and substantively important gains.

### **POWERSOURCE<sup>®</sup> Implementation Study 2007-08**

The core undertaking of our work during the 2007-08 school year was to conduct an extended, random assignment implementation study of our 6<sup>th</sup> grade POWERSOURCE<sup>®</sup> program. As with the 2006-07 field test, teachers were randomly assigned to either POWERSOURCE<sup>®</sup> or control conditions with the ultimate goal of determining program

impact on both students and teacher learning outcomes. The 2007-08 study, however, differed from the previous year's work in a number of important ways. Specifically:

- The experimental design incorporated both within- and between-school random assignment models. That is, for some of the districts the random assignment accomplished within each school (i.e., a given school had both POWERSOURCE<sup>®</sup> and control teachers), and for some the random assignment was between school (i.e., all teachers at a given school were POWERSOURCE<sup>®</sup> or control). Ultimately, a total of 112 6<sup>th</sup> grade teachers across seven school district participated in the study.
- Although the content focus of the four POWERSOURCE<sup>®</sup> modules remained the same (RNE, PA, SE, and RA), based on teacher feedback and our implementation experiences in 2006-07, the structure of each unit somewhat changed. In 2007-08, POWERSOURCE<sup>®</sup> teachers were provided with three *Checks for Understanding* for each unit—one prior to the first day's set of instructional materials, another in between the first and second day of instruction, and one after the second day of instruction. Thus, the students completed 12 *Checks for Understanding* (three for each of the four units) during the school year. The *Checks for Understanding* and instructional materials were also revised/ refined based on analysis of 2006-07 item level data.
- Unlike 2006-07, the control students did not complete any of the *Checks for Understanding* (i.e., the short formative assessments). Thus, the 2007-08 control students and teachers had no exposure to any of the POWERSOURCE<sup>®</sup> materials or concepts during the school year.
- All students (POWERSOURCE<sup>®</sup> and control) completed a test of prerequisite knowledge at the beginning of the school year and a transfer measures of math knowledge at the end of the school year. The test of prerequisite knowledge will serve as a baseline measure for later analyses; whereas, the transfer measure will serve as an independent student outcome measure (in addition to state test data).
- Based on district response and feedback, districts were offered the option of the control teachers receiving an alternative (i.e., non-POWERSOURCE<sup>®</sup>) professional development from the National Center for Research on Evaluation, Standards, and Student Testing (CRESST). The majority of participating districts selected this option.
- Additional teacher knowledge measures were included in the study, and a qualitative teacher observation/interview pilot study was undertaken.

Several supplementary strands of work were completed as part of CRESST activities during the 2007-08 school year as well. These studies include a validation study of teacher math knowledge measures, continued investigation of how computer-based assessment strategies can support mathematics learning, investigation of states' English language learner (ELL) assessment and accommodation practices (as part of CRESST supplementary grant),



investigation of district contexts for assessment, and international applications of the POWERSOURCE<sup>®</sup> work.

### **Plans for 2008-09**

The focus for project implementation during the 2008-09 school year will be continuing the experimental (random assignment) study of POWERSOURCE<sup>®</sup> impact begun in 2007-08. Specifically, in addition to continuing the study at the 6<sup>th</sup> grade level, we plan to add the 7<sup>th</sup> grade teachers in the participating districts to the study. The study will utilize a similar design and instrumentation to what was used in the 2007-08 study—with student and teacher outcome instruments adapted to reflect 7<sup>th</sup> grade content as applicable. During the 2008-09 school year we will also develop 8<sup>th</sup> grade materials (including *Checks for Understanding*, instructional activities, professional development resources), with the goal of expanding the study to 8<sup>th</sup> grade in the 2009-10 school year.

A planned set of supplemental activities for 2008-09 is part of the leadership strand of work. Our leadership activities intend to support states and districts in their desire to develop coherent instructional programs to engage in standards-based reform. The work focuses in two areas. First, it will focus on the collaborative development of methodology and annotated examples that practitioners and contractors can use to align instruction and assessment developmentally with key priorities for student capability in mathematics as well as with standards. Secondly, to support the ongoing quality current assessments, we propose a series of working conferences, connected to wider audiences through webinars, to clarify the specific criteria that states and local districts and schools should address in selecting and refining these assessments and to compare and learn from promising systems for which there are positive data. We propose also to make the criteria and examples of their application available interactively on the CRESST website. These efforts will concentrate on areas of recent CRESST research, formative assessment and English Language Proficiency (ELP) assessment, as well as provide a forum for sharing the best of current knowledge from CRESST and other leading efforts across the country.



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## **THE DEVELOPMENT AND IMPACT OF POWERSOURCE<sup>®</sup>: YEAR 3**

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### **Abstract**

The POWERSOURCE<sup>®</sup> intervention is a generalizable and powerful formative assessment-based strategy that can be integrated with any ongoing mathematics curriculum to improve teachers' knowledge and practice and, in turn, student learning. Our primary research objectives are based on our hypotheses that as a result of POWERSOURCE<sup>®</sup>, teachers will become more proficient in their subject matter knowledge, more skilled in their formative use of assessment, and better focus their instruction on key ideas. Ultimately, teachers will be more effective in helping students to improve their understanding—as shown by measures of student learning. This report provides a detailed overview of our study results from the 2006-07 school year. We found that a knowledge mapping task and a student response analysis task, could be used as a set, or the knowledge mapping task could be used separately from the other tasks, in order to efficiently investigate both content knowledge of teachers and knowledge of teaching algebra.

### **Introduction**

The POWERSOURCE<sup>®</sup> intervention is intended as a generalizable and powerful formative assessment strategy that can be integrated with any ongoing mathematics curriculum to improve teachers' knowledge and practice and, in turn, student learning. Combining theory and research in cognition, assessment and learning (for both adults and students) with design elements to support the transformation of practice within existing constraints, POWERSOURCE<sup>®</sup> includes both a system of learning-based assessments and an infrastructure to support teachers' use of those assessments to improve student learning. The current study focuses on middle school mathematics, starting in 6<sup>th</sup> grade, and on helping to assure that students possess key understandings they need for success in Algebra I. Such a focus is motivated by ample research showing the frequency and price of failure for subsequent academic performance, including high school graduation, college entry and preparation (e.g., Brown & Niemi, 2007). Our primary research objectives are based on our hypotheses that as a result of POWERSOURCE<sup>®</sup>, teachers will become more proficient in their subject matter knowledge, more skilled in their formative use of assessment, and better focus their instruction on key ideas; as a result, teachers will be more effective in helping students to improve their understanding, as shown by measures of student learning. Ultimately, we expect the improvements in student understanding to drive better performance

on No Child Left Behind (NCLB) mandated state tests, transfer measures, and future coursework.

### **Research on Formative Assessment**

The intervention builds on recent research showing formative assessment as a powerful strategy for improving learning. (Black & Wiliam, 1998a, 1998b; Bloom, 1968; Kluger & DeNisi, 1996). For example, Black and Wiliam's (1998a) landmark meta-analysis, based on a review of 250 studies, found effect sizes that ranged between .4 and .7, and found particularly large effect sizes for low-achieving students—including students with learning disabilities (Black and Wiliam, 1998b). This finding makes intuitive sense, as one of the major functions of formative assessment is to determine where students are relative to learning goals and to use this information to provide feedback and/or make necessary instructional adjustments—such as re-teaching, trying alternative instructional approaches, or offering more opportunities for practice. If students have already mastered the content, there is little need for subsequent adjustment and little room for learning improvement.

Yet even as research shows the rich potential of formative assessment, so too it suggests the limits of current practice. The quality of increasingly popular interim or benchmark testing, marketed as formative assessments to districts and schools, is uneven; in fact, assessment tends to be an afterthought rather than a core, quality element of current curriculum materials (Herman & Baker, 2006; Herman, Osmundson, Ayala, Schneider, & Timms, 2006; Wolf, Bixby, Glenn, & Gardner, 1991). Moreover, educators often have limited background and capacity to develop or engage in quality assessment practices (Heritage & Yeagley, 2005; Herman & Gribbons, 2001; Plake & Impara, 1997; Shepard, 2001; Stiggins, 2005). For many teachers, their current classroom assessment practices are almost exclusively summative, consisting, for example, of end-of-the-week, unit or semester tests.

Students receive grades or scores on these assessments and their teachers—who have neither the time nor the curriculum resources to remediate deficiencies—move on, disconnecting the assessments from any active function in learning. Yet as Black and Wiliam (1998a, 1998b) note, assessments can only become formative when information from them is used immediately to inform teaching and for the benefit of student learning. Teacher subject matter knowledge offers yet another challenge, as research and our own experiences in assessment development with teachers and districts suggest that many teachers do not have subject area knowledge sufficiently deep to teach or assess mathematics effectively (Ball & Bass, 2001; Ball, Lubienski, & Mewborn, 2001).

Learning to use assessment in a more formative way, thus, requires significant changes for many districts, teachers, and students. For districts, it will mean ensuring that teachers have the time and resources to act on the assessment information they receive. For teachers and students, it will involve learning to use assessment information diagnostically to determine the course of instruction and learning, and to deal with learning difficulties that are revealed by formative assessments. Given the challenges involved in changing assessment practices, a substantial part of our research and development, therefore, focuses on exploring the types and frequency of assessments and instructional supports that will be feasible to implement and most beneficial to teachers and students (e.g., helping teachers to understand mathematical concepts more deeply, monitor learning of key ideas and skills, and to figure out the best strategies to improve students' understanding).

### **Learning-Based POWERSOURCE<sup>®</sup> Strategy**

The POWERSOURCE<sup>®</sup> intervention involves not only the development of formative assessments, but also the development of professional development and instructional support resources to help teachers to understand the mathematical content, interpret assessment information, provide feedback to students, and adapt instruction as needed. Moreover, a striking innovation in POWERSOURCE<sup>®</sup> is its targeting of the big ideas—fundamental concepts and principles—and their interrelationships that underlie and define a field of knowledge, rather treating specific concepts and topics in isolation (as do traditionally developed tests). This innovation is motivated by ample evidence from a range of cognitive psychology perspectives, which suggest that for learning to be acquired efficiently and sustained it must enable students to connect to organizing principles what otherwise would be disconnected knowledge or procedures and to integrate and demonstrate their knowledge and skills in many situations, in near and far transfer, and across time (e.g., Atkinson & Shiffrin, 1968; Chi, Feltovich & Glaser, 1981; Ericson, 2003; Ericson & Simon, 1984; Hiebert & Carpenter, 1992; Mayer, 2003; Brown, Bransford, & Cocking, 1999; Newell, 1990, VanLehn, 1996, Catrambone & Holyoak, 1989).

Similarly, the specific item types used in POWERSOURCE<sup>®</sup> were developed based on cognitive research demonstrating the value of specific strategies for promoting transfer. Research, for example, suggests that learning and problem solving strategies can be successfully transferred if students are taught to focus on self-evaluation or metacognition (Moreno & Mayer, 2005; Palincsar & Brown, 1984; Pressley & Brainerd, 1985); the conditions for applying strategies (Judd, 1908; 1936; Kilpatrick, 1992); the building of principled representations of problem situations (Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Kilpatrick, Swafford, & Findell, 2001); worked-out examples as a way to build

problem schemas that generalize across a range of tasks (Chi & Bassok, 1989; Pawley, Ayres, Cooper, & Sweller, 2005); and lastly, the explanation and problem solving tasks requiring understanding of core concepts and principles that reoccur across arithmetic, pre-algebra, and algebra (Carpenter & Franke, 2001; Haverty, 1999; Ready, Edley, & Snow, 2002; Schmidt, McKnight, & Raizen, 1997). POWERSOURCE<sup>®</sup> not only uses item types that are positioned to uniquely foster learning, but it also purposively employs multiple formats to promote transfer, rather than focusing only on those representations adopted by test developers designing for accountability purposes (Richardson-Klavehn & Bjork, 2002).

### **Targeted Domains Operationalized in *Checks for Understanding***

The POWERSOURCE<sup>®</sup> intervention targets big ideas and related skills in four domains underlying success in Algebra I: a) rational number equivalence (RNE), b) properties of arithmetic (PA; the distributive property), c) principles for solving linear equations (SE); and d) application of core principles in these domains to other critical areas of mathematics, such as geometry and probability (RA). These domains were chosen because of their importance to later mastery of algebra and their significant place in state mathematics standards across 6<sup>th</sup> through 8<sup>th</sup> grade.

In each domain we have designed a series of short POWERSOURCE<sup>®</sup> assessments comprised of multiple item types, which are called *Checks for Understanding*, to help teachers assess their students understanding of basic mathematical principles and to connect their instruction and provide feedback to support deeper understanding. A set of instructional resources and targeted professional development activities were also developed for each of these domains. Thus, a POWERSOURCE<sup>®</sup> module around a given domain includes a set of *Checks for Understanding*, targeted instructional resources, and professional development opportunities. POWERSOURCE<sup>®</sup> materials are designed to complement existing curricula, but time for it must be found within tight district curriculum frameworks and timelines. It is thus important for POWERSOURCE<sup>®</sup> to integrate well and easily with existing initiatives, not add an unreasonable burden to the heavy testing requirements already imposed on teachers (e.g., weeks of state and district testing), and not replace large chunks of extant curricula.

More detailed information about the research foundations, content focus, initial development process, and program components of POWERSOURCE<sup>®</sup> can be found in the CRESST's 2006 and 2007 progress reports to the Institute of Education Sciences (Baker, 2007). The present report focuses on providing an update on project activities undertaken since the last progress reporting period (i.e., covering the 2007-08 school year). This update



is organized around four general areas. First, we provide updated results from the 2006-07 experimental (randomized) field test of POWERSOURCE<sup>®</sup> instructional sensitivity—including both item quality data for the *Checks for Understanding* and treatment/control differences on student and teacher outcomes. Second, we describe the experimental (randomized) study conducted during the 2007-08 school year, for which data collection is in its final stages. Third, we describe design and pilot testing of 7<sup>th</sup> grade materials conducted during the 2007-08 school year. Finally, we provide updates on supplemental/synergistic research studies, including technology-based activities and English Language Learner (ELL) assessment studies. Dissemination activities are also discussed, as well as an overview of planned activities for the 2008-09 school years.

### **Updated Results from 2006-07 POWERSOURCE<sup>®</sup> Field Test**

As described in previous progress reports (Baker, 2007) during the 2006-07 school year we conducted two types of inter-related studies building on the prior project years' work: a) continuing investigation of item quality of the *Checks for Understanding*, supplementing data we obtained on items in the previous year; and b) experimental tests (using random assignment) of the 6<sup>th</sup> grade POWERSOURCE<sup>®</sup> materials in four districts in two states. Detailed description of the methodology used and preliminary results were presented in these previous progress reports. Following is an updated summary of 2006-07 school year results from these two inter-related strands of work.

#### **Item Quality Analyses of 2006-07 Field Test Data**

Analyses of item and test data from 2006-07 field test continued and were completed over the course of the current year. Several kinds of item-level analyses were carried out: confirmatory factor analyses, reliability analyses, and Item Response Theory (IRT) analyses. Our typical scheme for analyzing each set of *Checks for Understanding* was to first calculate reliability coefficients (Cronbach's alpha) for the items comprising the set. Second, as another check of item quality, we conducted a principal component analysis and a confirmatory factor analysis for each test form to check whether the items exhibited the factor structure we expected; for example, whether the computation items loaded on the same factor. Third, IRT analyses based on Rasch models were conducted in order to obtain item parameters (difficulties) and item characteristic and information curves so that we could use them to select items for future testing. The model-data fit was investigated using two model fit indices. One is the  $G^2$  index which is the Chi-square ( $\chi^2$ ) statistic and provided in PARSCALE phase 2 outputs, and the other is the mean square fit (MNSQ) statistics.

As described in our previous progress report (Baker, 2007), we used 2005-06 and 2006-07 pilot test information to refine our assessment forms and develop *Checks for Understanding* that then were then administered to students as part of a 2006-07 randomized field test. This field test had two major objectives: a) to refine the design of our materials, and b) to provide some preliminary information about POWERSOURCE<sup>®</sup> effectiveness. In the following section, we provide some highlights of these findings.

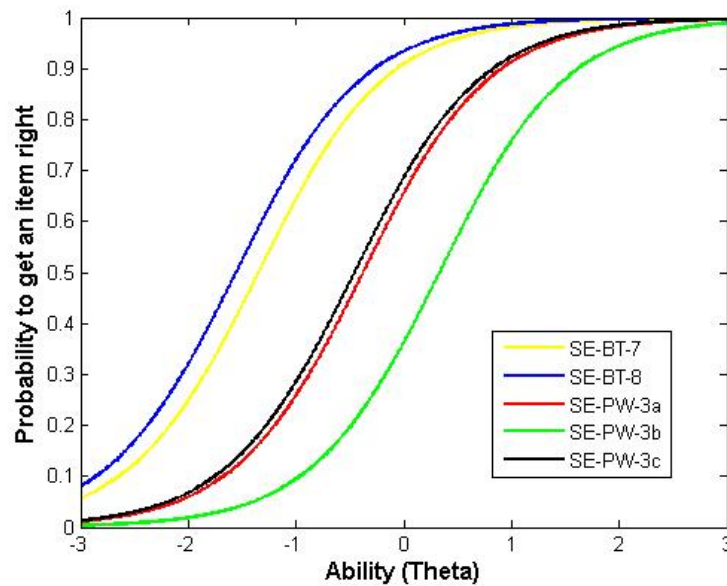
**Reliability and factor analysis.** Appendix A (Additional Item Analysis Results) contains detailed reliability coefficients and factor analysis results for *Checks for Understanding* forms in 2006-07 field testing. Each domain had two test forms, so there were total of 10 forms of RNE, PA, SE, RA, and FM. The range of reliability coefficients (Cronbach's alpha) were between 0.554 and 0.863. Based on the inter-correlations between the items, factor analysis further determines the theoretical constructs that might be represented by the set of items in a form. This analysis allowed us to look at the *Checks for Understanding* forms in each domain and determine whether the items exhibited the factor structure we expected. Values less than .05 for the RMSEA indicate a close fit, with values as high as .08 representing a reasonable fit (Joreskog & Sorbom, 1993, p. 124). The GFI provides a measure of the relative amount of variance and covariance accounted for by the model. Values greater than .90 for the GFI measure is required to indicate a good fit (Byrne, 1994). According to these criteria, the RMSEA and GFI values indicated that the items' variance in each test could be explained well by a single construct. In each form, the main component accounted for 27.529% through 50.195% of the total variance, which suggested the items measured a unidimensional trait (see Appendix A more detailed results).

**IRT analyses.** Appendix A contains item parameters of dichotomous and polytomous items estimated using the program PARSCALE as well as the item characteristic curve of every item in the five domains. The item parameter calibration for field testing was executed for each domain rather than for a form in each domain; whereby, the item parameter estimates of a domain could be considered being put onto a common scale. For example, item parameter estimates for FM in 06-07 field testing ranged from -2.82 (0.11) to 2.60 (0.09).

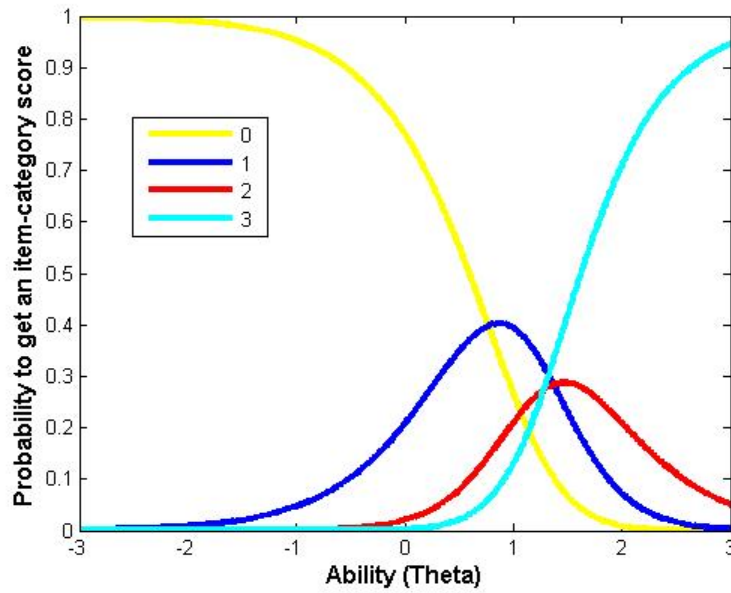
Using the SE items as an example, (see Figure 1 for item (category) characteristic curves and item information curves of items on the 2006-07 SE-11 field test form), SE-11 has five dichotomous and only one polytomous items. Figures 1a, 1b, and 1c respectively show item characteristic curves of the dichotomous items (SE-BT-7, SE-BT-8, SE-PW-3a, SE-PW-3b, and SE-PW-3c); item category characteristic curves of the polytomous item (SE-EX-9); and item information curves of the six items. We can interpret these item

characteristic and information curves under Rasch models to indicate that two of the assessment items, SE-BT-7 and SE-BT-8, are less difficult items and as such provide more information for low ability students than high ability students. Conversely, the item SE-PW-3b shows the opposite pattern. As depicted in Figure 1c, this item provides more information about the high ability students than for the less ability students. In Figure 1b, it could be interpreted that the students with ability less than about  $\Theta=0.8$  had high probability to get the item score of zero. As shown in Figure 1c, the polytomous item (SE-EX-9) was able to grant more information than any single dichotomous item as well as provide more information for students with high ability.

a) Item Characteristic Curves of the Five Dichotomous Items



b) Item Category Characteristic Curves of the Polytomous Item (SE-EX-9)



c) Item Information Functions of the Six Items

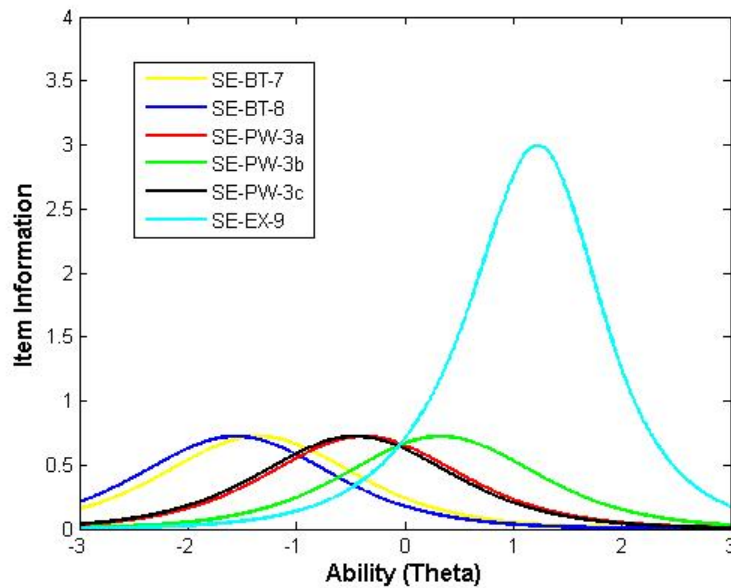


Figure 1. Item (category) characteristic curves and item information curves for 2006-07 SE field test items.

Detailed results of the model-data fit analyses for 2006-07 field testing are also provided in Appendix A. For example, only three items had  $p$ -values larger than 0.05 in the  $G^2$  statistic analysis among the total 14 items in RNE 2006-07 field test forms. The other items appeared to misfit the Rasch models. Because it is known that the  $G^2$  can control the

type I error rates only in very limited testing conditions (Orlando & Thissen, 2000), however, the fit of Rasch models seemed more appropriately interpreted using the concept of INFIT and OUTFIT. Using this approach, in the same form every item in RNE field test forms had MNSQ INFIT value less than 1.5 and only three items (RN-BT-7, EX-9, and EX-6b1) showed some misfit problems in terms of MNSQ OUTFIT. MNSQ INFIT values provide model item fit information around the difficulty parameter where the item is most informative, the appropriate fit is very important in this area. The items in RNE field testing appeared to have INFIT values less than 1.24. The similar pattern appeared in the other domains' 2006-07 field test forms.

Thus, even though a few items appeared not to be a strong fit by Rasch models, this finding does not invalidate the measure. It simply indicates that beyond the strong overall achievement measured by each domain's test forms, there are also some minor dimensions of achievement that impact the individual item scores of individual students. That the overall dimensions (or principal components) measured by each subject assessment are very strong is demonstrated by both (1) strong Cronbach's alpha internal consistency reliabilities (a measure of measurement precision of the overall dimension derived outside the IRT model) and (2) the positive results from the confirmatory factor analysis and principal component analysis.

### **POWERSOURCE<sup>®</sup> Field Test 2006-07: Experimental Comparison Findings**

As noted above, during the 2006-07 school year we field tested both the POWERSOURCE<sup>®</sup> assessments and associated instructional materials as part of a random assignment study. Analyses of experimental comparisons for all POWERSOURCE<sup>®</sup> units were completed during the current year. As noted in the previous annual report, sixty-six 6<sup>th</sup> grade teachers were recruited from middle-schools in Arizona (two districts-AZ-1 and AZ-2) and California (two districts—CA 1 and CA 2). Within each district, teachers were randomly assigned to experimental (POWERSOURCE<sup>®</sup>) and comparison groups. Experimental group teachers in all cases participated in initial summer professional development and after school follow-up sessions, and used project materials-such as *Checks for Understanding* and instructional supports (including teacher instructional handbooks);whereas, comparison group experiences varied slightly depending on district need and configuration. District CA-1 represented the simplest design for examining the effects of the POWERSOURCE<sup>®</sup> intervention in total. Here, the comparison group received no POWERSOURCE<sup>®</sup> professional development (though teachers did participate in usual district professional development for mathematics) and had no access to instructional supports, although teachers were asked to administer the *Checks for Understanding* for use as a dependent variable. In

the other three districts, the comparison group participated in some professional development with their POWERSOURCE<sup>®</sup> group colleagues and administered the *Checks for Understanding*, but had no access to the instructional supports, in effect providing an experimental test of the value added by the instructional support. All teachers gave eight *Checks for Understanding* throughout the school year—two for each of the four POWERSOURCE<sup>®</sup> modules (RNE, PA, SE, and RA). It is important to note that based on district curricula and needs a slightly modified module, FM, was used in the AZ districts in place of the RA module. Additional details about the 2006-07 field test design, materials, and sample can be found in Baker 2007. Preliminary results from this field test were presented in these progress reports based on the most complete data available at the time. We now present updated results from this field test, including student outcomes and an update on findings drawn from professional development activities.

**2006-07 field test: Student outcomes.** Table 1 presents the student characteristics for participating schools. Hispanic students are the most dominant ethnic group across all four districts. For instance, the percentage of the Hispanic students in CA District 1 was 75 percent at the time of the study. Furthermore, a high percentage of participants in the study were students that were economically disadvantaged, limited English proficient, and poorly performing in mathematics.

Table 1  
Student characteristics for participating schools

Student characteristics	CA district 1	CA district 2	AZ districts 3&4
Asian	3%	7%	0%
Black	15%	3%	6%
Hispanic	75%	38%	64%
Other White	6%	48%	26%
EL	38%	16%	23%
Below proficient in math, 2006	46%	57%	41%

Table 2 contains descriptive statistics for the student outcomes of interest in this random assignment study. Specifically, it contains the number of teachers and students; average score of the outcome (i.e., *Checks for Understanding*) in each domain; and standard deviation, for the POWERSOURCE<sup>®</sup> group and control group by each district. CA District 2 had the largest number of students and teachers among the four school districts. The teacher

participants in the POWERSOURCE<sup>®</sup> group in this district were ten for the POWERSOURCE<sup>®</sup> group and seven or eight (depending on the unit) teachers for the control group. The mean scores were calculated based on the total (sum) scores of two *Checks for Understanding* (1 and 2), which have total of 13 to 19 items depending on the domain. The *Checks for Understanding* included not only short-answer items but also extended-answer items.

Table 2  
Descriptive statistics of outcome by domain

Domain by district	Control				POWERSOURCE <sup>®</sup>			
	<i>n</i> teachers	<i>n</i> students	Mean	<i>SD</i>	<i>n</i> teachers	<i>n</i> students	Mean	<i>SD</i>
PA <sup>a</sup>								
CA dst1	8	162	7.75	4.08	7	131	12.44	4.80
CA dst2	7	139	7.96	4.40	10	210	12.75	4.29
AZ dst 3&4	11	204	5.30	3.73	13	230	11.42	4.43
Average	9	168	7.00	4.07	10	190	12.20	4.51
RNE <sup>b</sup>								
CA dst1	9	161	8.08	3.74	6	109	11.37	4.33
CA dst2	8	142	8.27	4.21	10	212	12.13	4.25
AZ dst 3&4	11	193	8.55	3.71	13	248	10.39	3.84
Average	9	251	7.98	3.93	10	190	11.52	4.23
SE <sup>c</sup>								
CA dst1	8	144	7.13	3.09	6	95	10.21	4.00
CA dst2	7	132	7.15	2.77	10	208	9.93	3.71
AZ dst3&4	11	207	6.33	2.61	13	230	7.98	3.10
Average	9	161	6.87	2.82	10	178	9.37	3.60
RA <sup>d</sup>								
CA dst1	3	62	7.71	4.57	3	39	21.23	3.17
CA dst2	6	120	11.20	5.93	8	169	15.69	5.99
Average	5	91	9.46	5.25	6	104	18.46	4.58
FM								
AZ dst 3&4	10	202	6.16	2.75	12	248	7.39	2.60

<sup>a</sup>17 items; 12 short; 5 extended. <sup>b</sup>14 items; 5 short; 9 extended. 13 items; 9 short; 4 extended. <sup>d</sup>19 items; 14 short; 5 extended.

As can be seen in this table, the most noticeable finding is that all of the mean scores for the POWERSOURCE<sup>®</sup> group in the five *Checks for Understanding* and four districts are considerably higher than those for the control group. The magnitude of the difference between the two groups ranges from a .45 pooled-standard deviation difference to 3.5 pooled-standard deviation. For example, the mean score for PA in the control group across the four districts is 7.00, compared to 12.2 for the POWERSOURCE<sup>®</sup> group. The difference between these scores is approximately as large as 1.2 of the pooled standard deviation. The largest difference was found in CA district 1 in RA content area (approximately 13.5).

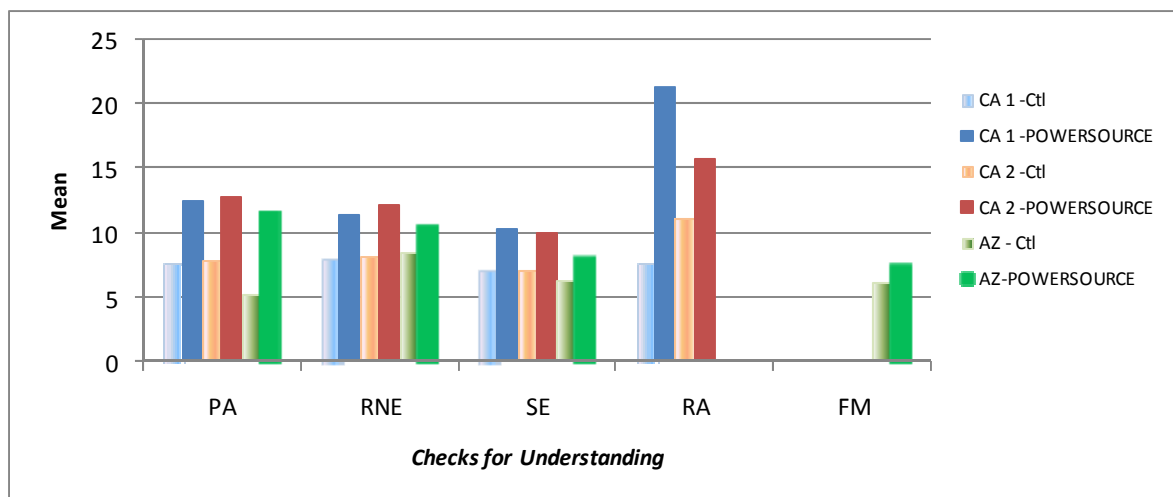


Figure 2. Mean scores of *Checks for Understanding* for the POWERSOURCE<sup>®</sup> group and the control group in each district.

Because students are nested within a teacher, two-level hierarchical modeling techniques were used to compare students' performance between the POWERSOURCE<sup>®</sup> group and the control group (Raudenbush & Bryk, 2002). Ignoring the nested structure of the data would have given rise to two major problems. First, ignoring the dependencies or similarities among observations (scores) for each class or teacher creates misleadingly small standard errors in estimating teacher effects—which creates a statistical problem for significance testing. Secondly, it leads to inability to detect between-teacher heterogeneity in the program effects. In other words, there might be variability among teachers within the POWERSOURCE<sup>®</sup> group depending upon fidelity of implementation, teacher characteristics, background characteristics of students, and many other factors that are related to the teacher effects.

The following Table 3 presents the Hierarchical Linear Modeling (HLM) results using the POWERSOURCE<sup>®</sup> measures (*Checks for Understanding*) as the outcome. The HLM estimate of difference corresponds to the estimate of  $\gamma_{01}$ . However, because data for different



experiments is collected in different scales, standardizing the data is important so that the results are meaningful to any researcher, not just someone who is familiar with a particular data set. A standardized effect size,  $\delta$ , is the population means difference of the two groups divided by the standard error of the outcome (Spybrook, Raudenbush, Liu, & Congdon, 2006). Standardized effect sizes between 0.50 and 0.80 are considered large and effect sizes as small as 0.20 to 0.30 are often considered worth detecting.

Table 3

Estimated difference and effect size between the POWERSOURCE<sup>®</sup> group and the control group

District	<i>n</i>	HLM estimate of diff. ( <i>p</i> -value)	Effect size ( $\delta$ )
PA			
CA dst1	293	4.86 (0.004)	1.08
CA dst2	349	5.06 (0.001)	1.12
AZ dst 3&4	434	5.92 (0.000)	1.43
RNE			
CA dst1	270	3.26 (0.016)	0.80
CA dst2	354	3.84 (0.009)	0.87
AZ dst 3&4	441	1.90 (0.015)	0.50
SE			
CA dst1	239	3.33 (0.026)	0.93
CA dst2	340	2.86 (0.029)	0.81
AZ dst 3&4	437	1.69 (0.009)	0.58
RA			
CA dst1	101	13.61 (0.000)	3.30
CA dst2	289	4.81 (0.055)	0.77
FM			
AZ dst 3&4	450	1.21 (0.067)	0.45

Significant differences were found between POWERSOURCE<sup>®</sup> and control students' performance for all districts and domains. The effect sizes ranged from .45 to 3.30 across the four districts and the five content areas, with a median effect size of 0.84 (mean effect size is 1.05). The median effect sizes by content area are 1.12, 0.80, 0.81, and 2.04, respectively for PA, RNE, solving equations, and review and applications. The median effect sizes by district

ranged from 0.54 to 1.01. These findings pertain to POWERSOURCE<sup>®</sup> tasks, which are broadly representative of fundamental pre-algebra content.

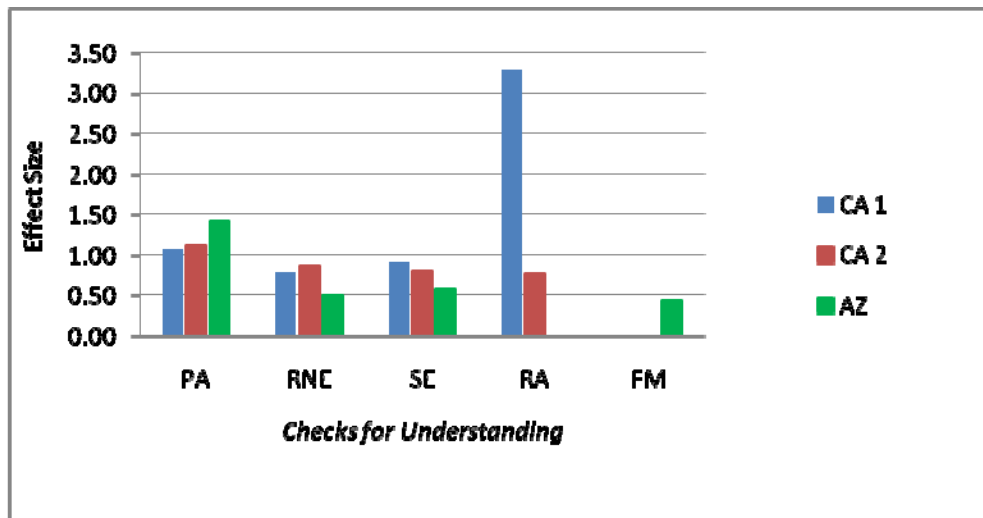


Figure 3. Estimated effect sizes for each *Checks for Understanding* and each district.

In terms of next steps, we are in the process of collecting and analyzing both prior and concurrent state standards test data (i.e., CST in CA, AIMS in AZ) in mathematics for both the POWERSOURCE<sup>®</sup> and control groups. The prior year's data would serve to document the initial comparability of the two groups and could be used as a covariate for analysis; the concurrent year data would serve as a partial transfer measure. However, we are concerned about the sensitivity of the state test measures to the intervention, in that the state tests cover a much broader domain than the four content areas addressed by POWERSOURCE<sup>®</sup>. We also are attempting to obtain item level data so that we can examine performance on the items related to POWERSOURCE<sup>®</sup> content, but even given the availability of this data, the reliability would need to be determined.

**2006-07 Field Test: Professional development and teacher outcomes.** As described in earlier progress reporting (Baker, 2007), the design of the professional development component of POWERSOURCE<sup>®</sup> is based on findings in the field of cognitive science about how students learn, on expert-novice literature that suggests how expertise in a subject like mathematics develops, on the role of formative assessment in facilitating this process, and how these components can be effectively combined to improve teacher practice.

As also detailed in our previous progress report, as part of the 2006-07 field test we examined changes in teacher knowledge across professional development participation based on an activity that asked teachers to rank a series of student assessment responses (Baker, 2007). This task requires teachers to rank five students' responses in order of difficulty. It

consists of two sets of items, the first intending to measure how well students can describe the algebra principle, while the second intended to measure students' knowledge of algebra principles. This activity was integrated into the POWERSOURCE<sup>®</sup> professional development meetings at both the beginning and end of the 2006-07 school year. We analyzed results from this measure both to provide some basic statistical quality information about the measures and to assess pre- and post- professional development differences in teacher knowledge.

Based on teacher responses to a background questionnaire, 43% of the participating teachers reported attaining a masters degree or higher, with the remaining 57% holding a Bachelor's degree. Prior years' teaching experience as reported by teachers, both in general and specific to mathematics, was quite varied with a mean of 9.2 and 7.7 for general and mathematics teaching experience, respectively. Table 4 summarizes the results of teacher responses related to self-evaluation of their algebra background knowledge. Overall approximately 50% to 70% of teachers felt they had high to very high relative knowledge; respondents were least confident about the distributive property.

Table 4  
Relative Knowledge in Specific Mathematics Knowledge Domains

Relative level of knowledge	Percent of respondents			
	Low	Average	High	Very high
RNE	2.8	26.4	47.2	23.6
Distributive property	2.8	31.9	38.9	26.4
Principles for solving equations	2.8	27.8	45.8	23.6

In general, there was relatively little correlation between teacher experience and their perceptions of relative knowledge ( $r < 0.25$ ). Teachers were fairly consistent, however, with inter-correlations of the knowledge self-evaluation domains in Table 4 ( $r = .80$ ).

A total of 37 teachers who participated in professional development activities offered during the 2006-07 school year completed the post-assessment<sup>1</sup>. The completion rate was lowered by one participating district's request that their teachers did not complete this follow-up knowledge measure.

<sup>1</sup> As noted earlier in this report, in some of the participating districts both POWERSOURCE<sup>®</sup> (i.e., using the POWERSOURCE<sup>®</sup> instructional materials) and control teachers (did not use POWERSOURCE<sup>®</sup> instructional materials) attended development activities.

In relation to task reliability, the two ranking tasks formed fairly consistent scales in terms of the response patterns. That is, responses formed a scale wherein the easiest item that was missed consistently related to subsequent more difficult items being answered correctly. For example, in set 1, if the easiest item was missed, it was extremely unlikely that any other item would have been ranked correctly. In other words, of those teachers who correctly answered the second most difficult item, answered the least difficult item correctly 98.3% of the time. The reliability of each of the tasks was evaluated using Cronbach's alpha. The reliability for set 1 is approximately,  $\alpha$  0.84. The reliability of set 2 was considerably lower ( $\alpha$  = .40), and this low reliability would translate into low reliability for assessing change. Hence, set 1 is examined independently for change as well as a combined, equally weighted composite of set 1 and set 2. The set1/2 composite reliability ( $\alpha$  = .73) is acceptable for examining pre-post change.

Before we evaluate changes in teacher knowledge, we briefly examine whether those teachers who have both pre- and post- scores are representative of all teachers with pre-scores. Two substantive differences arise: 1) teachers with higher set1/2 pre- composite scores tended not to complete the post-assessment, and 2) more teachers without advanced degrees (BA only) completed the post-assessment. Specifically, the average pre-test composite score among pre-test only teachers for set1/2 is 7.1, while it is 5.8 for teachers completing both the pre- and the post- test. Hence it is not tenable to assume that the teachers taking both assessments are representative of those taking the pre-test only; thus, results pertaining to change need to be interpreted with caution.

Given these caveats, we then evaluated changes in teacher knowledge for that subset of teachers that have both pre- and post- test scores. We examined change in teacher knowledge by utilizing a two level random effects model that models teacher ranking scores at level 1 and teachers at level 2. Similar to Bryk, Thum, Easton, and Luppescu (1998) we re-parameterized the level one model in order to estimate true initial status and gain.

Table 5 summarizes the results for set 1 rankings. The results indicate that among teachers who participated in both the pre- and post- assessments there was no systematic change in scores.

Table 5

Random Effects Model for Teacher Knowledge Set 1

Status	Estimate	<i>SE</i>	Approx <i>p</i> -value
Initial status	4.00	0.21	<.01
Gain	0.05	0.27	0.862

We next examine changes in the set 1/2 composite. The results are displayed in Table 6.

Table 6

Random Effects Model for Teacher Knowledge Set 1/2

Effects	Estimate	Variable	Approx <i>p</i> -value
Fixed effects		<i>SE</i>	
Initial status	4.1	0.22	<.01
Gain	1.2	0.22	<.01
Random effects		s.d.	
Initial status		1.10	<.01
Gain		0.91	<.01

The results in Table 6 indicate that teachers gained about 1.2 scale points (an effect size of about 0.8) pre- to post-. Further, the results indicate that both initial status and gain vary among teachers. Hence, we expanded the unconditional model to include teacher information that might account for pre-existing differences in initial status as well as factors related to growth (Osgood & Smith, 1995).

Table 7

Random Effects Model for Teacher Knowledge Set 1/2

Effects	Variables		
	Estimate	<i>SE</i>	Approx <i>p</i>
Fixed effects			
Initial status	4.08	0.20	0.00
Yrs. teach math	-0.05	0.04	0.23
Knowledge: RNE	-0.41	0.43	0.35
Knowledge: : Distr.	0.30	0.35	0.39
Knowledge: : Solve eq.	0.10	0.34	0.78
MA	1.09	0.50	0.04
Gain	1.22	0.17	0.00
Yrs. Teach math	0.07	0.03	0.06
Knowledge: RNE	0.76	0.37	0.05
Knowledge: : Distr.	-0.19	0.45	0.68
Knowledge: : Solve eq.	0.18	0.49	0.72
MA	-0.54	0.48	0.27
Random effects	<i>SD</i>	Approx <i>p</i>	Var. reduction
Initial status	0.97	0.02	19.8%
Gain	0.46	0	44.5%

The full model results presented in Table 7 indicate that only MA is significantly related to teachers' pre-test knowledge. Consistent with expectations, teachers with MA degrees score about 1 point higher than teachers with BA degrees. None of the self-perceived knowledge levels are related to teacher pre-assessment results.

There is some suggestive evidence ( $p < .10$ ) that teachers with more experience teaching mathematics gained slightly more in teacher outcomes pre- to post-. That is comparing a teacher who is one standard deviation above average in experience with one that is one standard deviation below average in mathematics teaching experience, the more experienced teacher is expected to gain approximately 1.3 points more than the least experienced teacher.

The results also indicate that self-perceived RNE knowledge is related to larger teacher knowledge gains. Comparing a teacher who reports low RNE knowledge to one that reports

very high knowledge, the teacher reporting a very high RNE knowledge level would be expected to gain about 2.3 points more than the teaching reporting a low level.

Overall, the full model is able to account for about 20% of the variation in pre-test results and about 45% of the variation in true gains.

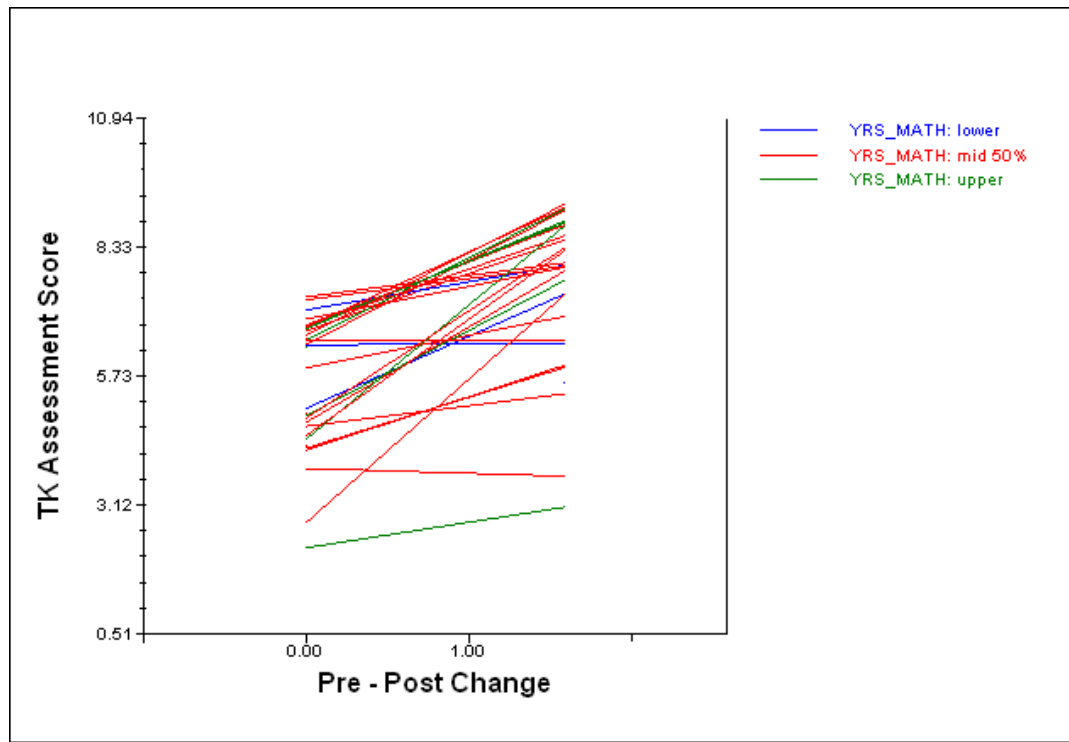


Figure 4. Effect of mathematics teaching experience.

Figure 4 highlights several points; for instance, it depicts the variability in pre-test scores, the variability of gains, and the effect of teacher experience teaching mathematics. It should be noted that the results presented in Figure 4 are re-scaled composite scores for set 1/2.

In summary, we found the teacher knowledge ranking task to be a short, relatively non-invasive tool for assessing teacher knowledge; further, it produces relatively reliable results as well as a scale that allows for cumulative interpretations. That is, the pattern of correct responses is very consistent as items move from easiest to most difficult. For this reason, scores are more than simply an index of a teacher being able to answer any three of five questions correctly; rather, a score of three indicates which three questions a teacher likely completed successfully. This provides valuable, as well as efficient, results from which mentors or trainers can identify the areas in which teachers are having difficulty.

Among teachers who took both the pre- and the post- Ranking assessment, the composite set 1/2 scores demonstrate statistically significant and substantively important gains. Moreover, gains are related to teachers' mathematics teaching experience as well as the self reported RNE knowledge. However, teachers demonstrate a fair amount of variability in pre-test scores that is not accounted for with the teacher information available in the data; this area certainly warrants further study.

It is important to note that, due to practical constraints, the gain results are based on a subset of teachers who took the pretest and that this subset is not representative of the entire sample. In fact, it appears as if those teachers with the lowest composite scores were exactly those that took the post-assessment, which highlights potential regression to the mean concerns. However, these results provide some initial support for the effectiveness of the professional development activities developed as part of POWERSOURCE<sup>®</sup>, as well as emphasize the need for additional systematic data regarding teacher outcomes of POWERSOURCE<sup>®</sup> participation. As described later in this report, the 2007-08 project activities included additional teacher outcome measures for both POWERSOURCE<sup>®</sup> and control teachers.

### **POWERSOURCE<sup>®</sup> Implementation Study 2007-08**

The core undertaking of our work during the 2007-08 school year was conducting an extended, random assignment implementation study of our 6<sup>th</sup> grade POWERSOURCE<sup>®</sup> program. As with the 2006-07 field test, teachers were randomly assigned to either POWERSOURCE<sup>®</sup> or control conditions with the ultimate goal of determining program impact on both students and teacher learning outcomes. The 2007-08 study differed from the previous year's work in a number of important ways, however. Specifically:

- The experimental design incorporated both within- and between-school random assignment models. That is, for some of the districts the random assignment accomplished within each school (i.e., a given school had both POWERSOURCE<sup>®</sup> and control teachers), and for some the random assignment was between school (i.e., all teachers at a given school were POWERSOURCE<sup>®</sup> or control). Additional background and rationale for this approach can be found in the CRESST supplement design report submitted to IES in August, 2007. Ultimately, a total of 112 6<sup>th</sup> grade teachers across seven school district participated in the study.
- Although the content focus of the four POWERSOURCE<sup>®</sup> modules remained the same (RNE, PA, SE, and RA), based on teacher feedback and our implementation experiences in 2006-07 the structure of each unit changed somewhat. In 2007-08, POWERSOURCE<sup>®</sup> teachers were provided with three *Checks for Understanding* for each unit – one prior to the first day's set of instructional materials, one in



between the first and second day of instruction, and one after the second day of instruction. Thus, the students completed 12 *Checks for Understanding* (three for each of the four units) during the school year. The *Checks for Understanding* and instructional materials were also revised/refined based on analysis of 2006-07 item level data.

- Unlike 2006-07, the control students did not complete any of the *Checks for Understanding* (i.e., the short formative assessments). Thus, the 2007-08 control students and teachers had no exposure to any of the POWERSOURCE<sup>®</sup> materials or concepts during the school year.
- All students (POWERSOURCE<sup>®</sup> and control) completed a test of prerequisite knowledge at the beginning of the school year and a transfer measures of math knowledge at the end of the school year. The test of prerequisite knowledge serves as a baseline measure for later analyses, while the transfer measure will serve as an independent, student outcome measure (in addition to state test data).
- Based on district response and feedback, districts were offered the option of the control teachers receiving an alternative (i.e., non-POWERSOURCE<sup>®</sup>) professional development from CRESST (as opposed to the control teachers not receiving any additional professional development than what the district already had planned). The majority of participating districts selected this option.

Below, we summarize changes made for the treatment and comparison conditions for the 2007-08 implementation study (including the alternative professional development offered to the control teachers), followed by brief descriptions of the design, measures and analysis plan for the study. Additional details about the plan and its rationale can be found in the supplemental design report submitted to IES in August, 2007. The data collection for these activities is in its final stages.

### **Revisions to the Treatment and Control Conditions for the 2007-08 Study**

**Revision of 6<sup>th</sup> grade *Checks for Understanding* and instructional materials.** The *Checks for Understanding* were revised based on data from the 2006-07 study, whose analyses were described in the earlier sections of this report as well as in Appendix A. Moreover, three *Checks for Understanding* were developed for each of the four 6<sup>th</sup> grade POWERSOURCE<sup>®</sup> domains,—instead of the two used the previous year, there were ultimately three *Checks for Understanding* per unit. The initial *Check for Understanding* consisted of between 8-10 items and was given prior to instruction in the relevant content. This *Check for Understanding* acted as a baseline assessment for each student and a) gave teachers information about their students initial knowledge and b) allowed us to compare students' performance before and after instruction, thus, providing information on the instructional sensitivity of the *Checks for Understanding*. Each subsequent *Check for Understanding* (of which there are two) consisted of 4-5 items (2 symbolic

representation/computation items and 2-3 open-ended problem solving and/or explanation tasks). Based on our research and development over the last two years, teachers' procedure for using the *Checks for Understanding* was as follows:

1. Administer an initial *Check for Understanding* big idea and its applications (15-20 minutes) and analyze results.
2. Present instructional activities (if necessary) addressing deficiencies in conceptual understanding identified in step 1 (one class period).
3. Administer a second *Check for Understanding* focusing on conceptual understanding (15 minutes); follow up instruction if necessary.
4. Present instruction on applications of the big idea to problem solving and symbolic representation and computation tasks (if necessary) (15 minutes).
5. Administer a third *Check for Understanding* focusing on conceptual understanding (15 minutes); follow up instruction if necessary.

Please see Appendix B for an example of a teacher handbook containing the revised 6<sup>th</sup> grade *Checks for Understanding* and instructional materials.

**Revision/addition of professional development resources for 2007-08.** While the overall program and structure of POWERSOURCE<sup>®</sup> professional development during the 2007 -08 school year was very similar to the program of professional development delivered in the 2006-07 school year, there were notable additions to enhance the strength of the study and to address the concerns and needs of the district partners in terms of the experiences of the control teachers.

As noted above, our participating districts insisted that if they were to participate in a random assignment study then their control teachers must receive some sort of professional development through their participation. The addition of such professional development adds strength to the study design, as well as limits some confounding of experimental and control group treatment in the prior year's study. Thus, in addition to the POWERSOURCE<sup>®</sup> professional development, we created two separate alternative (control) programs of professional development that control teachers participated in during the school year (two different programs were developed to meet the differing needs/agendas of the participating districts). The structure and length of these alternative professional development sessions – an initial one day institute plus follow-up after school meetings—was identical to that of the POWERSOURCE<sup>®</sup> professional development.

We created the first of the two alternative professional development programs for districts that use the Data Director<sup>™</sup> assessment data management system. Control teachers in these districts received an identical amount of professional development as the

POWERSOURCE<sup>®</sup> teachers, but this professional development concerned information and reports provided by Data Director<sup>™</sup>. Specifically, this professional development focused on providing a method for users to analyze the technical quality of district benchmark assessment forms and items. While the focus of these sessions was on assessment, the researchers provided no actual guidance on how to use the information from the assessments or guidance on how to create instructional plans.

The second complete alternative program of professional development was developed for use in districts where Data Director<sup>™</sup> was not available. It is an educational psychology-based, four unit program centered on motivational factors in the classroom and their role in student learning. It introduces teachers to research-based motivational theories and highlights their connections to everyday classroom practices. The units each emphasize a specific area of research and offer some practical ways to incorporate the research findings into daily practice. Issues covered included personal responsibility, self-efficacy, attributions and affect, and teacher expectations.

Although these alternative programs of professional development were quite different from one another and from the POWERSOURCE<sup>®</sup> professional development, they were designed to be credible, stand-alone offerings based on current educational theory and research. Sample activities from both of these alternative sessions are presented in Appendix C.

As with the 2006-07 pilot test, the POWERSOURCE<sup>®</sup> professional development for 2007-08 aimed to provide an intellectually stimulating and supportive environment, that builds teacher knowledge and pedagogical content capacity, provides time for reflection, and monitors the effectiveness and the impact of our activities.

Teachers were asked to evaluate each professional development meeting they attended using a satisfaction survey. This survey asked teachers to rate the organization of the professional development, presenter, content, benefit to the teacher, and their overall impression on a 4- point scale. The teachers were also asked short-answer response questions concerning the helpfulness of the content presented, and the percent of new content contained within the presentation. Finally, the survey solicited any additional comments teachers may have about the professional development meeting. These surveys were filled out by each teacher at the end of each professional development meeting and were collected before teachers departed.

The data from the 2006-07 POWERSOURCE<sup>®</sup> teacher satisfaction surveys were highly positive and we expect this year's results to reveal similar findings. Specifically, 77% of the

teachers found the meeting presentation to be “excellent” while another 20% rated the session as “good”. When rating the session presenter, 76% found the presenter to be “excellent” and 21% found the presenter “good.” On the content presented, 63% found the content “excellent” and 32% found it “good.” Sixty percent of the teachers rated the benefit of the professional development to themselves as an educator as “excellent,” while 31% found it to be “good.” Overall, 67% of teachers rated their overall experience at POWERSOURCE<sup>®</sup> meetings to be “excellent” and 30% rated it as “good.” Teacher open-ended remarks exemplified their happiness with the professional development meetings. As one teacher stated, “These sessions are the best professional development I have experienced during my time as a teacher. The presenters help me think outside the box and get creative about how to bring understanding to students.” Another teacher commented that, “I have never taught this [content] so completely. This will help me as much as the students.”

In order to supplement the face-to-face professional development, researchers also created a password protected professional development website with additional resources. Each of the four major sections of the website addresses one or more of the big ideas that are the focus of POWERSOURCE<sup>®</sup>. Every section, in turn, discusses why a particular big idea or its integration is important; how that big idea appears across the 6<sup>th</sup> grade curriculum and will be used in future grades; misconceptions that students often hold about that idea; and frequently asked questions that teachers have about teaching the idea. It is intended that this website will serve two primary purposes. First, it will provide teachers in the treatment group with an additional site to acquire the information that we present at professional development sessions. In other words, teachers can visit the site and repeatedly review the material they heard at professional development. Secondly, we intend to use the website to help us build capacity in the participating districts themselves. Eventually, we will allow all teachers (treatment and control) in participating districts to gain access to our POWERSOURCE<sup>®</sup> materials. The website will allow these teachers and districts to access and download the POWERSOURCE<sup>®</sup> materials. In this way, these materials can form a core for ongoing professional development in the future. Sample materials from the professional development website can be found in Appendix D. Our project professional development experiences over the last three years have provided some important lessons that we learned and case studies of how formative assessment contributes to teacher development and student learning.

### **Overview of 2007-08 Implementation Study Design/Sample**

As described in previous progress reports (Baker, 2007) the project had to make some sample adaptations due to the decision of Los Angeles Unified School District not to continue in the implementation (i.e., random assignment) study beyond the 2006-07 school

year. CRESST spent considerable time and effort to secure additional school district participation in the project for the 2007-08 school year—meeting with representatives of close to 20 districts—and continues to do so in an ongoing effort to strengthen and expand the sample. Ultimately, seven small-to-medium sized school districts participated in the random assignment 2007-08 implementation study and all but one are in California. As compared to the larger districts that we spoke with, districts of this size seemed to have fewer algebra initiatives already in progress; they also appeared to be more open to participating in an experimental study that would result in not having all teachers in the district using the same program and materials.

Although the inclusion of a larger number of districts than originally planned was driven by practical considerations, methodologically it is also a definite strength. It will provide us with an opportunity to see how well the program works in a variety of settings—both in terms of analysis of quantitative data and more qualitative/descriptive indicators. Furthermore, it will be possible to investigate factors that might magnify or dampen the effects of the program. This facet of the study adds the qualities of a rich, mixed methods multi-site case study. With one large district, we might get a very precise estimate of the treatment, but it would pertain to only one set of district conditions.

As noted earlier, overall there were 112 (63 POWERSOURCE<sup>®</sup> and 49 control) teachers across seven school districts (totaling 28 schools) participating in the 2007-08 study. As also described previously, we used two designs—within- school and between- school, which was based on district needs and configuration. In districts with strong grade level team collaborative initiatives, for instance, a within-school design would not be tenable (as both treatment and control teachers would be regularly collaborating on curricular issues, thereby potentially exposing the control teachers to POWERSOURCE<sup>®</sup> concepts and materials). Ultimately, three of the districts used a within-school design, where random assignment was accomplished within each school (i.e., a given school had both treatment and control teachers). Four districts used a between-school design, where schools within a district were randomly assigned to treatment or control conditions. The total number of students in the 2007-08 POWERSOURCE<sup>®</sup> and control samples are approximately 3,600 and 2,900, respectively (although post-measure data collection is still underway).

### **Special Measures for the 2007-08 Study**

In addition to revisions to the POWERSOURCE<sup>®</sup> materials for the 2007-08 implementation, there were also some additions and refinements to the project outcome measures. These are described below.

**Student transfer measure.** While we were pleased with the very strong effect sizes from the 2006-2007 study based on POWERSOURCE<sup>®</sup> measures, we recognized the need for demonstrating intervention effects on an independent, transfer measure. We hypothesized that students in the POWERSOURCE<sup>®</sup> group would possess a better understanding of the basic mathematical principles contained within each domain. We also hypothesized that students would be able to apply concepts they have learned, solve complex problems and transfer the principles covered by the POWERSOURCE<sup>®</sup> domains outside of the specific program materials. For example, having received instruction and formative assessment on RNE, students should understand the multiplicative identity principle and be able to use it to: a) demonstrate that a set of rational numbers are equivalent, b) find equivalent fractions, c) find missing numbers in proportions and d) solve proportional reasoning problems. In order to answer these questions we developed a transfer measure to compare the POWERSOURCE<sup>®</sup> and control groups on novel items related to our four POWERSOURCE<sup>®</sup> domains. This instrument will provide an additional student outcome measure to compliment results from student state standardized tests (which will be provided to CRESST by the participating school districts).

The transfer measure was developed using items from several sources including Trends in International Mathematics and Science Study (TIMSS); the National Assessment of Educational Progress (NAEP); the Qualifications and Curriculum Authority (QCA) Key Stage 3 exam; the Programme for International Student Assessment (PISA); and benchmark tests used in one of our pilot districts (see Appendix E for additional item source information). An initial set of 44 items were selected from the various sources. Items were selected based on their relevance to the POWERSOURCE<sup>®</sup> domains and their appropriateness for a transfer task (i.e., related to POWERSOURCE<sup>®</sup> content but not exact replicas of item types used in the *Checks for Understanding*). A final set of items (29) were selected from the initial 44 items. Of these items 19 were multiple choice, 9 short answer and one explanation task. Items were selected based on their representation in the California state standards and relevance to POWERSOURCE<sup>®</sup> items (see Appendix F for alignment of items to California standards and the National Council of Teachers of Mathematics (NCTM) Focal points). Some of the initially developed items were deemed more appropriate for 7<sup>th</sup> grade and will be used for the 7<sup>th</sup> grade transfer measure.

The transfer measure was given to all participating students (POWERSOURCE<sup>®</sup> and control) at the culmination of the 2007-08 school year. Data are still being collected and will be analyzed beginning in the summer 2008.

**Teacher outcome measures.** As described in the August 2007 design supplement, several teacher outcome measures were included in the 2007-08 implementation study. These included:

- **Knowledge map task:** The knowledge map task asks teachers to complete a graphical network of key concepts in algebra, including their links and interconnections. These maps are scored in comparison to expert maps of the same concepts. All teachers (POWERSOURCE<sup>®</sup> and control) completed this task twice, at the beginning and end of the school year.
- **Teacher pre/post survey:** All teachers (POWERSOURCE<sup>®</sup> and control) completed a survey measure at the beginning and end of the school year. This measure asked teachers about their expertise in a variety of mathematics areas, their instructional attitudes/beliefs, as well as asked them to evaluate samples of student work. The pre measure also included background items about teachers' prior math experience and training. These measures are included in Appendix G.
- **Teacher implementation surveys:** All teachers completed a series of short surveys during the school year about their classroom math practices and assessment use. These surveys were timed to coincide with the POWERSOURCE<sup>®</sup> modules (e.g., RNE, PA, SE). The POWERSOURCE<sup>®</sup> teachers' implementation surveys included additional questions about their POWERSOURCE<sup>®</sup> materials use and feedback. The surveys are included in Appendix G.

**Observation/interview study.** As part of the 2007-08 POWERSOURCE<sup>®</sup> implementation study we conducted a pilot study of a classroom observation and teacher interview measure. The purpose of this study was twofold: 1) to investigate the relationship between teacher self-report (survey) measures and actual practice and, 2) to provide more detailed information and feedback about the POWERSOURCE<sup>®</sup> materials and how they are being used in the classroom. A total of 12 teachers participated in this qualitative study and they were selected to represent a range of districts and prior teaching experiences. All participating teachers were observed implementing the RA instructional materials in their classroom; moreover, they participated in an in-depth, semi-structured interview. The RA unit was selected for the observations as it represents an integration of all of the POWERSOURCE<sup>®</sup> units across the school year. The observation protocol attempts to obtain information about how teachers actually use formative information, how concepts are presented (e.g. algorithmically, conceptually or both), and how the big ideas are integrated with instruction on other concepts in the 6<sup>th</sup> grade math curriculum. The interview protocol focused on information about the teachers' thinking on the use and integration of formative assessment information and their rationale about how to incorporate the big ideas into their instructional planning and teaching. In addition, the interview was designed to solicit teacher feedback about the POWERSOURCE<sup>®</sup> materials and professional development. The

interviews and observations were conducted by pairs of researchers. Prior to data collection, the researchers participated in one day training about use of the interview measures, and reached concordance on observation ratings. The interview and observation protocol are in Appendix H.

### **Analysis Plan for 2007-08 Implementation Study**

We have adapted our original analysis plan given two different research designs and samples described in the previous section. Specifically, there are two changes that affect the original analysis plan. First, we included schools from multiple districts in the sample (seven different school districts). Due to this change, we need to consider variability in the effects of POWERSOURCE<sup>®</sup> across districts. Second, there are two different research designs mixed depending upon district idiosyncrasies. In other words, there are one groups of schools/districts in which only between school (B-S) design should be employed, while there also the other groups of school/districts where only within-school (W-S) design (addition details about design and analysis considerations can be found in the design supplement submitted to IES in August 2007). In the following section, we will describe our current plans/considerations for analyses of the 2007-08 implementation study data—with the understanding that some aspects of the analyses will be refined depending on the characteristics of the data once it has all been collected.

**Within-schools (W-S) and between-school (B-S) design considerations.** An advantage of the W-S design is that each participating school provides us with a mini-experiment, that is, each school provides us with an estimate of the effects of POWERSOURCE<sup>®</sup>. Drawing on the implementation data and other background information that is available, this will provide some possibilities to generate insight into why the program seemed to work very well in some schools but not as well in others.

Another advantage of W-S designs is that they can be fairly powerful. In the case of B-S designs, a key source of variability that shows up in the standard error of the estimate of the treatment effect is the variability between treatment schools in their post-test scores and the variability between comparison schools in their post-test scores. In a district where, for example, the intake characteristics considerably differ from school to school, this source of variability could be substantial. In a very large urban school district, the likelihood of variability across schools in terms of their intake characteristics is very high. But if intake characteristics and other school characteristics do not differ a great deal (which might be the case of a small suburban school district), this source of variability is likely to be fairly small. In the case of W-S designs, the variability between schools in their post-test scores does not



enter in the standard error of the estimate of the average treatment effect of the sample of schools; hence, this is a large advantage in terms of precision. However, the variance in the effects of the treatment across schools matters in the standard error of estimate. All things being equal, the more the effect of treatment varies across the schools, the larger the standard error.

**By district analyses.** The first analysis option is to employ a three-level hierarchical model in each district. District-by-district analysis yields a fairly precise estimate of the treatment. In certain smaller districts the precision of the estimate of the treatment may be lowered due to the small number of teachers/schools, but this may be tempered a bit by the fact that in these districts, schools generally do not differ widely from each other in terms of student and school background characteristics. We will also consider some pooling of information in these analyses where it seems sensible. For example, a pooled estimate of between-school variance could be computed using the data from two small districts employing between-school design. Note that this kind of pooling would be similar to what programs like HLM (Raudenbush, Bryk, Cheong & Congdon, 2004) do regularly when estimating variance components. In districts where a W-S design is employed, again precision will likely be lower in the case of smaller districts. As with the B-S design, the pooling technique will also be applied as appropriate.

In sum, how precisely a district's treatment effect is estimated will depend on a variety of factors: where a W-S or B-S design is employed in the district; the number of schools, teachers, and classes in a district that participate; and the magnitude of the various sources of variance discussed.

**Combining information across the districts.** One challenge is how to proceed in combining this information to estimate an "Intent-to-Treat (ITT)" effect. One strategy would be to employ two-stage meta-analytic technique. First, we analyze data for each district separately and obtain an estimate of the POWERSOURCE<sup>®</sup> effect for each district and its standard error. As mentioned previously, we might need to do a little pooling of information to estimate certain variance components. Then the district estimates of the effects of POWERSOURCE<sup>®</sup> and their standard errors could be read into HLM program or WinBUGS (Spiegelhalter, Thomas, Best, & Lunn, 2003) and synthesized, as in a meta-analysis. This would give us the estimate of the ITT effect, its standard error, and, importantly, an estimate of the variance in the effects of POWERSOURCE<sup>®</sup> across districts. HLM will also provide us with a test of the heterogeneity in treatment effects across districts. With only seven districts, the estimate of the between-district variance in treatment effects would not be very precise, but still it will provide useful information about its magnitude. Note that an

advantage of using WinBUGS to do this analysis is that it will give us a posterior distribution for the between-district variance in treatment effects, which will give us a sense of how probable it is (given the data) that this parameter exceeds values that are deemed substantively meaningful.)

Note that there are some precedents in the literature for doing these sorts of two-stage analyses. A classic example is Rubin's synthesis of the results from eight parallel randomized experiments of the effects of coaching for the Scholastic Achievement Test (SAT; Rubin, 1980). In each of eight schools, kids were randomly assigned to coaching or to a control condition. The data were analyzed separately from school to school in order to obtain an estimate of the coaching effect for each school and its standard error. Rubin then synthesized these eight estimates in an analysis much like the WinBUGS analysis mentioned previously. For other examples in the literature, see Raudenbush & Bryk, 1985, 2002.

### *3-Level Hierarchical Model for Within-School Design*

$$Y_{ijk} = \pi_{0jk} + \pi_{1jk}X_{1ijk} + \varepsilon_{ijk} \quad \varepsilon_{ijk} \sim n(0, \sigma^2) \quad (1)$$

$$\pi_{0jk} = \beta_{00k} + \beta_{01k}POWERSOURCE^{\circledast}_{jk} + r_{0jk} \quad r_{0jk} \sim n(0, \tau_{00}) \quad (2a)$$

$$\pi_{1jk} = \beta_{10k} + r_{1jk} \quad r_{1jk} \sim n(0, \tau_{11}) \quad (2b)$$

$$\beta_{00k} = \gamma_{000} + \gamma_{001}W_{1k} + u_{00k} \quad u_{00k} \sim n(0, \tau_{u00}) \quad (3a)$$

$$\beta_{01k} = \gamma_{010} + \gamma_{011}W_{1k} + u_{01k} \quad u_{01k} \sim n(0, \tau_{u01}) \quad (3b)$$

$$\beta_{10k} = \gamma_{100} + \gamma_{101}W_{1k} + u_{10k} \quad u_{10k} \sim n(0, \tau_{u10}) \quad (3c)$$

### *3-Level Hierarchical Model for Between-School Design*

$$Y_{ijk} = \pi_{0jk} + \pi_{1jk}X_{1ijk} + \varepsilon_{ijk} \quad \varepsilon_{ijk} \sim n(0, \sigma^2) \quad (4)$$

$$\pi_{0jk} = \beta_{00k} + \beta_{01k}W_{1jk} + r_{0jk} \quad r_{0jk} \sim n(0, \tau_{00}) \quad (5a)$$

$$\pi_{1jk} = \beta_{10k} + r_{1jk} \quad r_{1jk} \sim n(0, \tau_{11}) \quad (5b)$$

$$\beta_{00k} = \gamma_{000} + \gamma_{001}POWERSOURCE^{\circledast}_k + u_{00k} \quad u_{00k} \sim n(0, \tau_{u00}) \quad (6a)$$

$$\beta_{01k} = \gamma_{010} + u_{01k} \quad u_{01k} \sim n(0, \tau_{u01}) \quad (6b)$$

$$\beta_{10k} = \gamma_{100} + u_{10k} \quad u_{10k} \sim n(0, \tau_{u10}) \quad (6c)$$

## **Development and Pilot Testing of 7<sup>th</sup> Grade Materials**

In addition to the implementation study at 6<sup>th</sup> grade, during the 2007-08 school year we developed and pilot tested materials for 7<sup>th</sup> grade POWERSOURCE<sup>©</sup> modules, towards the goal of adding 7<sup>th</sup> grade teachers to the experimental study in 2008-09. This work included

the development and testing of 7<sup>th</sup> grade *Checks for Understanding* as well as the development of instructional and professional development support materials.

### **Development of 7<sup>th</sup> Grade Items**

During the 2007-08 year, we pilot tested a set of new 7<sup>th</sup> grade assessment items. Working with expert math teachers, and using data from the 6<sup>th</sup> grade *Checks for Understanding* we developed a set of 7<sup>th</sup> grade items reflecting the three conceptual POWERSOURCE<sup>®</sup> domains culled from our big ideas list—rational number equivalence, principles for solving equations, the distributive property and applications of these big ideas in other critical areas of mathematics. Items were reviewed by math experts as well as a group of five experienced middle-school math teachers.

The three conceptual domains chosen for inclusion in the POWERSOURCE<sup>®</sup> study were selected because they are: a) heavily represented in state standards and state and district test blueprints; b) historically difficult for students to master; and c) important prerequisites for learning and mastering algebra. Given that the conceptual domains for POWERSOURCE<sup>®</sup> remain constant across the grade levels, we paid close attention to how the concepts develop across grades. Thus, in developing the 7<sup>th</sup> grade assessment items, we looked at the learning trajectory for each domain. In the 6<sup>th</sup> grade, students were learning how to solve one-step linear equations; whereas, in 7<sup>th</sup> grade they begin to solve more complex, two-step equations. The big ideas underlying the concepts remain the same; yet, the skills students are mastering are different and build on what was previously learned. Our POWERSOURCE<sup>®</sup> materials and *Checks for Understanding* assessments reflect this trajectory and make the necessary connections between 6<sup>th</sup> and 7<sup>th</sup> grade content and the big ideas (see Table 8 for an example of the California state standards that relate to solving equations).

Table 8

CA Standards relating to Principles for Solving Equations (6<sup>th</sup>-8<sup>th</sup> grades)

6 <sup>th</sup> Grade	7 <sup>th</sup> Grade	8 <sup>th</sup> Grade
1.1 Write and solve one-step linear equations in one variable.	4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.	4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$ .
1.2 Write and evaluate an algebraic expression for a given situation, using up to three variables.	1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).	5.0 Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

### Pilot Testing of 7<sup>th</sup> Grade Items

Over 150 7<sup>th</sup> grade items have been developed and 72 items pilot tested in 27 classrooms with nine teachers in four schools. Using the same assessment model as the 6<sup>th</sup> grade items, we have developed six types of assessment: basic computation tasks, partially worked problems, explanation tasks, word problems and problems involving graphics. Items were grouped together (within domains) to create the *Checks for Understanding* assessment forms. We used an overlapping design to allow us to compile item data and conduct IRT analyses on all items. The 72 items that we have pilot tested to date were compiled into 46 forms (17 forms for solving equations, 19 forms for PA, and 10 forms for RNE).

**Pilot testing process.** For pilot testing, the tasks described were assembled into forms that students should be able to complete in about 15 minutes. This time frame was imposed by the districts with whom we collaborated. Any assessment that was longer, they felt, would be viewed by teachers as a test, and would evoke complaints about too much district testing. However as it has turned out, this time frame actually has a number of advantages; for instance, it focused teachers and students' attention on students' understanding of a single concept as well as encouraged deep assessment without being too intrusive into or engendering teacher hostility about intrusion into instructional time.

Each teacher participating in pilot tests received at least two different test forms, each focusing on the same big idea, with each form containing between 3-5 tasks. The forms were

randomly assigned to students within classrooms; each teacher administered the assessments to all of their 7<sup>th</sup> grade students. In all cases the first two to three items on the test forms were basic computation items. The subsequent items were either open-ended explanation tasks, partially worked problems, word problems, or problems with a graphic prompt. Forms containing explanation tasks did not contain any other tasks besides the basic computational items.

All pilot data from the closed-ended responses were entered by a group of undergraduate and graduate student workers and other CRESST staff. Three-point scoring rubrics were developed for the open-ended items. Training was conducted for scoring session participants.

**Selecting items for inclusion in the 7<sup>th</sup> grade *Checks for Understanding*.** From the set of 7<sup>th</sup> grade items piloted in the 2007-2008 year, we will choose items to include on our *Checks for Understanding* forms and instructional materials for the extension of the POWERSOURCE<sup>®</sup> study in 7<sup>th</sup> grade (to be conducted in the 2008-2009 school year). Data from the pilot test are currently being analyzed and we will use the same procedures for analyzing data and selecting items as we used for the 6<sup>th</sup> grade *Checks for Understanding*. That is, as indicated earlier, we will employ several criteria to evaluate the items used in the pilot-testing phase. These include: confirmatory factor analyses, reliability analyses, and IRT analyses. Specifically, our typical analysis scheme for each extended set of *Checks for Understanding* from each pilot test form was to first calculate reliability coefficients (Cronbach's alpha) for items representing each domain. Second, as another check of item quality, we conducted a principal component analysis and a confirmatory factor analysis for each test form to check whether the items exhibited the factor structure we expected (e. g., whether the computation items loaded on the same factor, etc). Third, IRT analyses based on Rasch models were conducted in order to obtain item parameters (difficulties) and item characteristic and information curves so that we could use them to select items for future testing. The model-data fit was investigated using two model fit indices. One is the G2 index, which is the Chi-square ( $\chi^2$ ) statistic and provided in PARSCALE phase 2 outputs, and the other is the MNSQ statistics.

**7<sup>th</sup> grade instructional materials and professional development design.** Concurrent to the development of the *Checks for Understanding* items in 7<sup>th</sup> grade, we are developing instructional materials and professional development supports. To date these materials are in draft form and have been developed with input and advice from five expert middle-school math teachers as well as reviewed by a panel of mathematics experts recruited specifically for this task. As with the 6<sup>th</sup> grade materials, knowledge from teaching experience, research

on teaching in these areas, and information gathered during the pilot testing year all play a role in developing these instructional materials.

### **Supplementary Research Activities**

Following is a brief update of several supplementary strands of work undertaken as part of CRESST activities during the 2007-08 school year. This work includes validation of teacher math knowledge measures, continued investigation of how computer-based assessment strategies can support mathematics learning, investigation of states' ELL assessment and accommodation practices (as part of CRESST supplementary assessment center grant), investigation of district contexts for assessment, and international applications of the POWERSOURCE<sup>®</sup> work. Each of these strands is discussed separately.

#### **Validation Study of Teacher Math Knowledge Measures**

A validation study was conducted on several measures of teacher knowledge with respect to math. Validity was examined by comparing the performance of math (non-teaching) experts, expert math teachers, experienced math teachers, and novice math teachers. Group differences were explored for each task, and the relation among tasks was investigated.

**Participants.** Eighty-six participants were recruited from several states through a teacher list-serve. Participants were recruited from several groups including two types of experts, math experts ( $n = 13$ ) and teacher experts ( $n = 10$ ), along with two groups of non-experts, novice teachers ( $n = 17$ ) and experienced teachers ( $n = 46$ ). The purpose of the different groups of participants was to ensure expertise in certain types of knowledge.

**Measures.** All participants completed five tasks and a background survey. These tasks included the Study of Instructional Improvement (SII) instrument (Hill, Schilling, & Ball, 2004), a ranking of student work (Baker, 2007), analysis of student work (Heritage, Kim, & Vendlinski, 2007), as well as a two-part knowledge mapping task (Delacruz et al., 2007) that was designed to measure content knowledge (KMP1) and math teaching knowledge (KMP2). The SII instrument, the student work ranking task, analysis of student work, and the first part of the knowledge mapping task were all designed to measure knowledge for teaching algebra.

**Summary of findings.** Overall, we found evidence that math experts consistently scored highest on tasks that required high content knowledge and teacher experts scored highest on the student response analysis task, a measure of knowledge for teaching algebra. In addition, we found evidence that the knowledge mapping task was sensitive to differences

in both math content knowledge and math teaching knowledge, consistent with prior results (Delacruz et al., 2007).

**Implications.** One important implication of this work for POWERSOURCE<sup>®</sup> was that this set of tasks can be used for evaluation of teachers participating in professional development. The SII, the knowledge mapping task, and the student response analysis task could be used as a set, or the knowledge mapping task could be used separately from the other tasks in order to efficiently investigate both content knowledge of teachers (KMP1) and knowledge of teaching algebra (KMP2). These findings supported the integration of the Knowledge Map task into the teacher outcome measures for the 2007-08 POWERSOURCE<sup>®</sup> implementation study.

A second important implication of this work is the potential application of these measures beyond the POWERSOURCE<sup>®</sup> study by both researchers and practitioners looking for measures of teacher knowledge of math at the middle school level, with the suggestion that non-traditional approaches to teacher knowledge assessment, such as the knowledge mapper, can provide useful information about teacher learning.

### **Assessment and Accommodation Practices for ELLs**

As described in our earlier progress report (Baker, 2007), CRESST received a supplemental center grant focused on state ELL assessment practices. This research project addresses the validity of assessments used to measure the performance of ELLs, such as those mandated by the NCLB Act of 2001 (2002). The goals of the research are to help educators understand and improve ELL performance by investigating the validity of their current assessments, and to provide states with much needed guidance to improve the validity of their English language proficiency (ELP) and academic achievement assessments for ELL students. The research has three phases. In the first phase of the study, the researchers analyzed existing data and documents in three states to understand the nature and validity of states' current assessment practices and their priority needs. The analyses and results of phase 1 focused on the following six areas: (1) the language demands exhibited on state content-area and ELP assessments, (2) the identification of items that function differentially for ELL subgroups and the characteristic of those items, (3) achievement gaps among the subgroups of ELL students (e.g., ELLs, re-designated ELLs) compared to non-ELL students, (4) the relationship between the ELP and content-area assessment scores, (5) the factors related to the redesignation status, and (6) accommodation practices.

The first phase of the project was exploratory in that the researchers identified key validity issues by examining the existing data and formulating research areas where further

investigation is needed. In the second phase of the research, currently underway, the researchers have been working with two states to develop and implement research studies to support the validity of ELL assessment practices within the states. Results from the first phase of the project and plans for the second phase were detailed in the phase 1 project report previously provided to IES. In the third phase of the research, the researchers will use what was learned in the first two phases of the project to develop specific guidelines on which states may base their ELL assessment policy and practice.

### **Technology-Based Studies**

Several inter-related studies were undertaken during the 2007-08 project year that focused on the use of computer technology as part of the math assessment process. Some of these activities specifically included POWERSOURCE<sup>®</sup> materials (i.e., instructional materials, *Checks for Understanding*), and others focused on the development and testing technology applications that could be integrated into subsequent iterations of POWERSOURCE<sup>®</sup> and POWERSOURCE<sup>®</sup>'s web-based support materials.

Another unifying aspect of these studies' design was that they all incorporated a computer-based classroom response system technology. The software, called 3I (Interactive, Individualized Instruction), was originally developed in collaboration with the UCLA School of Engineering to provide the capability for students to respond (by typing) to instructor prompts (Encarnacao, Espinosa, Au, Chung, Johnson, & Kaiser, 2008). The general goal is to promote student engagement and problem solving with the content; to promote student interaction with the instructor; and lastly, to provide instructors with real-time student response data feedback about students' understanding of the material.

This approach incorporates key elements of an effective formative assessment system: (a) the capability of the system to provide good information about what students know and do not know; (b) the use of that information by instructors to provide feedback to students about their performance; and (c) the use of that information by instructors to adjust instruction. As with effective tutoring (Bloom, 1984) and instructional techniques that promote interaction and engagement (Hake, 1998), the fundamental enabling capability is the bidirectional flow of accurate information between student and instructor. Students solve problems and ask questions—while the instructor observes performance, interprets student actions, questions, and performance, and remediates as needed.

Two research initiatives were undertaken as part of this technology strand of work during the current reporting period. The first initiative involved studies testing the response system in various middle school settings to examine which aspects of 3I—originally



developed for use in a higher education setting—would need to be adjusted for middle school mathematics use. Our middle school settings varied with respect to setting (summer school, regular school), scheduling (block schedule, normal schedule), teachers’ experience with technology (very inexperienced, typical level of experience), and experience in teaching the content (all experienced). The second initiative was to gather validity evidence for a student survey that could be used to measure students’ perceptions of classroom processes afforded by such a response system.

### **Use of Interim Assessment Data/District Contexts**

This research activity takes a broader contextual approach to interim assessment use—examining the ways in which middle school mathematics teachers use the data provided by POWERSOURCE<sup>®</sup> and other types of interim assessments, and how the features of the interim assessments are related to data use. The project is being conducted simultaneously at three sites—Central Colorado, Southern California, and Northern California (with lead researchers including Dr. Lorrie Shepard of the University of Colorado; Dr. Hilda Borko of Stanford University, and Dr. Brian Stecher of RAND). Districts were selected that had invested in teacher professional development around formative assessment or had installed formal interim assessment systems. Although three sites are involved, the project operates as a single study with a common design and procedures.

During the past project year, the team has engaged in four major activities: instrument development, pilot testing, recruitment, and data collection. Three interview guides were developed for use in collecting data from district-level administrators, school principals and classroom teachers. The classroom teacher interview took place in two stages: initially by telephone and subsequently in person. Pilot testing was completed for these instruments with appropriate respondents in California and Colorado, and revisions were made based on the results of those pilots. After the pilot testing, the team recruited up to three middle schools in each district to participate in the study; within each school we recruited up to three mathematics teachers. The number of schools and teachers varied from site to site, based on local conditions. In January, principal and teacher interviews began; by the end of the school year, the team had completed approximately 20 interviews with school administrators and about 20 interviews with mathematics teachers. This summer the team will begin analyzing the interview transcripts (and the classroom artifacts we obtain during our second school interviews). In the fall will continue the interviews with the remaining school administrators and teachers.

## **International Applications of POWERSOURCE<sup>®</sup>**

As described in our previous progress report (Baker, 2007), through the extension of previous professional relationships, a collaboration with Korean educators who wish to test out POWERSOURCE<sup>®</sup> in their country has been developed. In 2007, these Korean partners translated POWERSOURCE<sup>®</sup> materials into Korean, including teacher handbooks, assessments, and instructional materials (Choe, 2007). Based on these materials, in late 2007 teacher training was carried out by Korean researchers in Korea with 25 middle school teachers and 5 elementary school teachers. Over the spring of 2008, Korean researchers selected 21 teachers from those who attended the 2007 teacher training to implement POWERSOURCE<sup>®</sup> in their grade classrooms; the same number of teachers was selected as a control group from the same school as the experimental group in this study. For the experimental group, four translated POWERSOURCE<sup>®</sup> modules will be implemented over the summer 2008 (including *Checks for Understanding* and instructional materials); whereas, for the control group the teachers' pre-existing formative assessment tools will be used.

### **Plans for 2008-09**

Currently, we are beginning data analysis of data collected during the 2007-08 school year, including the student transfer measure and the multiple teacher outcomes described in this paper. Additionally, we will analyze the *Checks for Understanding* completed by the POWERSOURCE<sup>®</sup> group teachers, both in terms of statistical quality of the items and to track student scores across the school year. We will also analyze state test data outcomes as they are made available by the districts including, when available, subscale scores of state mathematics items.

The focus for project implementation during the 2008-09 school year will be continuing the experimental (random assignment) study of POWERSOURCE<sup>®</sup> impact begun in 2007-08. Specifically, in addition to continuing the study at the 6<sup>th</sup> grade level, we plan to add the 7<sup>th</sup> grade teachers in the participating districts to the study (note that, depending on district configuration, there may be some overlap in sample in cases where the same teachers teach both 6<sup>th</sup> and 7<sup>th</sup> grade math). The study will utilize a similar design and instrumentation to what was described above regarding the 2007-08 study, with student and teacher outcome instruments adapted to reflect 7<sup>th</sup> grade content as applicable. We will also continue our recruiting efforts to add districts to the project and increase overall sample size. During the 2008-09 school year we will also develop 8<sup>th</sup> grade materials (including *Checks for Understanding*, instructional activities, and professional development resources), with the goal of expanding the study to 8<sup>th</sup> grade in the 2009-10 school year.

Data collection and analysis for the supplemental ELL assessment study will continue. Another core planned set of supplemental activities is our leadership strand of work. Our leadership activities intend to support states and districts in their desire to develop coherent instructional programs to engage in standards-based reform; moreover, the work will focus in two areas. First, it will focus on the collaborative development of methodology and annotated examples that practitioners and contractors can use to align instruction and assessment developmentally with key priorities for student capability in mathematics as well as with standards. The methodology seeks deeper understanding and communication of the learning demands, inherent standards, and the developmental progressions that are essential to accomplishing key standards. The methodology lays out a systematic framework describing these learning demands and progression, rather than simply working backward from one existing test. Products from the proposed effort will include software with embedded tutorials for conducting alignment analyses, paper and poster illustrations, and the results of workshops and webinars held with experts in math, math education, test developers, and other researchers as well as with the practitioner and policy communities.

Secondly, to support the ongoing quality of current assessments, we propose a series of working conferences—connected to wider audiences through webinars—to clarify the specific criteria that states, local districts, and schools should address in selecting and refining these assessments and in order to compare and learn from promising systems for which there are positive data (e.g., POWERSOURCE<sup>®</sup>, assessments of the Center for Data-Driven Reform in Education). We also propose making the criteria and examples of their application available interactively on the CRESST website; using web 2.0 interactive capability to promote attention and dialogue on the quality of existing assessment practices; and lastly responding to state, district, and local district queries on improving their systems. These efforts will concentrate on areas of recent CRESST research—formative assessment as well as ELP assessment—and provide a forum for sharing the best of current knowledge from CRESST and other leading efforts across the country.



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**Appendix A:**  
**Additional Item Analysis Results**

# 1. RNE: 05-06 Pilot Study

FORM	ITEM	Short/Extended	etc	reliability	Alpha if Item Deleted
RNE-5 (N=55)	rnebt4	short item		0.687	0.647
	rnebt6	short item			0.661
	rnpw2a	short item			0.696
	rnpw2b	short item			0.611
	rnpw2c	short item			0.612
	<i>rnepwe1_a</i>	<i>extended item</i>			0.664
	<i>rnepwe1_b</i>	<i>extended item</i>			0.670
RNE-7-5 (N=142)	rnebt1	short item		0.535	0.518
	rnebt6	short item			0.504
	<i>rneex6_a</i>	<i>extended item</i>	<i>rneex6_a(i)</i>		0.480
	<i>rneex6_b</i>	<i>extended item</i>	<i>rneex6_a(ii)</i>		0.466
	<i>rneex6_c</i>	<i>extended item</i>	<i>rneex6_b(i)</i>		0.508
	<i>rneex6_d</i>	<i>extended item</i>	<i>rneex6_b(ii)</i>		0.460
RNE-8 (N=77)	rnebt2	short item		0.597	0.453
	rnebt5	short item			0.567
	<i>rnewp2_a</i>	<i>extended item</i>			0.582
	<i>rnewp2_b</i>	<i>extended item</i>			0.493
RNE-A4 (N=127)	rnebt1	short item		0.674	0.635
	rnebt2	short item			0.622
	<i>rneex1_a</i>	<i>extended item</i>			0.560
	<i>rneex1_b</i>	<i>extended item</i>			0.612
RNE-B4 (N=129)	rnebt2	short item	not "rnebt3"	0.707	0.635
	rnebt4	short item			0.686
	<i>rneex2_a</i>	<i>extended item</i>			0.679
	<i>rneex2_b</i>	<i>extended item</i>			0.565
RNE-C3 (N=124)	rnebt5	short item		0.718	0.669
	rnebt6	short item			0.717
	<i>rneex3_a</i>	<i>extended item</i>			0.665
	<i>rneex3_b</i>	<i>extended item</i>			0.627
	<i>rneex3_c</i>	<i>extended item</i>			0.669

## 2. RNE: 06-07 Pilot Study

FORM	ITEM	Short/Extended	etc	reliability	Alpha if Item Deleted
RNE-13 (N=107)	RN-BT-10	short item		0.613	0.534
	RN-BT-4	short item			0.311
	RN-BT-8	short item			0.655
RNE-14 (N=98)	RN-BT-1	short item		-0.094	
	RN-BT-8	short item			
RNE-15 (N=101)	RN-BT-5	short item		0.788	0.798
	RN-BT-8	short item			0.768
	RN-PW-2a	short item			0.731
	RN-PW-2b	short item			0.710
	RN-PW-2c	short item			0.716
	RN-BT-7	short item	not "RN-EX-12a"		0.799
RNE-16 (N=74)	RN-BT-6	short item		0.711	0.692
	RN-BT-8	short item			0.711
	RN-PW-3a	short item			0.624
	RN-PW-3b	short item			0.661
	RN-PW-3c	short item			0.619
RNE-17 (N=88)	RN-BT-1	short item		0.741	0.742
	RN-BT-2	short item			0.678
	RN-PW-4a	short item			0.640
	RN-PW-4b	short item			0.746
	RN-PW-4c	short item			0.661

### 3. PA: 05-06 Pilot Study

FORM	ITEM	Short/Extended	reliability	Alpha if Item Deleted
PA-A v3 (PA-1 v3) (N=58)	PA-BT-3	short item	0.443	0.260
	PA-BT-4	short item		0.320
	PA-PW-3a	short item		0.268
	PA-PW-3b	short item		0.224
	<i>PA-PWE-1</i>	<i>extended item</i>		<i>0.682</i>
PA-2 v4 & PA-B v4 (N=114)	PA-BT-3	short item	0.689	0.675
	PA-BT-4	short item		0.693
	PA-PW-2a	short item		0.576
	PA-PW-2b	short item		0.550
	PA-PW-2c	short item		0.568
	<i>PA-PWE-2</i>	<i>extended item</i>		<i>0.765</i>
PA-C v2 (PA-3 v2) (N=52)	PA-BT-3	short item	0.558	0.540
	PA-BT-4	short item		0.448
	<i>PA-EX-1a</i>	<i>extended item</i>		<i>0.521</i>
	<i>PA-EX-1b</i>	<i>extended item</i>		<i>0.426</i>
PA-F v3 (PA-6 v6) (N=25)	PA-BT-5	short item	0.558	0.537
	PA-BT-6	short item		0.692
	PA-PW-4a	short item		0.287
	PA-PW-4b	short item		0.264
	PA-PW-4c	short item		0.255
	<i>PA-PWE-1</i>	<i>extended item</i>		<i>0.719</i>
PA-G v3 (PA-7 v3) (N=55)	PA-BT-3	short item	0.551	0.486
	PA-BT-4	short item		0.510
	<i>PA-EX-2a</i>	<i>extended item</i>		<i>0.408</i>
	<i>PA-EX-2b</i>	<i>extended item</i>		<i>0.506</i>
PA-H v3 (PA-8 v3) (N=57 )	PA-BT-3	short item	0.371	0.398
	PA-BT-7	short item		0.136
	PA-PW-5a	short item		0.332
	PA-PW-5b	short item		0.370
	<i>PA-PWE-3</i>	<i>extended item</i>		<i>0.331</i>
PA-9 v3 (N=54)	PA-BT-8	short item	0.682	0.642
	PA-BT-9	short item		0.734
	PA-PW-4a	short item		0.538
	PA-PW-4b	short item		0.538
	PA-PW-4c	short item		0.546
	<i>PA-PWE-3</i>	<i>extended item</i>		<i>0.760</i>
PA-10 v3	PA-BT-10	short item	0.725	0.667

(PA-J v3) (N=135)	PA-BT-11	short item		0.627
	<i>PA-EX-2a</i>	<i>extended item</i>		0.700
	<i>PA-EX-2b</i>	<i>extended item</i>		0.657
PA-11 v4 (PA-K v4) (N=131)	PA-BT-3	short item	0.583	0.461
	PA-BT-7	short item		0.455
	<i>PA-EX-3a</i>	<i>extended item</i>		0.652
	<i>PA-EX-3b</i>	<i>extended item</i>		0.534
	<i>PA-EX-3c</i>	<i>extended item</i>		0.505
PA-12 v1 (N=43)	PA-BT-10	short item	0.785	0.753
	PA-BT-11	short item		0.759
	PA-PW-6a	short item		0.764
	PA-PW-6b	short item		0.699
	PA-PW-6c	short item		0.747
PA-13 v2 (N=84)	PA-BT-10	short item	0.526	0.303
	PA-BT-11	short item		0.380
	<i>PA-EX-6</i>	<i>extended item</i>		0.569
PA-14 v2 (N=82)	PA-BT-10	short item	0.794	0.817
	PA-BT-11	short item		0.767
	PA-PW-7a	short item		0.726
	PA-PW-7b	short item		0.723
	PA-PW-7c	short item		0.728
PA-15 v1 (N=28)	PA-BT-10	short item	0.643	0.609
	PA-BT-11	short item		0.643
	PA-PW-8a	short item		0.544
	PA-PW-8b	short item		0.627
	PA-PW-8c	short item		0.511

#### 4. SE: 05-06 Pilot Study

FORM	ITEM	Short/Extended	reliability	Alpha if Item Deleted
SE-1 v3 (SE-A3) (N=87 )	SE-BT-1	short item	0.470	0.531
	SE-BT-2	short item		0.429
	SE-PW-1-a	short item		0.340
	SE-PW-1-b	short item		0.361
	<i>SE-PWE-1</i>	<i>extended item</i>		0.387
SE-B v4 (SE-2-4) (SE-B7) (N=80)	SE-BT-1	short item	0.561	0.604
	SE-BT-2	short item		0.473
	SE-PW-2-a	short item		0.424
	SE-PW-2-b	short item		0.603
	<i>SE-PWE-2</i>	<i>extended item</i>		0.371
SE-3 v1 (N=17)	SE-BT-1	short item	-0.226	-0.402
	SE-BT-2	short item		0.118
	<i>SE-EX-1</i>	<i>extended item</i>		-0.201
SE-D v4 (SE-4-4) (N=59)	SE-BT-3	short item	0.545	0.549
	SE-BT-4	short item		0.518
	SE-PW-1-a	short item		0.523
	SE-PW-1-b	short item		0.415
	<i>SE-PWE-1</i>	<i>extended item</i>		0.426
SE-5 v4 (N=78)	SE-BT-5	short item	0.512	0.445
	SE-BT-6	short item		0.556
	SE-PW-1-a	short item		0.388
	SE-PW-1-b	short item		0.407
	<i>SE-PWE-1</i>	<i>extended item</i>		0.467
SE-7 v4 (N=121)	SE-BT-1	short item	0.712	0.756
	SE-BT-5	short item		0.711
	<i>SE-WP-2-a</i>	<i>extended item</i>		0.623
	<i>SE-WP-2-b</i>	<i>extended item</i>		0.622
	<i>SE-WP-2-c</i>	<i>extended item</i>		0.663
	<i>SE-WP-2-d</i>	<i>extended item</i>		0.668
	<i>SE-WP-2-e</i>	<i>extended item</i>		0.687
SE-8 v1 (N=109 )	SE-BT-5	short item	0.687	0.654
	SE-BT-6	short item		0.737
	SE-PW-3-a	short item		0.616
	SE-PW-3-b	short item		0.617
	SE-PW-3-c	short item		0.589
	<i>SE-EX-3</i>	<i>extended item</i>		0.643



5. SE: 06-07 Pilot Study (extended items are not scored)

FORM	ITEM	Short/Extended	reliability	Alpha if Item Deleted
SE-15 (N=73)	SE-BT-7	short item	-0.266	
	SE-WP-7	<i>extended item</i>		
	SE-EX-10-a	short item		
	SE-EX-10-b	<i>extended item</i>		
	SE-EX-10-c	<i>extended item</i>		
SE-16 (N=48)	SE-BT-8	short item	-0.216	
	SE-WP-8	<i>extended item</i>		
	SE-EX-11-a	short item		
	SE-EX-11-b	<i>extended item</i>		
	SE-EX-11-c	<i>extended item</i>		
SE-17 (N=35)	SE-BT-9	short item	0.442	0.140
	SE-WP-9-a	short item		0.404
	SE-WP-9-b	<i>extended item</i>		
	SE-WP-9-c	<i>extended item</i>		
	SE-EX-12-a	short item		0.459
	SE-EX-12-b	<i>extended item</i>		
	SE-EX-12-c	<i>extended item</i>		
SE-18 (N=18)	SE-BT-7	short item	0.064	0.264
	SE-WP-9-a	short item		-0.052
	SE-WP-9-b	<i>extended item</i>		
	SE-WP-9-c	<i>extended item</i>		
	SE-EX-13-a	short item		-0.127
	SE-EX-13-b	<i>extended item</i>		
	SE-EX-13-c	<i>extended item</i>		
SE-19 (N=39)	SE-BT-8	short item	0.592	
	SE-WP-7	<i>extended item</i>		
	SE-EX-14-a	short item		
	SE-EX-14-b	<i>extended item</i>		
	SE-EX-14-c	<i>extended item</i>		
SE-20 (N=80)	SE-BT-9	short item	0.045	
	SE-WP-8	<i>extended item</i>		
	SE-EX-15-a	short item		
	SE-EX-15-b	<i>extended item</i>		
	SE-EX-15-c	<i>extended item</i>		

SE-21 (N=36)	SE-BT-7	short item	0.072	
	SE-WP-8	<i>extended item</i>		
	SE-EX-16-a	short item		
	SE-EX-16-b	<i>extended item</i>		
	SE-EX-16-c	<i>extended item</i>		
SE-22 (N=32)	SE-BT-8	short item	0.596	0.307
	SE-WP-9-a	short item		0.812
	SE-WP-9-b	<i>extended item</i>		
	SE-WP-9-c	<i>extended item</i>		
	SE-EX-10-a	short item		0.221
	SE-EX-10-b	<i>extended item</i>		
	SE-EX-10-c	<i>extended item</i>		
SE-23 (N=75)	SE-BT-8	short item	0.462	
	SE-WP-7	<i>extended item</i>		
	SE-EX-11-a	short item		
	SE-EX-11-b	<i>extended item</i>		
	SE-EX-11-c	<i>extended item</i>		

## IRT Tables

### 1. RNE 05-06 & 06-07 Pilot

item	b	SE(b)	tau1	tau2	tau3	tau4	G <sup>2</sup>	df	p-value	INFIT (MNSQ)	OUTFIT (MNSQ)
RNBT1	-1.52	0.09					11.72	5	0.04	1.07	3.46
RNBT2	-1.11	0.08					35.91	6	0.00	1.00	1.05
RNBT4	-0.65	0.08					10.38	6	0.11	1.17	1.75
RNBT5	-1.01	0.09					28.74	5	0.00	0.96	1.07
RNBT6	-0.72	0.07					38.96	7	0.00	1.04	1.34
RNBT7	-1.48	0.19					0.00	0	0.00	0.97	0.74
RNBT8	-0.25	0.08					25.33	8	0.00	1.11	1.10
RNBT10	-3.10	0.54					0.00	0	0.00	0.99	1.13
RNPW2A	0.70	0.12					10.36	6	0.11	1.00	1.36
RNPW2B	0.78	0.12					27.31	7	0.00	0.79	0.62
RNPW2C	0.81	0.13					26.49	7	0.00	0.86	0.71
RNPW3A	-0.41	0.16					1.73	2	0.42	0.87	0.96
RNPW3B	-1.06	0.19					0.00	0	0.00	0.96	0.84
RNPW3C	-0.50	0.16					4.07	2	0.13	0.83	0.80
RNPW4A	0.02	0.15					9.05	4	0.06	0.82	0.62
RNPW4B	-1.07	0.16					0.20	2	0.90	1.06	0.86
RNPW4C	-0.48	0.15					6.80	3	0.08	0.74	0.67
RNEEX1_A	-0.61	0.07	-0.53	-0.01	0.55		10.06	3	0.02	0.61	0.82
RNEEX1_B	1.06	0.08	1.39	0.34	-1.73		92.64	11	0.00	0.82	0.67
RNEEX2_A	0.24	0.08	-0.17	0.17			18.87	7	0.01	1.18	1.19
RNEEX2_B	0.05	0.05	-0.37	0.11	0.06	0.20	8.72	6	0.19	0.49	0.53
RNEEX3_A	0.50	0.07	-0.05	0.90	-0.85		16.85	12	0.16	1.04	1.00
RNEEX3_B	0.06	0.08	-0.26	0.26			21.46	6	0.00	0.76	0.80
RNEEX3_C	1.42	0.09	0.63	0.50	-1.14		19.27	9	0.02	0.96	0.71
RNEEX6_A	0.74	0.12					20.13	4	0.00	0.91	2.81
RNEEX6_B	1.29	0.11	-0.47	0.47			18.68	6	0.01	0.86	6.02
RNEEX6_C	0.94	0.13					9.27	5	0.10	1.25	2.12
RNEEX6_D	1.61	0.15	-0.74	0.74			11.00	6	0.09	0.89	0.15
RNEPWE1_	0.94	0.16	0.44	-0.44			3.45	4	0.49	1.03	1.02
RNEPWE_1	1.02	0.15	0.23	-0.77	0.55		3.54	4	0.47	1.11	0.86
RNEWP2_A	0.71	0.17					10.77	4	0.03	1.17	4.61
RNEWP2_B	1.48	0.13	0.02	0.70	-0.71		8.93	5	0.11	0.90	0.23

## 2. RNE 06-07 Field

item	b	SE(b)	tau1	tau2	tau3	G <sup>2</sup>	df	p-value	INFIT (MNSQ)	OUTFIT (MNSQ)
RNBT7	-2.48	0.08				8.50	5	0.13	1.19	2.15
RNBT8	-0.57	0.02				258.57	10	0.00	0.88	0.79
RNBT9	-0.50	0.02				275.73	10	0.00	0.87	0.78
RNBT4	-0.83	0.03				85.63	9	0.00	0.99	0.94
RNBT2	-0.83	0.03				157.39	9	0.00	0.94	0.83
EX8	0.76	0.02	0.24	-1.03	0.79	113.72	25	0.00	1.05	1.22
EX9	-0.22	0.01	-1.26	0.45	0.81	96.68	23	0.00	1.24	2.10
EX1A	-0.11	0.01	-0.66	-0.02	0.68	117.15	26	0.00	0.94	0.98
EX1B	0.97	0.02	0.60	-0.19	-0.41	330.23	27	0.00	0.89	0.79
PWE_2	0.81	0.02	-0.20	0.44	-0.24	32.37	25	0.15	1.03	1.22
EX6a1	0.15	0.04				44.15	10	0.00	1.12	1.28
EX6a2	1.08	0.03	-0.50	0.02	0.48	25.07	20	0.20	1.08	1.50
EX6b1	0.70	0.04				66.66	10	0.00	1.12	1.75
EX6b2	1.26	0.04	-0.76	0.37	0.39	31.88	16	0.01	0.93	1.23

### 3. PA 05-06 Pilot

item	b	SE(b)	tau1	tau2	tau3	G <sup>2</sup>	df	p-value	INFIT (MNSQ)	OUTFIT (MNSQ)
pabt3	-0.85	0.07				9.67	4	0.05	0.99	1.41
pabt4	0.10	0.08				10.93	3	0.01	1.19	2.35
pabt5	-0.63	0.29				0.00	0	0.00	0.96	0.82
pabt6	0.29	0.28				0.40	2	0.82	1.56	1.59
pabt7	-0.37	0.10				14.01	3	0.00	0.82	0.77
pabt8	-0.34	0.19				1.28	2	0.53	0.97	0.74
pabt9	0.27	0.19				0.92	2	0.64	1.24	1.56
pabt10	-0.65	0.08				23.11	4	0.00	1.05	1.16
pabt11	-0.27	0.07				33.46	4	0.00	0.98	1.1
papw2a	0.69	0.14				7.52	3	0.06	0.68	0.46
papw2b	0.66	0.14				13.91	3	0.00	0.57	0.38
papw2c	0.80	0.15				15.10	3	0.00	0.6	0.37
papw3a	-0.12	0.18				0.74	1	0.39	0.8	0.7
papw3b	-0.41	0.19				0.95	1	0.33	0.84	0.59
papw4a	-0.35	0.16				10.78	2	0.01	0.7	0.51
papw4b	-0.01	0.16				9.95	2	0.01	0.74	0.61
papw5a	0.29	0.18				0.41	2	0.82	1.25	1.42
papw6a	0.91	0.28				0.02	2	0.98	1.27	1.54
papw6b	0.56	0.25				4.45	2	0.11	0.74	0.59
papw6c	0.67	0.26				0.37	2	0.83	1.08	1.11
papw7a	-0.06	0.16				17.09	4	0.00	0.84	0.73
papw7b	0.15	0.16				18.32	4	0.00	0.83	0.66
papw7c	-0.29	0.16				19.90	4	0.00	0.76	0.7
papw8a	-0.04	0.26				1.03	2	0.60	0.82	0.79
papw8b	0.46	0.28				0.03	2	0.98	1.13	1.26
papw8c	0.91	0.32				1.01	2	0.61	0.77	0.49
paex1_a	1.59	0.14	0.59	0.19	-0.79	1.85	4	0.77	0.82	0.73
paex1_b	0.50	0.09	-0.69	-0.73	1.42	0.83	2	0.66	0.47	0.96
paex2_a	1.20	0.10	0.13	-0.13		15.62	5	0.01	1.2	1.12
paex2_b	0.59	0.06	-0.13	-1.08	1.21	5.04	4	0.28	0.68	0.94
paex3_a	-1.60	0.18				2.40	1	0.12	1.36	1.58
paex3_b	1.18	0.15				2.85	3	0.42	1.05	0.88
paex3_c	1.32	0.10	1.34	-0.35	-0.99	12.34	7	0.09	1.08	0.99
paex6	1.15	0.11	0.62	-1.13	0.51	4.43	4	0.35	1.08	1.46
papwe1	1.67	0.18	-0.82	0.82		23.77	3	0.00	1.22	9.9
papwe2	2.06	0.19	-0.21	0.21		15.42	4	0.00	1.41	5.89
papwe3	1.81	0.13	-0.25	0.96	-0.71	6.09	4	0.19	0.79	3.09

#### 4. PA 06-07 Field

item	b	SE(b)	tau1	tau2	tau3	G <sup>2</sup>	df	p-value	INFIT (MNSQ)	OUTFIT (MNSQ)
PABT1_1R	-0.25	0.02				51.04	10	0	1.09	1.22
PABT13_1	-0.05	0.02				81.43	10	0	0.96	0.99
PABT9_1R	-0.39	0.02				33.23	9	0	1.08	1.1
PAPW8A_1	0.32	0.02				108.31	10	0	1.07	1.31
PAPW8B_1	0.57	0.02				195.87	10	0	0.85	1.02
PAPW8C_1	0.56	0.02				163.49	10	0	0.89	1.25
PABT2_2R	-0.98	0.02				46.02	8	0	0.93	0.94
PABT3_2R	-1.15	0.02				72.56	8	0	0.86	0.75
PABT12_2	-1.00	0.02				49.50	8	0	0.93	0.78
PAPW4A_2	-1.21	0.02				89.94	7	0	0.85	0.58
PAPW4B_2	-0.67	0.02				233.04	9	0	0.71	0.58
PAPW4C_2	-0.58	0.02				184.22	9	0	0.76	0.75
PAEX1a	1.61	0.03	-0.71	0.71		85.05	13	0	1.22	5.3
PAEX1b	0.34	0.02	-0.36	0.36		25.33	13	0.02	1.14	1.6
PAPWE3	1.84	0.02	0.67	0.80	-1.47	148.80	20	0	1.46	2.64
PAEX6a	1.59	0.02	0.77	0.65	-1.42	39.04	21	0.01	1.12	1.45
PAEX6b	-0.37	0.03				137.70	8	0	1.58	2.01

## 5. SE 05-06 & 06-07 Pilot

item	b	SE(b)	tau1	tau2	tau3	G <sup>2</sup>	df	p-value	INFIT (MNSQ)	OUTFIT (MNSQ)
sebt1	-2.12	0.18				0.02	1	0.87	1.31	1.49
sebt2	-0.31	0.10				0.38	2	0.83	1.05	0.98
sebt3	-1.80	0.33				0.00	0	0.00	1.10	3.25
sebt4	-2.01	0.37				0.00	0	0.00	0.92	0.71
sebt5	0.13	0.08				2.74	4	0.61	1.25	1.63
sebt6	-1.75	0.17				1.55	1	0.21	1.27	1.80
sebt7	-1.15	0.15				10.49	2	0.01	1.05	2.23
sebt8	-1.38	0.15				14.37	2	0.00	0.78	0.56
sebt9	-0.33	0.15				4.08	2	0.13	0.93	0.88
sepw1a	-0.81	0.10				19.29	2	0.00	0.80	0.80
sepw1b	-0.77	0.10				20.70	2	0.00	0.73	0.64
sepw2a	-0.01	0.15				3.44	2	0.18	0.95	1.13
sepw2b	-0.72	0.17				2.01	2	0.37	1.25	1.26
sepw3a	-0.29	0.13				1.26	2	0.54	0.90	0.78
sepw3b	0.44	0.14				2.70	3	0.44	1.04	0.92
sepw3c	-0.44	0.13				4.27	2	0.12	0.76	0.61
seex1	0.49	0.25	1.67	-0.63	-1.04	0.59	1	0.45	0.85	0.80
seex3	0.36	0.07	0.07	0.52	-0.59	2.57	5	0.77	1.12	0.93
seex10a	-1.09	0.17				9.78	2	0.01	1.20	0.87
seex11a	-1.62	0.20				0.00	0	0.00	0.94	0.61
seex12a	-2.03	0.45				0.00	0	0.00	0.94	0.49
seex13a	-0.49	0.33				0.01	1	0.87	1.34	1.79
seex14a	-1.09	0.27				0.00	0	0.00	0.99	0.98
seex15a	-1.56	0.22				0.00	1	0.91	1.13	1.02
seex16a	-1.85	0.38				0.00	0	0.00	1.00	1.00
sepwe1	0.69	0.10				20.05	3	0.00	1.06	1.21
sepwe2	0.78	0.12	0.09	-0.09		6.51	4	0.16	0.79	0.70
sewp2a	-0.93	0.13	0.93	-0.93		2.50	2	0.29	0.55	0.45
sewp2b	-0.91	0.13	0.95	-0.95		2.92	2	0.23	0.56	0.46
sewp2c	1.38	0.20				2.75	3	0.43	0.94	0.74
sewp2d	1.53	0.22				1.41	3	0.71	1.34	9.90
sewp2e	0.07	0.11	0.65	-0.65		2.36	5	0.80	1.21	1.12
sewp9a	1.25	0.20				23.12	3	0.00	1.01	0.70

## 6. SE 06-07 Field

item	b	SE(b)	tau1	tau2	tau3	G <sup>2</sup>	df	p-value	INFIT (MNSQ)	OUTFIT (MNSQ)
sebt2	-0.21	0.03				246.52	10	0.00	0.94	0.92
sebt3	-3.35	0.14				0	0	0.00	1.11	2.44
sebt7	-1.35	0.03				28.5557	9	0.00	1.14	1.91
sebt8	-1.56	0.04				51.6484	8	0.00	1.08	1.90
sepw2a	0.01	0.02				162.165	10	0.00	1.12	1.31
sepw2b	-0.67	0.03				161.148	10	0.00	0.99	1.03
sepw3a	-0.38	0.03				352.773	10	0.00	0.85	0.83
sepw3b	0.33	0.03				262.473	10	0.00	0.99	1.11
sepw3c	-0.46	0.03				382.581	10	0.00	0.83	0.87
seex6a	-0.12	0.03	0.90	-0.90		206.559	17	0.00	0.83	0.82
seex6b	1.52	0.03	0.44	0.10	-0.55	156.45	20	0.00	0.80	0.60
seex9	1.14	0.03	0.37	-0.23	-0.14	53.9213	24	0.00	1.07	1.36
sepwe2	1.58	0.04	0.46	-0.12	-0.34	45.2868	19	0.00	1.08	1.02



## 7. RA 06-07 Field

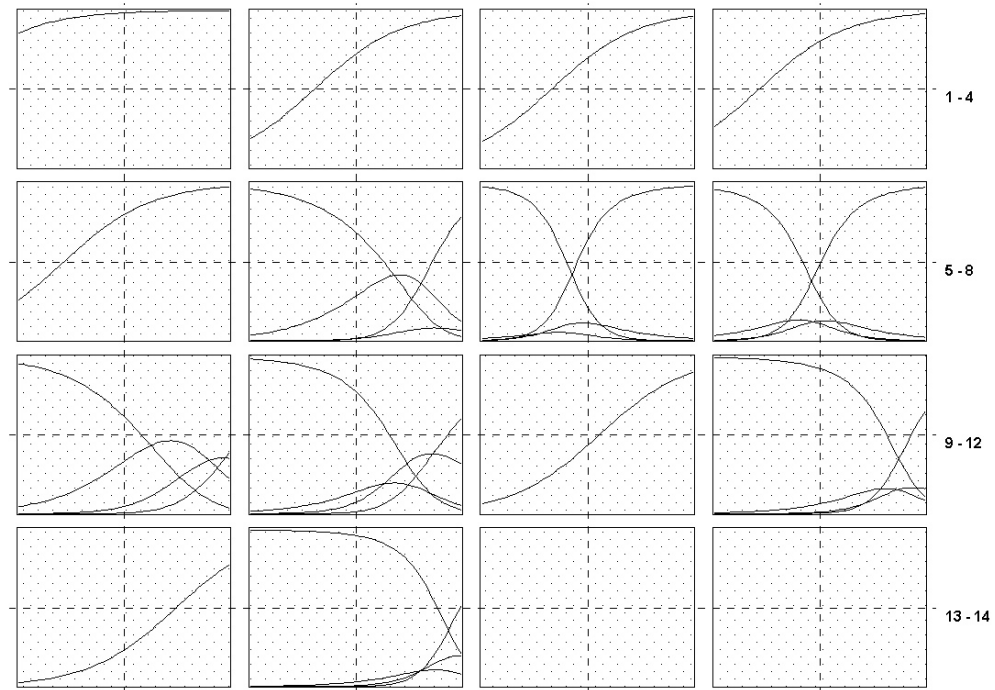
item	b	SE(b)	tau1	tau2	tau3	G <sup>2</sup>	df	p-value	INFIT (MNSQ)	OUTFIT (MNSQ)
rasebt9	-1.03	0.05				41.28	7	0.00	1.30	1.98
rasebt10	-1.65	0.06				6.00	5	0.31	1.00	1.53
rasebt11	-0.41	0.04				15.98	8	0.04	0.97	1.03
rpabt14a	-0.66	0.04				46.30	8	0.00	0.84	0.69
rpabt14b	-0.64	0.04				20.95	8	0.01	1.02	0.95
rpabt14c	-0.24	0.04				78.73	9	0.00	0.78	0.68
rpabt15a	-0.58	0.04				25.15	8	0.00	0.90	0.75
rpabt15b	-0.42	0.04				26.25	8	0.00	1.10	1.15
rarnbt11	-1.62	0.06				26.07	5	0.00	0.86	0.87
rarnbt13	-1.40	0.05				30.73	7	0.00	0.83	0.72
rarnwp3	-0.41	0.02	-0.55	0.49	0.06	60.81	17	0.00	1.25	1.61
rarnwp4	0.75	0.05				44.99	10	0.00	1.04	0.99
rarngp1	-0.80	0.05				5.68	8	0.68	1.01	1.16
rarnbt14	0.01	0.04				22.24	9	0.01	1.01	0.96
rapabt16	0.14	0.07				43.56	8	0.00	0.69	0.6
rasewp10	0.63	0.04	0.61	-0.60	-0.01	16.08	15	0.38	1.10	1.31
rasegp1	-0.08	0.06	1.19	-1.19		21.43	14	0.09	1.23	1.27
rapabt18	0.55	0.05	0.21	-0.21		18.76	14	0.17	0.95	0.93
rapabt17	0.28	0.07				49.03	9	0.00	0.78	0.79

## 8. FM 06-07 Field

item	b	SE(b)	tau1	tau2	G <sup>2</sup>	df	p-value	INFIT (MNSQ)	OUTFIT (MNSQ)
rnbt7	-2.82	0.11			3.54	2	0.17	1.17	1.74
rnbt8	-0.52	0.04			73.94	4	0.00	1.36	1.66
rn timer3a	0.09	0.04			41.04	4	0.00	0.91	0.79
rn timer3b	-1.01	0.05			7.60	4	0.11	1.07	5.8
rn timer3c	0.07	0.04			43.42	4	0.00	0.87	0.72
rn timer10	-0.54	0.04			35.59	4	0.00	0.81	0.74
rn timer2	-0.70	0.05			15.54	4	0.00	0.87	0.78
rn timer4a	0.01	0.04			21.25	4	0.00	0.97	1.06
rn timer4b	-0.93	0.05			24.95	4	0.00	1.13	1.05
rn timer4c	-0.14	0.04			56.72	4	0.00	0.8	0.66
rn timer3	2.60	0.09	0.69	-0.69	54.29	8	0.00	1.15	2.46

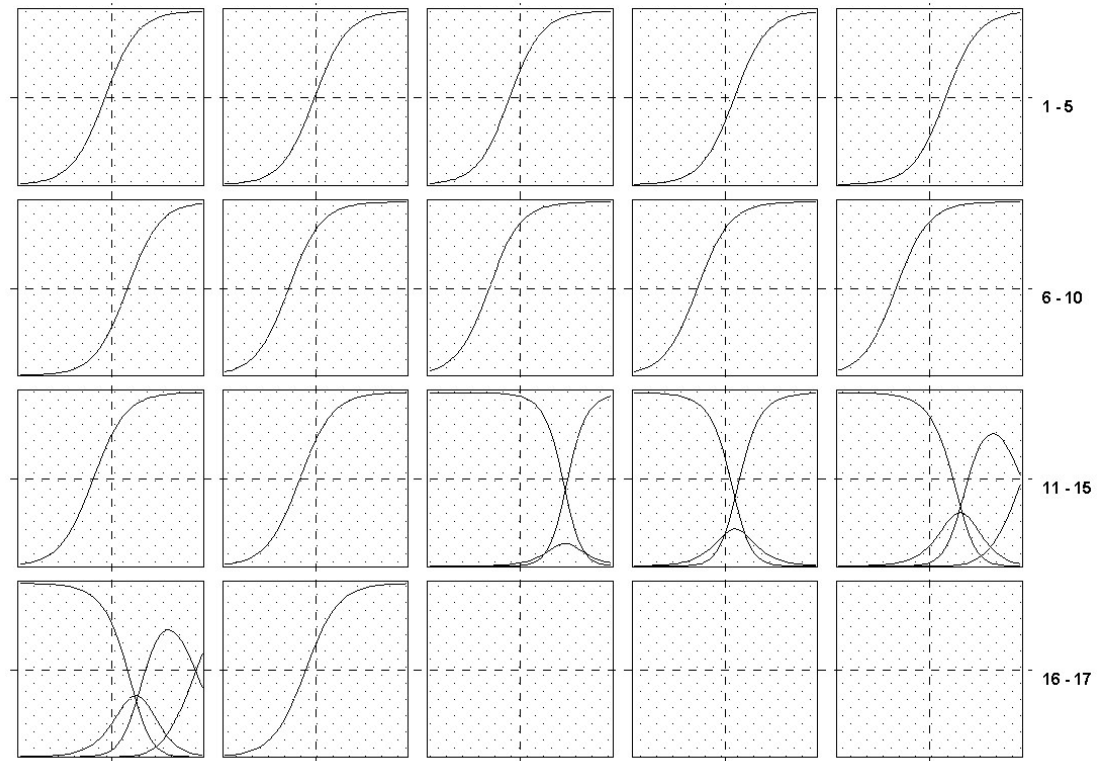
## Item (Catetory) Characteristic Curves

### 1. Items in RNE 06-07 Field Tests



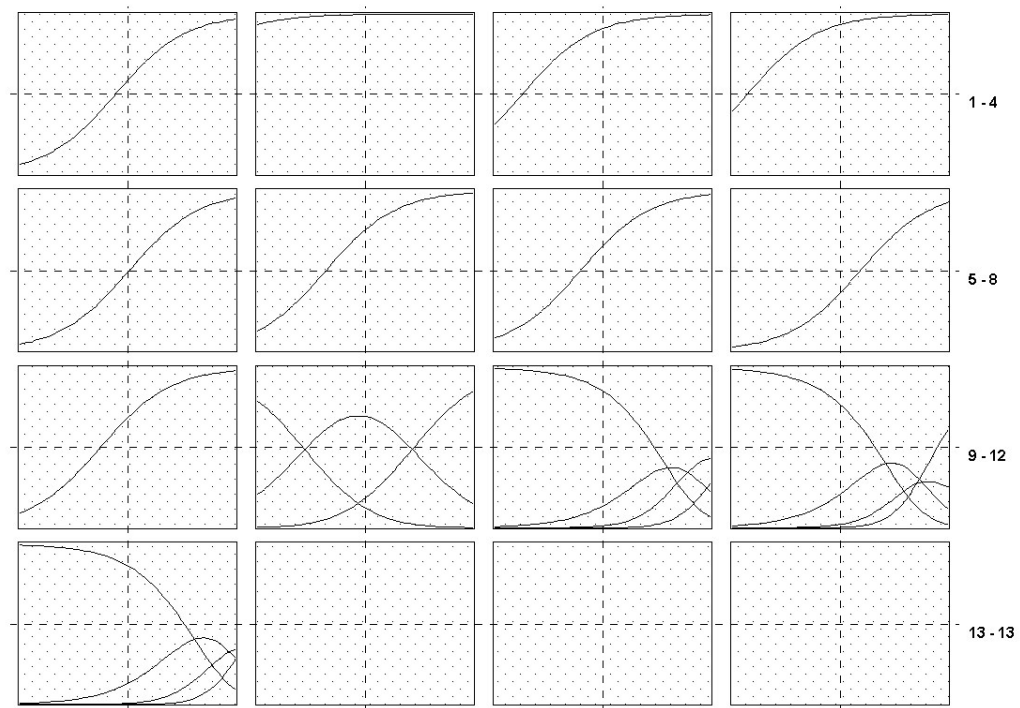
RNBT7	RNBT8	RNBT9	RNBT4
RNBT2	EX8	EX9	EX1A
EX1B	PWE_2	EX6a1	EX6a2
EX6b1	EX6b2		

## 2. Items in PA 06-07 Field Tests



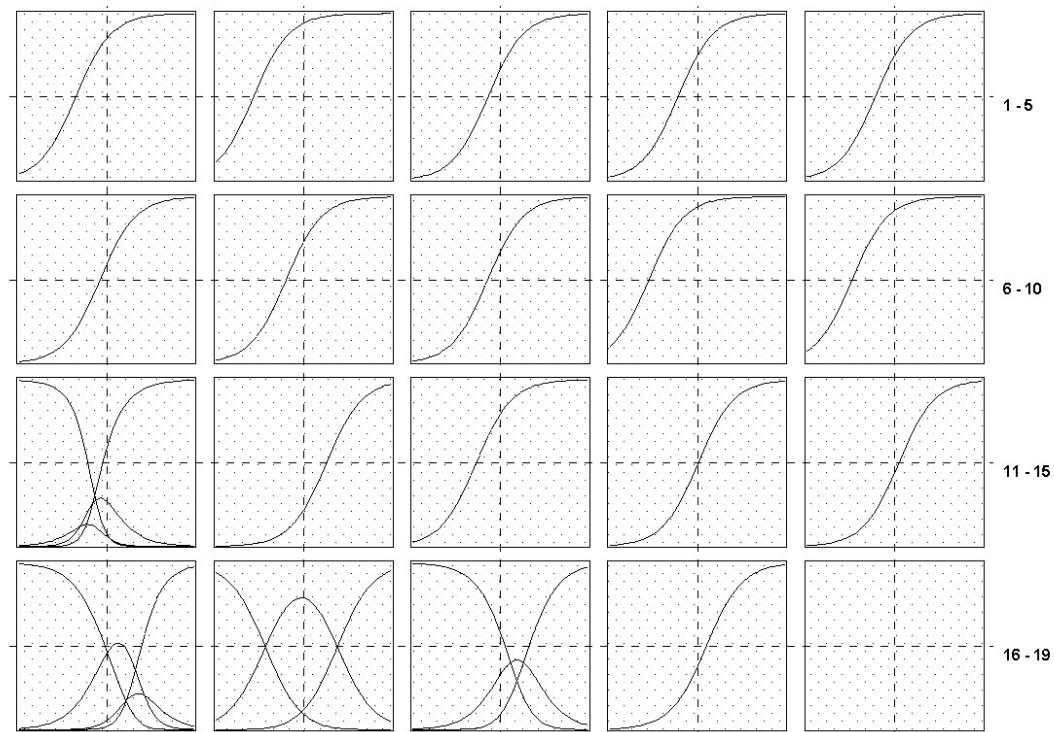
PABT1\_1R   PABT13\_1   PABT9\_1R   PAPW8A\_1   PAPW8B\_1  
 PAPW8C\_1   PABT2\_2R   PABT3\_2R   PABT12\_2   PAPW4A\_2  
 PAPW4B\_2   PAPW4C\_2   PAEX1a   PAEX1b   PAPWE3  
 PAEX6a   PAEX6b

### 3. Items in SE 06-07 Field Tests



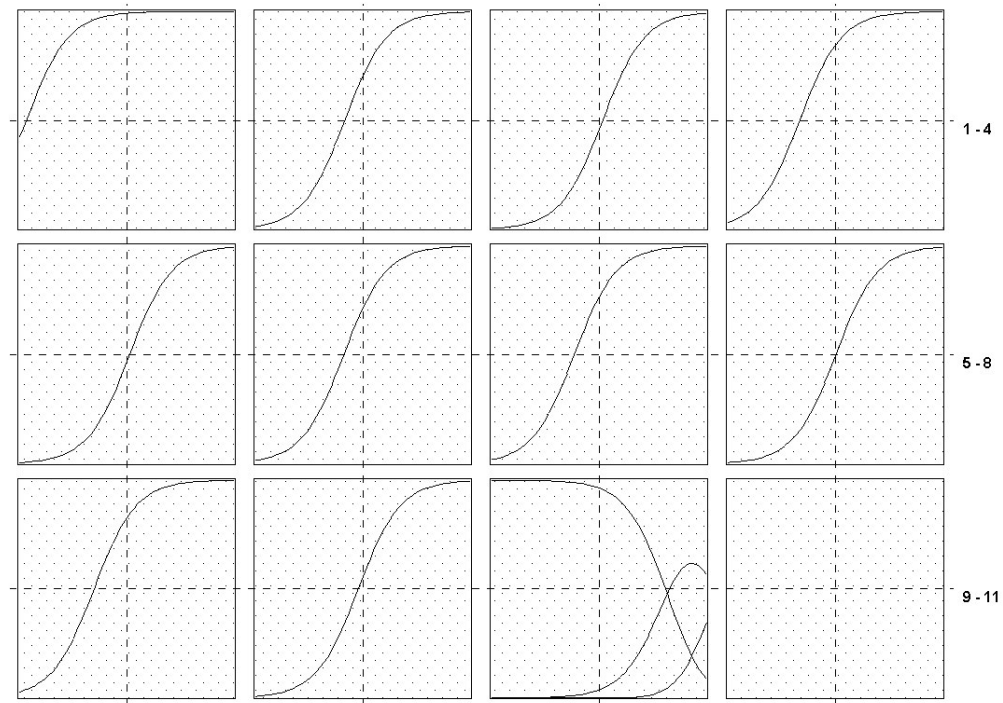
sebt2      sebt3      sebt7      sebt8  
 sepw2a    sepw2b    sepw3a    sepw3b  
 sepw3c    seex6a    seex6b    seex9  
 sepwe2

#### 4. Items in RA 06-07 Field Tests



rasebt9   rasebt10   rasebt11   rpabt14a   rpabt14b  
 rpabt14c   rpabt15a   rpabt15b   rarnbt11   rarnbt13  
 rarnwp3   rarnwp4   rarngp1   rarnbt14   rapabt16  
 rasewp10   rasegp1   rapabt18   rapabt17

## 5. Items in FM 06-07 Field Tests



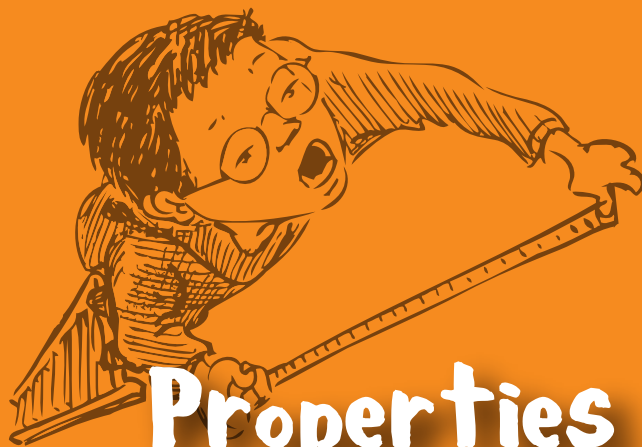
rnbt7	rnbt8	rn timer3a	rn timer3b
rn timer3c	rn timer10	rn timer2	rn timer4a
rn timer4b	rn timer4c	rn timer5e3	





**Appendix B:**  
**6<sup>th</sup> Grade, Teacher Handbook, Properties of Arithmetic: The Distributive Property**

# Teacher Handbook



## Properties of Arithmetic

The Distributive Property



grade **6**

ies



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# **Properties of Arithmetic**

## **The Distributive Property**

**David Niemi, Julia Phelan, Bryan Hemberg,  
Terry Vendlinski, Keith Howard, Laura Vinyard,  
Annabel Martell and Jennifer Casper**

# Properties of Arithmetic

## The Distributive Property

### Contents

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Understanding the Results 2 .....	9
Lesson 1 .....	18
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Understanding the Results 2a .....	28
Lesson 2.....	33
Administering Checks for Understanding 2b .....	39
Understanding the Results 2b .....	41

## Introduction

---

This handbook is divided into several parts. These parts reflect what you will be doing in each of the units. This handbook accompanies the second Powersource topic – properties of arithmetic. You will notice that all assessments begin with the number 2. So the first assessment, given prior to instruction is *Checks for Understanding 2*, the second one is *Checks for Understanding 2a* and the third is *Checks for Understanding 2b*.

### 1. Day 1: Checks for Understanding 2

The first *Checks for Understanding* for properties of arithmetic has ten questions covering the distributive property. You will administer this assessment to your students on Day 1. It should take your students about 20-25 minutes to complete this *Checks for Understanding*.

### 2. Understanding the Results 2

Once your students have completed *Checks for Understanding 2*, you can tally their scores and enter them into the table on page 13. Information in this table will allow you to easily see which problems were answered incorrectly by the most students. You can look at the total incorrect for each item, and use the information about the items on pages 15 through 16 to help you plan your teaching of the content in Lesson 1.

### 3. Day 2: Teach Lesson 1

The instructional text on pages 18 and 33 is your guide to the content for each of two lessons on the distributive property. These lessons are designed as an example of how a teacher might teach a lesson on the distributive property. Please do not feel as if you have to adhere exactly to this example lesson. You do not have to say or do exactly what is laid out in the lesson. Rather this is an example of one way that you could teach this content. This is only a guideline and if you have your own ideas about how to teach this content, please feel free to use them.

## Introduction

The important thing is to make sure you are covering the concepts presented in the sample lesson. We hope that some of the examples and overheads will be useful as you help students to understand these concepts, and we recommend the use of these examples somewhere within your instruction.

The lesson consists of the following:

### Material to say to the class:

say Have you heard of the distributive property? Can anyone say how it works?

### Material to write on the board:

write  $2(3 + 4) = 2 \cdot 3 + 2 \cdot 4$

### Overheads to show to the class:

show ▶▶▶▶▶▶▶▶ Overhead 1

### 4. Checks for Understanding 2a

Once you have gone over the content in Lesson 1, you will administer *Checks for Understanding 2a*. This assessment contains items based on content taught in Lesson 1. The items focus on the distributive property.

### 5. Understanding the Results 2a

Using the chart on page 30, you can again record how many students in your class answered incorrectly on each item. If you have students who

## Introduction

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did not understand some of the concepts in Lesson 1, you may choose to go back over some of the ideas presented in that lesson. The items on *Check for Understanding 2a* are comparable to items 1, 3, 4 and 8 on the *Checks for Understanding 2*.

### **6. Day 3: Teach Lesson 2**

Lesson 2 is designed to be used either by individual students, as a group exercise, or as a lesson taught by you. It is up to you to decide how to use Lesson 2. You might have some students who need extra time on the concepts presented in Lesson 1.

### **7. Checks for Understanding 2b**

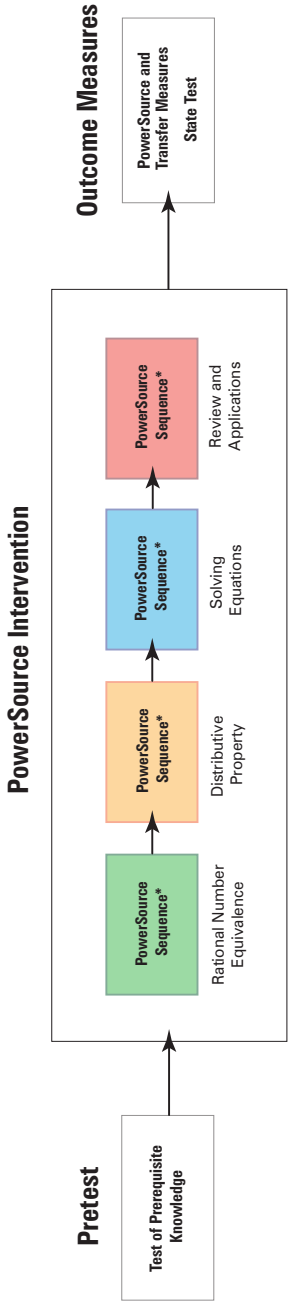
Once you have gone over the content in Lesson 2, you will administer *Checks for Understanding 2b* to your students. This assessment is based on material covered in Lesson 2 on the distributive property. Students solve problems, and write explanations about the distributive property in *Checks for Understanding 2b*.

### **8. Understanding the Results 2b**

Using the chart on page 44 you can record how many of your students answered incorrectly to a particular problem. If you have students who did not understand some of the concepts in Lesson 2, you may choose to go back over some of the ideas presented in that lesson. The items on *Checks for Understanding 2b* are comparable to items 1, 2, 5, 6 and 7 on *Checks for Understanding 2*.

\* See the diagram on the next page for an overview of the study.

# Overview of PowerSource™ Group



## \*PowerSource Sequence:

- a. Initial Checks for Understanding
- b. Lesson 1
- c. Checks for Understanding on Lesson 1
- d. Lesson 2
- e. Checks for Understanding on Lesson 2





# **Administering Checks for Understanding 2**

## **Administering Checks for Understanding 2**

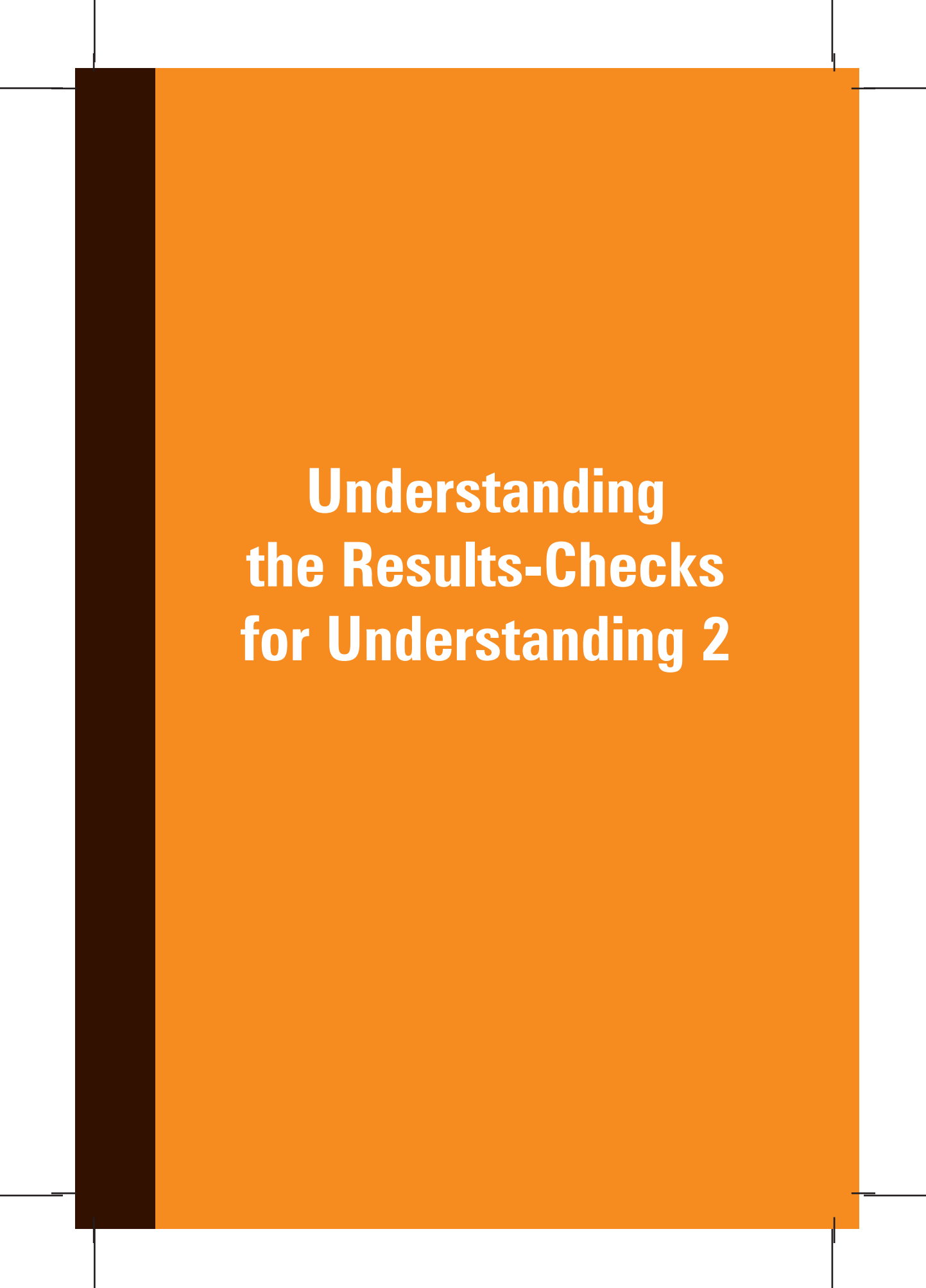
Please distribute the *Check Your Understanding* booklets to the students in your class in the order you found them in the envelope. Place any unused booklets back in the original envelope.

Today we are going to do some short problems that will take about 20-25 minutes for you to complete. Some problems may ask you to figure out what the missing numbers are, and some may ask you to explain your thinking.

say Read each problem and write the answer that you think is best. If there are any words that you are confused about, circle them and put a question mark next to the circle.

- \* These problems are intended to last no more than 20-25 minutes. Please stop your students after 20-25 minutes, even if they are not finished.
- \* When the students are working on the problems, if they ask a question, please make a note of it on the pages labeled *Teacher Notes of Student Questions*. Please do not provide information to a student that would help him or her answer a problem.
- \* In the CRESST folder, you will find yellow copies of the *Check Your Understanding* forms your students are taking. If you have suggestions for changes, please write them directly on these forms.

After students have worked on the problems for 20-25 minutes, please put all of the students' booklets back in their original envelope.



# **Understanding the Results-Checks for Understanding 2**

## Understanding the Results 2

*Checks for Understanding 2* is based on material you will be covering in Lessons 1 and 2. These lessons focus on the distributive property. Below is the answer key to *Checks for Understanding 2*.

### Check Your Understanding

Answer all questions below. Be sure to show all your work.

①  $7(4 + 2) =$  7  $\cdot 4 + 7 \cdot 2$

②  $4(y + 2)$

For this problem write only the first step that you would write if you were simplifying this problem.

possible answers:  $4y$

$$(4 \cdot y) + (4 \cdot 2)$$

$$4 \cdot y + 8$$

$$4y + 8$$

Go to next page.

## Understanding the Results 2

### Check Your Understanding

- ③ Show how you can use the distributive property to rewrite this equation:

$$3(2 + 4) = \boxed{3 \cdot 2 + 3 \cdot 4}$$

④  $6(3 + 1) = 6 \cdot 3$   $\boxed{+}$   $6 \cdot 1$

Go to next page.

## Understanding the Results 2

### Check Your Understanding

Complete the number sentence below.

⑤  $3(x + 5) = ( \boxed{3} \cdot x ) + ( 3 \cdot 5 )$

Using what you know about mathematical principles, explain why your answer is correct.

**This problem is an example of the distributive property that illustrates how to multiply a series of addends (in this case  $x + 5$ ) by the same quantity (3). 3 rows of  $x + 5$  is the same as 3  $x$ s + 3 5s, which can be written as  $(3 \cdot x) + (3 \cdot 5)$**

- ⑥ To get ready for science class, a student bought 8 pencils that cost \$3 each and 5 rolls of tape that also cost \$3 each. How much was his total bill?

Answer the question by using the distributive property, be sure to show all of your work.

$$\begin{aligned} & 3(8 + 5) \\ &= (3 \cdot 8) + (3 \cdot 5) \\ &= 24 + 15 \\ &= 39 \end{aligned}$$

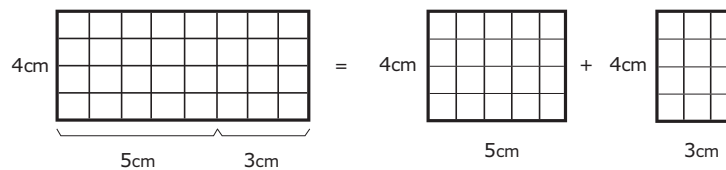
**Total bill = \$39.00**

Go to next page.

## Understanding the Results 2

### Check Your Understanding

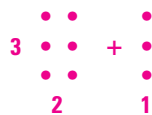
- ⑦ A student drew these two diagrams to show how the distributive property works. Write a number sentence that goes with these diagrams. Be sure your number sentence shows the distributive property.



$$4(5 + 3) = (4 \cdot 5) + (4 \cdot 3)$$

- ⑧ Write a number sentence that shows the distributive property. Draw a picture to show what your number sentence represents. Explain how your picture represents the number sentence.

$$3(2 + 1) = (3 \cdot 2) + (3 \cdot 1)$$



This problem is an example of the distributive property that illustrates how to multiply a series of addends (in this case  $2 + 1$ ) by the same quantity (3). 3 rows of  $2 + 1$  is the same as 3 2s + 3 1s, which can be written as  $(3 \cdot 2) + (3 \cdot 1)$

## Understanding the Results 2

Use the following chart to record the number of incorrect responses to each problem in *Checks for Understanding 2*:

	Problem	Incorrect Tally	Total
1	$7(4 + 2) = \square \cdot 4 + 7 \cdot 2$		
2	$4(y + 2)$ For this problem write only the first step that you would write if you were simplifying this problem.		
3	Show how you can use the distributive property to rewrite this equation: $3(2 + 4) =$		
4	$6(3 + 1) = 6 \cdot 3 \square 6 \cdot 1$		
5	$3(x + 5) = (\square \cdot x) + (3 \cdot 5)$ Using what you know about mathematical principles, explain why your answer is correct.		
6	A student bought 8 pencils that cost \$3 each and 5 rolls of tape that also cost \$3 each. How much was his total bill?		
7	Write a number sentence that goes with these diagrams. Be sure your number sentence shows the distributive property.		
8	Write a number sentence that shows the distributive property.		

Examine the total incorrect for each problem to see which concepts your students have the most difficulty with. The following pages provide more detailed information about these problems that may help inform your subsequent teaching. Some of this information may have been covered in your initial professional development session and is provided here by way of review.



## **Understanding the Results 2**

We have included, where appropriate, some results from our pilot testing of these items in the 2006-2007 school year. These results can provide useful information on patterns of responses we have observed in a sample of over 2000 students. Clearly, there will be some students for whom these *Checks for Understanding* pose no problems. Others may have more difficulty. The specific information on each item may help you focus your instruction on distributive property.

## Understanding and Using the Results of Checks for Understanding 2

---

The lessons you will be going over with your students in the next few days explore content centered on the big idea of the distributive property. You will use the ideas underlying the distributive property to help students develop a deeper understanding of how the property can be used to both simplify expressions and equations and solve other types of problems.

The first Checks for Understanding (given before any instruction) focuses on students' level of understanding of using and explaining the distributive property.

- \* **Item 1** assesses students' understanding as to which term is being distributed across the terms in the parentheses. During pilot testing, 69% of students correctly answered this with a 7. Other errors – although only occurring in around 5% of students each – were 2 or 42. Clearly, the student who answered 42 does not understand the purpose of this type of problem and is simply solving the left hand side of the equation and placing the answer in the box. Experience tells us that once students have some experience with problems set up in the way this one is, we see a decrease in the tendency to simply solve the problem.
- \* **Item 2** asks students to provide the first step they would write when solving this problem. They are not asked to solve the problem, but give the first step. The approach of asking students for the next step they would use in solving a problem comes from several research studies focused on effectively accessing what students know in a domain. Studies have found that this technique is highly predictive of performance on other tasks where students were asked to solve a whole problem. These results suggest that a “next step” approach may be an efficient assessment tool.
- \* **Item 3** assesses students' ability to solve a distribution problem given no scaffolding at all.

## Understanding and Using the Results of Checks for Understanding 2

---

- \* **Item 4** assesses students' understanding as to which operation is missing in the equation. This item is similar to one we used in pilot testing. Students correctly responded with a + sign in over 90% of cases. The most common incorrect answer was a multiplication sign.
- \* **Item 5** assesses students' ability to understand which term is being distributed across the terms in the parentheses, this time with a variable as part of the problem. Students are also asked to explain why their answer is correct.
- \* **Item 6** is a word problem that asks students to answer the question posed in the problem by using the distributive property. By explaining the process, the students demonstrate that they are not simply remembering the process, but are also understanding it as well.
- \* **Item 7** assesses the students' abilities to evaluate the application of the process, demonstrating a yet deeper understanding.
- \* **Item 8** asks students to write a number sentence that shows the distributive property and to draw a picture that shows what the number sentence represents. This is followed by an explanation.

# Lesson 1

# Lesson 1

say

Have you heard of the distributive property? Can anyone say how it works?

show



## Overhead 1

**Distributive Property:**  
 $2(3+4)$   
 $= (2 \cdot 3) + (2 \cdot 4)$   
 $= 6 + 8$   
 $= 14$

say

This equation shows how the distributive property works.  $2(3 + 4) = 2 \cdot 3 + 2 \cdot 4$

show



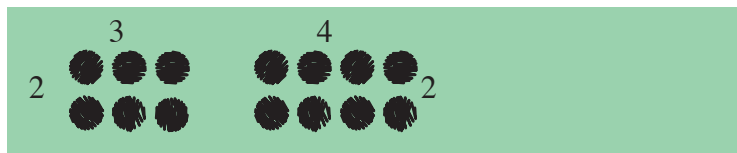
## Overhead 2

The **distributive property** states that you multiply both (or each of the) numbers inside the parentheses by the number outside the parentheses then add (or subtract) the products. We can use pictures to see why the distributive property works.

say

The distributive property states that you multiply both (or each of the) numbers inside the parentheses by the number outside the parentheses then add (or subtract) the products. We can use pictures to see why the distributive property works.

draw



say

Besides counting, how can we find the number of dots on the board? We could do it two ways. We could add the 3 and the 4 across the top to get the number of columns in this diagram.

## Lesson 1

write  $3 + 4$

say

Then we could multiply this by how many rows there are.  
How many rows are there?  
Right, 2. So we can write  $2(3 + 4)$

write  $2(3 + 4)$

say

What does this equal? Yes, 14. How did you get that?  
Yes, you can use order of operations. Add 3 and 4 to get 7,  
then multiply that by 2, and you have 14.

say

Now let's see another way you can figure out the number  
of dots. You could find out how many dots there are in  
each part of Figure 1, then add them together. So,  $2 \cdot 3$   
for the left part, and  $2 \cdot 4$  for the right part.

write  $2 \cdot 3$   $2 \cdot 4$

say

$2 \cdot 3 = 6$  and  $2 \cdot 4 = 8$ , so we have 14 when we add  
them together. That is the same number we found before.  
So at first we did this:  $2(3 + 4)$ , then we did this  $2 \cdot 3 +$   
 $2 \cdot 4$  and now we see that they are equal.

write  $2(3 + 4) = 2 \cdot 3 + 2 \cdot 4$

say

These pictures show you that you can separate the dots  
or put them together, and the number stays the same.

## Lesson 1

If necessary, repeat this procedure with different dot diagrams, such as  $3(4 + 1)$ .

say Let's look at another example.

say The distributive property shows us how to multiply a series of numbers being added by the same quantity. Let's see how it works and how we can use it.

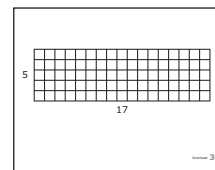
write  $5(17)$

say One way to figure out the answer here would be to use mental math and split 17 apart. We can break 17 into  $10 + 7$ .

write  $5(10 + 7)$

say We can solve this type of problem using order of operations. But we can also use an area model to show how the distributive property works.

show  **Overhead 3**

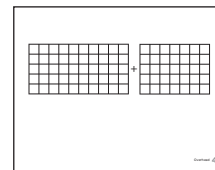


# Lesson 1

say How can we find the area of the large rectangle without counting the squares?

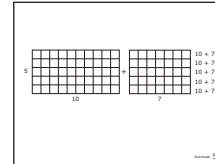
say We know we can break up 17 into  $10+7$ .

show ▶▶▶▶▶▶▶▶ **Overhead 4**



Make sure students understand that breaking up the 17 does not change the area because  $10 + 7 = 17$ . Also be sure the students recognize that the pictures are the same rectangle drawn in different ways.

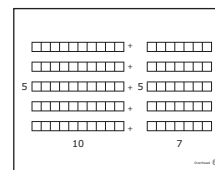
show ▶▶▶▶▶▶▶▶ **Overhead 5**



write  $5(10 + 7) = 5 \cdot 10 + 5 \cdot 7$

say We can show, using these models, that the distributive property shows that multiplication is repeated addition of the same thing. In this case  $(10 + 7) \cdot 5$  is the same as  $(10 + 7) + (10 + 7) + (10 + 7) + (10 + 7) + (10 + 7)$

show ▶▶▶▶▶▶▶▶ **Overhead 6**





## Lesson 1

say As you can see we can break up the model into 5 sets of  $10 + 7$  which can be rewritten as  $5(10 + 7)$ .

say Let's look at another example.

say Do you think the distributive property works with large numbers?

write  $8(351)$

say How could we use the distributive property to help solve this problem?

write  $8(300 + 50 + 1)$

say And so we can use distribution to rewrite this as:

write  $(8 \cdot 300) + (8 \cdot 50) + (8 \cdot 1) = 2400 + 400 + 8 = 2808$

say Do you think we could use the distributive property to help us solve  $8 \cdot 99$ ?

Ask students for ideas.

write  $8(100-1) = (8 \cdot 100) - (8 \cdot 1) = 792$

say This strategy will help us a great deal with mental math.

## Lesson 1

say

All of the above problems use the distributive property. How can we write a general formula for the distributive property?

write

$$a(b + c) = (a \cdot b) + (a \cdot c) \text{ or} \\ a(b - c) = (a \cdot b) - (a \cdot c)$$

draw



say

Can you write an expression that shows how you find the area of the rectangle?

Guide students to discovering the response of  $2(12 + b)$

say

How can we use this picture to explain the distributive property? Help me explain each step as we do them.

write

$$2(12 + b) =$$

Along with your students continue until the expression is distributed in the following manner:

$$\begin{aligned} 2(12 + b) &= (2 \cdot 12) + (2 \cdot b) \\ &= 24 + 2b \\ &= 24 + 2b \end{aligned}$$

Create and discuss additional examples if necessary.

## Lesson 1

---

**Administer the second  
*Check Your Understanding* assessment.  
See directions on page 26.**



# **Administering Checks for Understanding 2a**

## **Administering Checks for Understanding 2a**

Please distribute the *Check Your Understanding* booklets to the students in your class in the order you found them in the envelope. Place any unused booklets back in the original envelope.

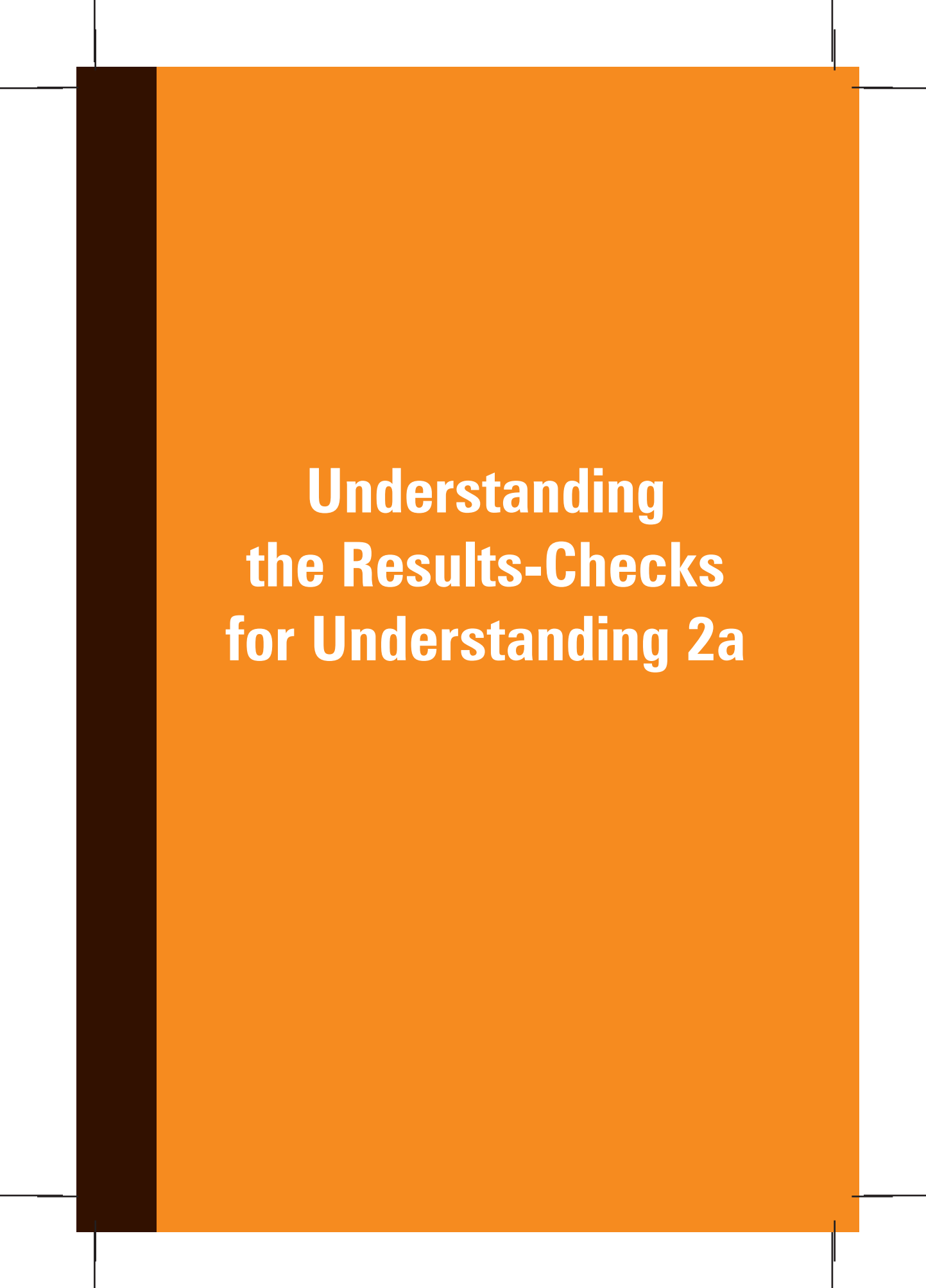
say

Today we are going to do some short problems that will take about 15 minutes for you to complete. Some problems may ask you to figure out what the missing numbers are, and some may ask you to explain your thinking.

Read each problem and write the answer that you think is best. If there are any words that you are confused about, circle them and put a question mark.

- \* These problems are intended to last no more than 15 minutes. Please stop your students after 15 minutes, even if they are not finished.
- \* When the students are working on the problems, if they ask a question, please make a note of students' questions on the pages labeled *Teacher Notes of Student Questions*. Please do not provide information to a student that would help him or her answer a problem.
- \* In the CRESST folder, you will find yellow copies of the *Check Your Understanding* forms your students are taking. If you have suggestions for changes, please write them directly on these forms.

After students have worked on the problems for 15 minutes, please collect and score the assessments, and then put all of the students' booklets back in their original envelope.



# **Understanding the Results-Checks for Understanding 2a**

## Understanding the Results 2a

*Checks for Understanding 2a* is based on the material covered in Lesson 1 on the distributive property. This includes using the distributive property to simplify expressions and solve problems. The following pages contain the answer key for this assessment.

### Check Your Understanding

Answer all questions below. Be sure to show all your work.

①  $8(4 + 2) = \boxed{8} \cdot 4 + 8 \cdot 2$

②  $6(3 + 1) = 6 \cdot \boxed{3} + 6 \cdot 1$

Go to next page.

## Understanding the Results 2a

### Check Your Understanding

- ③ Show how you can use the distributive property to rewrite this equation:

$$5(3 + 2) = 5 \cdot 3 + 5 \cdot 2$$

- ④ Write a number sentence that shows the distributive property. Draw a picture to show what your number sentence represents. Explain how your picture represents the number sentence.

$$3(2 + 1) = (3 \cdot 2) + (3 \cdot 1)$$

$$\begin{array}{ccc} \cdot & \cdot & \cdot \\ 3 & \cdot & + & \cdot \\ & \cdot & & \cdot \\ & \cdot & & \cdot \\ 2 & & & 1 \end{array}$$

**This problem is an example of the distributive property that illustrates how to multiply a series of addends (in this case  $2 + 1$ ) by the same quantity (3). 3 rows of  $2 + 1$  is the same as 3 2s + 3 1s, which can be written as  $(3 \cdot 2) + (3 \cdot 1)$**



## Understanding the Results 2a

Use the following chart to record the number of incorrect responses to each problem in *Checks for Understanding 2a*:

Problem		Incorrect Tally	Total
1	$8(3 + 2) = \square \cdot 3 + 8 \cdot 2$		
2	$6(3 + 1) = 6 \square + 3 + 6 \cdot 1$		
3	Show how you can use the distributive property to rewrite this equation: $5(3 + 2) =$		
4	Write a number sentence that shows the distributive property.		

Examine the total incorrect for each problem to see which concepts and problems need to be reviewed and/or re-taught.

If you have many students who did not understand some of the concepts in Lesson 1, you may choose to go back over the relevant ideas in Lesson 1. While you are doing this, if you have students who mastered everything in Lesson 1, you can have them go over Lesson 2 in pairs and fill in the worksheet.

## Understanding and Using the Results of Checks for Understanding 2a

---

*Checks for Understanding 2a* focuses directly on the concepts presented in the first instructional unit—how the distributive property is represented visually and how the distributive property can be used to simplify expressions and help solve problems. The key here is to have multiple representations available to students. Our work with teachers suggests that many students learn the distributive property as a rote procedure, and that they simply perform the procedure mechanically when they determine that it is needed. When students are asked to define the distributive property, they often can quickly recite “ $a(b+c) = ab+ac$ ” from memory. However, when asked to demonstrate understanding of the property, such as in problems using area models, it becomes clear that many do not understand the underlying logic.

This unit transitions from an area model using squares, to a model with a variable. It is critical that students understand that multiplication can be distributed across multiple terms, even though their inclination may be to use the order of operations in a problem with no variables.

The items in *Checks for Understanding 2a* are comparable to items 1, 3, 4 and 8 on the *Checks for Understanding 2*. You can use the data from *Checks for Understanding 2a* to determine how well students grasped the ideas in Lesson 1. You can also look at the descriptions of the items on pages 15-16 to determine exactly which concepts and principles your students may be having problems with (if any).



# Lesson 2

## Lesson 2

---

Lesson 2 is designed to be used either by individual students, groups, or as a lesson taught by you. It is up to you to decide how to use Lesson 2.

say

In the last lesson we talked about why the distributive property allows you to multiply both (or each of the) numbers inside the parentheses by the number outside the parentheses then add (or subtract) the products.

Hand out the Lesson 2 worksheets to your students. As mentioned earlier, the way you decide to organize this lesson will be determined by how well your students did on the first assessment.

The following pages contain teacher versions of the worksheets, which include the expected responses for each question.

## Lesson 2

### Lesson 2 Distributive Property

We can use the distributive property to help us solve problems like this:

$$3(5 + 6)$$

We know that because we are multiplying 3 by both 5 and 6 that we are going to distribute the 3 to the other numbers. See if you can complete the problem:

$$3(5 + 6)$$

$$= (\bigcirc \times \square) + (\bigcirc \times \triangle)$$

$$= (15) + (18)$$

$$= \square$$

1) What is the missing number that belongs in the  $\square$ ? 5 or 6

2) What is the missing number that belongs in the  $\triangle$ ? 5 or 6

3a) What is the missing number that belongs in the  $\bigcirc$ ? 3

b) Why is the number in the  $\bigcirc$  multiplied in both parentheses?

So it is distributed to both numbers

4) What is the final answer? 33

5) In your own words explain the distributive property.

Multiply both numbers inside a parentheses by the  
number outside the parentheses then add the products.

## Lesson 2

### Lesson 2

### Distributive Property

A student bought 7 pens that cost \$2 each and 4 note books that cost \$2 each. How much was her total bill?

6a) Can you use the distributive property to solve this problem?

☒ Yes ☐ No

6b) If yes, answer the question by using the distributive property, be sure to show your work.

$$\begin{aligned} &2(7 + 4) \\ &= (2 \cdot 7) + (2 \cdot 4) \\ &= 14 + 8 \\ &= 22 \\ &\text{Total bill} = \$22 \end{aligned}$$

Let's practice using the distributive property!

7a)  $3(2 + 4)$

$$\begin{aligned} &= (3 \cdot 2) + (3 \cdot 4) \\ &= 6 + 12 \\ &= 18 \end{aligned}$$

b)  $4(5 + 3)$

$$\begin{aligned} &= (4 \cdot 5) + (4 \cdot 3) \\ &= 20 + 12 \\ &= 32 \end{aligned}$$

c)  $6(1 + 7)$

$$\begin{aligned} &= (6 \cdot 1) + (6 \cdot 7) \\ &= 6 + 42 \\ &= 48 \end{aligned}$$

## Lesson 2

### Lesson 2 Distributive Property

Look at this example:

$$\begin{aligned} &2(8 + a) \\ &= (2 \diamond 8) + (2 \diamond a) \\ &= (\underline{\quad}) + (\underline{\quad}) \\ &= \end{aligned}$$

8) What mathematical symbols are missing from the  $\diamond$ ? **x or •**

9) What numbers are missing from the blanks? **16 and 2a**

10) What is the final answer? **16 and 2a**

11) Using the variables a, b, c, write a general equation for the distributive property.

$$\mathbf{a(b + c) = (a \cdot b) + (a \cdot c)}$$

More practice with the distributive property

12a)  $4(x + 3)$

$$\begin{aligned} &= (4 \cdot x) + (4 \cdot 3) \\ &= 4x + 12 \\ &= 4x + 12 \end{aligned}$$

12b)  $6(4 + b)$

$$\begin{aligned} &= (6 \cdot 4) + (6 \cdot b) \\ &= 24 + 6b \\ &= 24 + 6b \end{aligned}$$

12c)  $f(7 + 3)$

$$\begin{aligned} &= (f \cdot 7) + (f \cdot 3) \\ &= 7f + 3f \\ &= 10f \end{aligned}$$

## Lesson 2

---

**Administer the third  
*Check Your Understanding* assessment.  
See directions on page 39.**





# **Administering Checks for Understanding 2b**

## **Administering Checks for Understanding 2b**

Please distribute the *Check Your Understanding* booklets to the students in your class in the order you found them in the envelope. There may be more than one type of form. Place any unused booklets back in the original envelope.

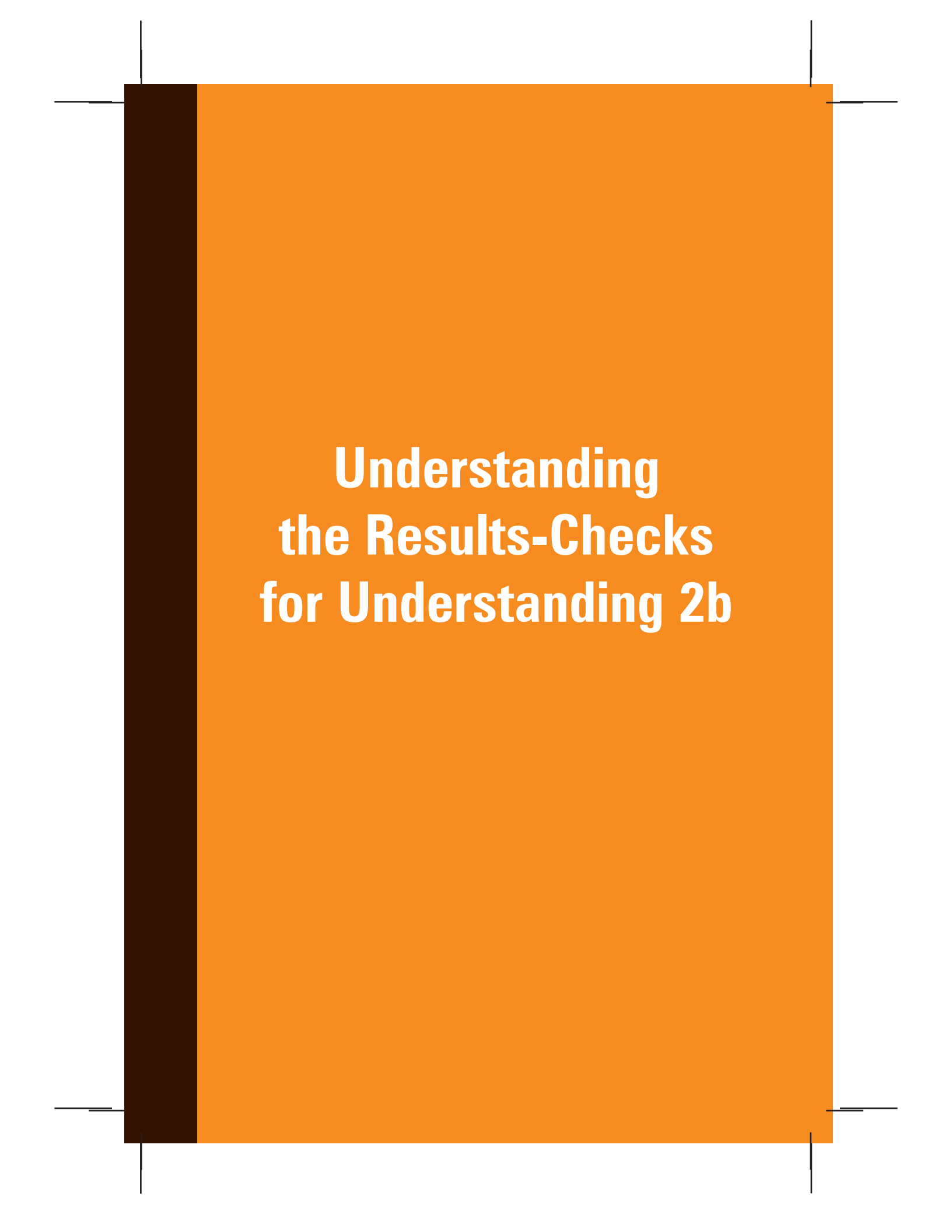
say

Today we are going to do some short problems that will take about 15 minutes for you to complete. Some problems may ask you to figure out what the missing numbers are, and some may ask you to explain your thinking.

Read each problem and write the answer that you think is best. If there are any words that you are confused about, circle them and put a question mark next to the circle.

- \* These problems are intended to last no more than 15 minutes. Please stop your students after 15 minutes, even if they are not finished.
- \* When the students are working on the problems, if they ask a question, please make a note of it on the pages labeled *Teacher Notes of Student Questions*. Please do not provide information to a student that would help him or her answer a problem.
- \* In the CRESST folder, you will find yellow copies of the *Check Your Understanding* forms your students are taking. If you have suggestions for changes, please write them directly on these forms.

After students have worked on the problems for 15 minutes, please put all of the students' booklets back in their original envelope.



# **Understanding the Results-Checks for Understanding 2b**

## Understanding the Results 2b

*Checks for Understanding 2b* is based on the material covered in Lesson 2 on the distributive property. This includes using the distributive property to simplify expressions and solve problems. The following pages contain the answer key for this assessment.

### Check Your Understanding

Answer all questions below. Be sure to show all your work.

①  $6(y + 3)$

For this problem write only the first step that you would write if you were simplifying this problem.

possible answers:  $6y$

$$(6 \cdot y) + (6 \cdot 3)$$

$$6 \cdot y + 18$$

$$6y + 18$$

②  $6(3 + 1) = 6 \cdot \boxed{3} + 6 \cdot 1$

Go to next page.

## Understanding the Results 2b

### Check Your Understanding

Complete the number sentence below.

③  $y(15 + 5) = (\boxed{y} \cdot 15) + (y \cdot 5)$

Using what you know about mathematical principles, explain why your answer is correct.

**This problem is an example of the distributive property that illustrates how to multiply a series of addends (in this case  $15 + 5$ ) by the same quantity ( $y$ ).  $y$  rows of  $15 + 5$  is the same as  $y$   $15$ s +  $y$   $5$ s, which can be written as  $(y \cdot 15) + (y \cdot 5)$**

- ④ Frank was washing cars over the summer. On one day he washed 12 cars in the morning and 6 cars in the afternoon. It took him 10 minutes to wash each one. How much time did he spend washing cars?

Answer the question by using the distributive property, be sure to show all of your work.

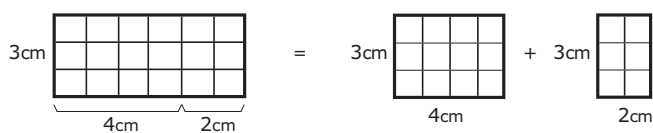
$$\begin{aligned} &10(12 + 6) \\ &= 10 \cdot 12 + 10 \cdot 6 \\ &= 120 + 60 \\ &= 180 \\ &= 180 \text{ minutes or 3 hours} \end{aligned}$$

Go to next page.

## Understanding the Results 2b

### Check Your Understanding

- 5 A student drew these two diagrams to show how the distributive property works. Write a number sentence that goes with these diagrams. Be sure your number sentence shows the distributive property.



$$3(4 + 2) = (3 \cdot 4) + (3 \cdot 2)$$

## Understanding the Results 2b

Use the following chart to record the number of incorrect responses to each problem in *Checks for Understanding 2b*:

Problem		Incorrect Tally	Total
1	$6(y + 3)$		
2	$6(3+1) = 6 \cdot \square + 6 \cdot 1$		
3	$y(15 + 5) = (\square \cdot 15) + (y \cdot 5)$		
4	Answer the question by using the distributive property, be sure to show all of your work.		
5	Write a number sentence that goes with these diagrams. Be sure your number sentence shows the distributive property.		

Examine the total incorrect for each problem. This total will give you a sense of how many of your students have mastered the concept of the distributive property.

These concepts will come up again in future units, so students will have more opportunities to practice what they have learned.

## **Understanding the Results 2b**

*Checks for Understanding 2b* is based on the material covered in Lesson 2 on the distributive property. This includes solving word problems using the distributive property and also provides extra practice for students to practice using the property in different contexts.

The items in *Checks for Understanding 2b* are comparable to items 1, 2, 5, 6, and 7 on *Checks for Understanding 2* (see page 9). You can use the data from *Checks for Understanding 2b* to determine how well your students have grasped the ideas in Lesson 2. You can also look at the descriptions of items 1, 2, 5, 6 and 7 on page 15 to determine exactly which concepts and principles your students maybe having problems with (if any).



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**Appendix C:**  
**Sample Alternative Professional Development Materials**

# **Using Data to Make Instructional Decisions**

**Terry Vendlinski**  
**Bryan Hemberg**

Norwalk – La Mirada USD  
October 1, 2007

# Generating Data

- Assessment in California Education
  - *Entry-level*
  - *Summative*
  - *Progress Monitoring*
- Standards and Blueprints
- California Standards Tests
- Benchmarks

# Accessing the Data (Data Director)

- Login to Data Director  
(<https://www6.achievedata.com/nlmsd>)
- Tabs vs. Menus
  - *Assessments*
  - *Exams*
  - *Reports*
- Other Tabs

# Refining Assessment Dataset

- Last year's 6<sup>th</sup> grade CST results for Math
- Make a report
  - *Delete Cluster 6, Test Taken, Test Taken Text, Items Attempted, and Include Indicator.*
  - *Modify columns so we can sort on grade, period and teacher (first and last name).*
  - *Refine data set to include only your first period (even if not period one)*
  - *Choose Summary to type summary and include graph (color or grayscale) of proficiency level*

# Cluster Scores (handmade)

- Does one cluster better predict overall performance? (Sort by each cluster score)
- Will weighting make a difference?
  - *What does each cluster mean?*
  - *How many items tested in each cluster?*
- Other explanations for variability in student CST performance?
- Can this focus instructional improvement?



# Cluster Scores (pre-made)

- Click on Report tab and choose CST Cluster Scores
- Modify data set for your first period in 2006 – 2007
- Sort by cluster
- Helpful in making instructional decisions?
- Helpful in focusing on specific students?

# Cluster Scores (modifying)

- Make a duplicate report, provide summary and save
- Modify column to add 2005 – 2006 Raw Cluster Scores.
- Reorder scores so '06 and '07 Clusters are side by side
- Are 5<sup>th</sup> and 6<sup>th</sup> grade clusters associated?
- Did same clusters result in same ordering?

# Cluster Scores (Pivot Table)

- Click on Report Tab and choose Pivot Table
- Use 2005 – 2006 as first data set
- Use 2006 – 2007 as second data set
- Use linear scaling
- What do circles mean?
- Choose one group above and one group below diagonal.

# Summative Conclusions

- What might summative data tell you?
- What are limitations of the data we analyzed?
  - *Data Set limitations (what data is available)*
  - *Analytical limitations (what tools are available)*
  - *Administration limitations (when data is available)*

# Refining Examination Dataset

- Final ALS Benchmark 2 for 6<sup>th</sup> grade Math in 2006 – 2007
- View Classroom Performance Summary menu item for your first period.
- Sort by clusters
- Would you identify the same cluster as most important instructional focus?

# Examination Details

- Examine mean, median and range in Test Statistics.
- Meaning of quartiles
- Meaning of mean and standard deviation
- Confidence intervals
- Reliability measures

# Examination Item Details

- Item Statistics
  - *P-values*
  - *Point Biserial Statistic*
- Item Analysis
  - *Distractors (Q5, Q18, Q24)*
- Differential Item Functioning

# Conclusions from Progress Monitoring

- What might progress data tell you?
- What are limitations of the data we analyzed?
  - *Technical Quality limitations?*
  - *Administration limitations?*
  - *Correlation with summative measures?*



# Implications

- What type of data did you find most useful?
  - *What is your purpose?*
  - *How will you use the data?*
- Proposal for future meetings
  - *Benchmark analysis*
  - *Implications for instruction (TLCs)*



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# **Self-Regulated Success**

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## **Metacognitive Strategies toward Improved Academic Achievement**

Keith Howard, Jessica Ulloa

# Roadmap

## 1. Self-Regulated Learning

- Students taking responsibility for learning
- Self-directed behavior & metacognition

## 2. Self-Efficacy & Goals

- **Student beliefs and their impact**
- **Mastery vs. Performance Goals**

## 3. Attributions & Affect

- Attributing causes for successes
- Anxiety & stereotypical beliefs

## 4. Teacher & Classroom Influences

- Rewards
- Feedback
- Expectations



# **Advance Organizer**

## **1. Feedback**

- **Calendar, Structured Worksheet, etc.**

## **2. Math Class Vignette**

- **Building Self-Efficacy**

## **3. Self-Efficacy**

- **Sources**
- **How Instruction Can Influence**

## **4. Mastery vs. Performance Goals**

- **Defined**
- **Social Comparisons**
- **Views on Ability**

# Self-efficacy

- “Beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments” (Bandura, 1997;2003)
- Cognitive Appraisal of Capabilities
- Context or Domain Specific
- Affects future actions (i.e. choice of tasks, persistence, effort)



# Self-Efficacy & Academic Motivation

- Self-efficacious students:
  - Take on difficult and challenging task more readily than do inefficacious students
  - Exert more mental effort in academic tasks
  - Demonstrate increased persistence
  - Exhibit less anxiety
  - Demonstrate better self-regulation of learning through goal setting, monitoring, self-evaluation, and strategy use

(Zimmerman, B. J. 2000)

# Self-Efficacy Sources

- Vicarious experiences can be very influential if student identifies with model
- Social persuasion can influence, but this influence is not enduring
- Mastery experiences are the most powerful influences

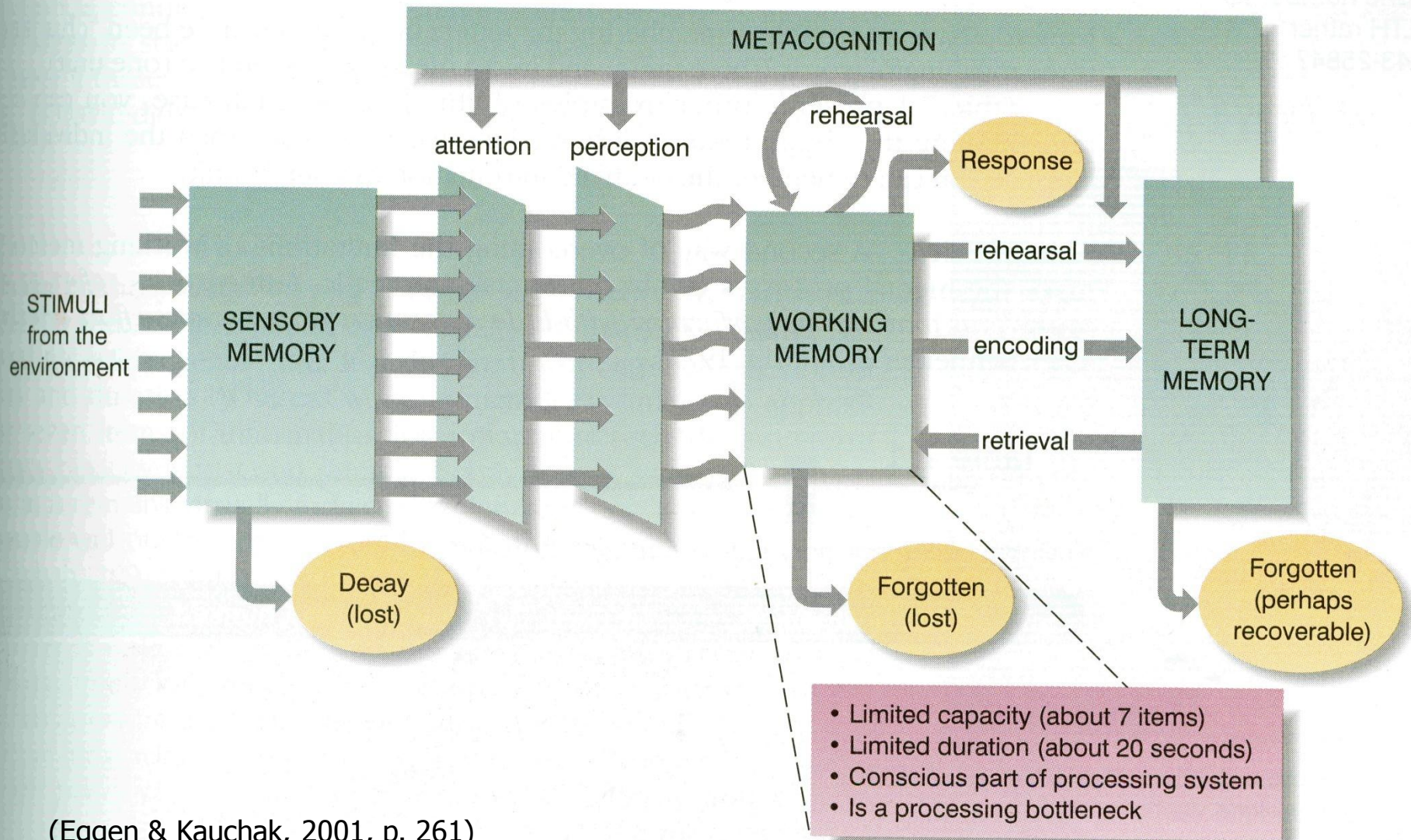
(Woolfolk, A. 2007)



# Self-Efficacy for Specific Tasks

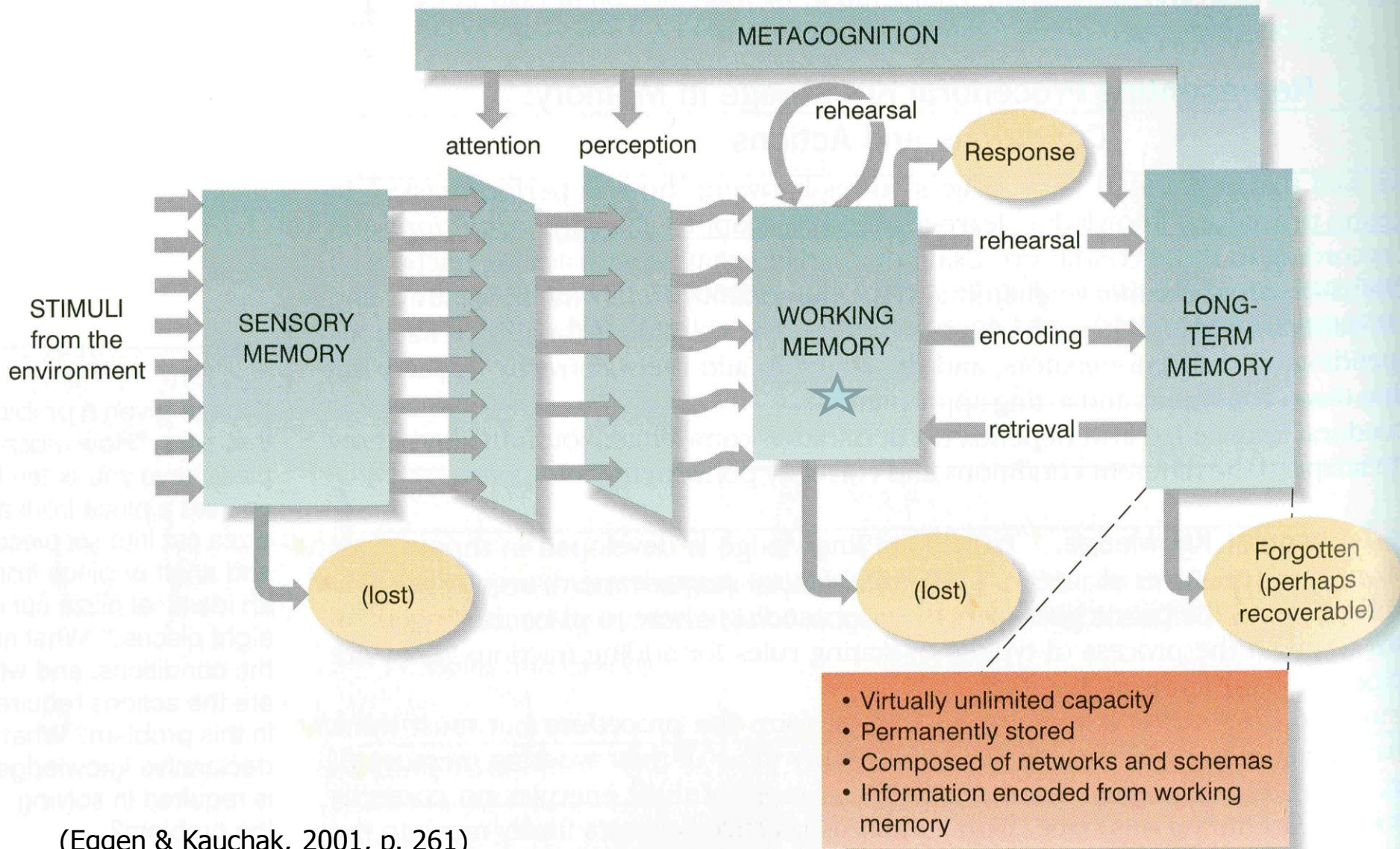
- Solve for x:  $2 + x = 5$
- Solve for x:  $2x = 10$
- Solve for x:  $12 = x - 12^2$
- Evaluate:  $5 \times 7 - 6 \times 2 + 3^2$
- Add:  $(9x^3 + 12x) + (16x^3 - 4x + 2)$
- Rewrite with positive exponents:  $\frac{1}{2x^8y^{-5}}$

# Information Processing Model





# Information Processing Model



(Eggen & Kauchak, 2001, p. 261)

# Transition to Long-Term Memory

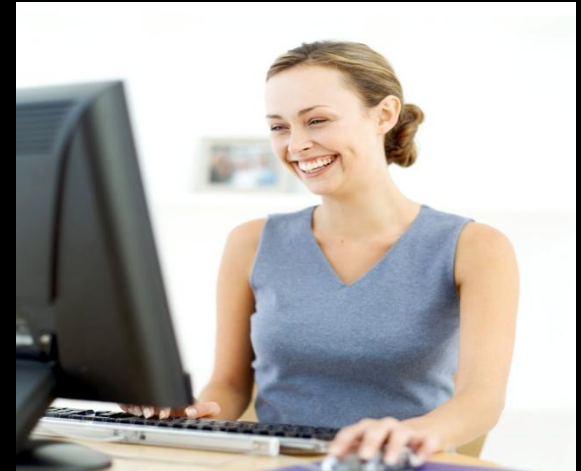
- Organization
  - Knowledge structures (schemas)
- Elaboration
  - Increase meaningful connections in context
- Rehearsal
  - Maintain information in working memory long enough to allow linking to meaning in long-term memory
- Automaticity
  - Frees up processing resources

# Automaticity

- What types of knowledge should be automated?
- Advantages of Automation?
- Long-Term Working Memory?
- Process of Automation:
  - Cognitive Stage - thinking about each step
  - Associative Stage - chunking into larger units
  - Autonomous -procedure requires little conscious processing

# Goal Theory

- Mastery Goal – learning for learning's sake; a desire for self improvement, irrespective of the performance of others
- Performance Goal – focuses on competition and social comparisons; the main objective is to outperform others



# Mastery vs. Performance Goals

- Mastery Goal
  - Positive attitude toward task
  - Self-monitoring
  - Makes connections with prior learning
- Performance Goal
  - Focus on memorization/rehearsal strategies
  - Less engaged
  - More focused on getting done than learning

# Different Views of Performance: Goal Orientation

	<b>Mastery Orientation</b>	<b>Performance Orientation</b>
<b>Success defined as...</b>	Improvement, progress, mastery, innovation	High grades, high performance compared with others
<b>Error viewed as...</b>	Part of the learning process, informational	Failure, evidence of lack of ability
<b>Ability viewed as...</b>	Developing through effort	Fixed



# **Performance Approach & Avoidance**

- Performance Approach – desire to look favorable in comparison to others
- Performance Avoidance – desire to avoid looking bad in comparison to others
- Performance goals are encouraged by school practices such as posting of grades, grading on curves, and norm-referenced tests

# Social Comparison

- Comparison information can enhance students' motivation, but not necessarily their self-efficacy or learning
- The perception that one is improving is hypothesized to raise self-efficacy
- The posting and public celebration of grades can encourage performance goals if learning is not emphasized

er Mrs. Wood

s: A+ — Excellent, Independent work.  
erage; B — Satisfactory Work of Average  
is Passing But Can Be Improved; D —  
E — Absolute Failure.

Days Absent	Department	English	Mathematics	Social Science	Spelling	History
	John Doe	A	C			CA
	Jane Smith	BB				BB
	Bob Johnson	BB				BA
	Emily White	BC				CB
	Michael Brown	BC				CB
	Sarah Green	BC				CA
	David Black	BC				

# Encouraging Mastery Orientation

- Connect subject matter to useful future application in life
- Emphasize understanding more than grades
- Suggest incremental view of intelligence
- Acknowledge mistake-making as part of learning process





**Appendix D:**  
**Examples of Professional Development Website Materials**


[Overview](#)
[Calendar](#)
[Surveys](#)
[Content Map](#)
[Measures](#)
[Message Board](#)

The purpose of the PowerSource study is to try out new Grade 6 formative pre-algebra assessments and associated instructional materials. CRESST researchers propose to determine the effectiveness of our materials, which are based on helping teachers to understand whether students are learning the fundamental mathematical principles underlying mastery in algebra. This project is designed to develop assessment and instructional materials that can be used throughout the school year to evaluate student understanding of foundational concepts in pre-algebra and algebra. The topics covered in this material are likely to be covered in tests required by the State.

Over the course of the school year, PowerSource study participants will be introduced to the four **Big Ideas** found below: Rational Number Equivalence, Properties of Arithmetic, Solving Equations, and Review and Applications.

On the menu to the left you will find resources related to the overall scope of the study, including a calendar, surveys, and a content map which visually connects the Big Ideas to Algebra I.

Within each of the Big Ideas sections found below are resources that will aid and assist participating PowerSource teachers. Descriptions of the units, study materials, research links, lesson explanations, and frequently asked questions are just some of the many useful items provided.

You will be able to access each section after it is presented to you at the Professional Development Institutions that preface each unit.

[Rational Number Equivalence](#)
[Properties of Arithmetic](#)
[Solving Equations](#)
[Review and Applications](#)



Overview

FAQs



Big Ideas

Content Map

Explanation

Connections

Teaching Resources

Misconceptions

Teaching Aids

Research Links

Results

Teacher Handbook

Using the Checks

What You'll Need

Scoring

Next Steps

1. When will my students ever use this?

- Your students probably already use the distributive property when multiplying multiple-digit numbers by other non-zero numbers; however, they probably aren't aware that is what they are doing. We will use their experience of multiplying such numbers as a beginning point in our professional development and a beginning point in instruction.
- In the Properties of Arithmetic unit, students will apply the concept of multiplication to develop an understanding of the distributive property and adding "like terms."
- Eventually, students will use this concept to solve linear and quadratic equations, and to factor expressions so they can simplify rational expressions (these topics will appear in later grades).

2. My students are confused as to what an operator is.

- An operator, as used in the *Checks for Understanding* materials, is referring to one of the basic mathematical operations; addition (+), subtraction (-), multiplication ( $\cdot$ ), or division ( $\div$ ).

3. Why do we have to use the distributive property when the order of operations can get the same answer?

- At times it is faster and easier to use the distributive property. For example, when multiplying 37 times four mentally, it is probably easier to think 30 times 4 is 120 and 7 times 4 is 28, and then add 120 and 28 to determine the final product is 148. This same principle can be used when trying to calculate a 15% tip. It is often easier to calculate 10% of the bill, then take half of that (5%) and add the two results, rather than multiplying fifteen times the total.
- As students begin to study algebra and pre-algebra, they will encounter situations when they can't combine terms inside the parentheses (for example  $[x + 2]$ ), but must multiply the quantity in order to easily solve the problem. The ability to distribute in such situations will become even more important when students try to solve equations like  $(x + 2)(x + 3) = 7$ .
- Non-graphical solutions to such an equation (such as factoring, completing the square or the quadratic formula) will require students to understand and use distribution if they are to comprehend, rather than just algorithmically manipulate (e.g., F.O.I.L.) a solution strategy.
- Order of operations is not always obvious, such as when you multiply  $2 \cdot 2\%$ . In this case you are actually adding  $2\% + 2\%$  or adding two 2s and two %s or adding  $2 \cdot 2$  and  $2 \cdot \%$  (distribution).

4. My students are not used to the formatting of the questions, what should I do?

- The construction of the questions in the *Check for Understanding* forms follow a few different formats. There are "basic" items that require students to fill in the blank or provide a solution to a problem, and "explanation" problems that require students to write out an explanation for their solution, sometimes in conjunction with mathematical work associated with the item and/or previous items.
- Other types of problems are ones where students just have to complete the first step. The approach of asking students for the next step they would use in solving a problem comes from several research studies focused on effectively assessing what students know in a domain. Studies have found that this technique is highly predictive of performance on other tasks where students were asked to solve a whole problem. These results suggest that a "next step" approach may be an efficient assessment tool. This type of item is usually the most unfamiliar to students. Be sure to remind students that, although there may be multiple steps within a single problem, they only need to write in the first step they would do—they don't need to solve the whole problem.
- It is also important to have students carefully read the instructions that precede each item, so that they know what is being asked and what type of response is expected. You may also highlight or explain to students what the problem is asking them to do without actually answering the question for them.

5. My students are having difficulty understanding the terms contained in the questions on the *Checks for Understanding*. What can I do?

- To ensure that students are able to understand what is being asked by each question, it may be beneficial to ask students if there are any mathematical terms contained in the questions that they do not recognize or know the definition for. By going through the materials before you administer them to students you may be able to pick out any terms your students may not recognize and introduce them beforehand as part of your instruction.

6. My students are having difficulty providing a written explanation to some of the questions. They know the answer, just not how to put it into words.

- Have your students say aloud how they would answer the question, in the most accurate and concise way they can. This may take more than one attempt. Then have them write the answer they have verbalized.
- As part of instruction, have students justify their thinking to a partner by explaining the logic they used to solve a question that you pose. This justification can be verbal or written.

7. How can I help my students avoid making careless errors?

- Students and teachers will make careless errors in their thinking and computation. A way to avoid these errors is to ask students to show their work. Teachers should also remind students to carefully read the question, complete their work slowly, and spend time practicing the type of problem they are having difficulty with.
- Encourage students to check their answers after solving a short answer question to ensure their solution works in the problem as it was originally presented.

8. There were many points during the lesson where I could perceive that my students were unfamiliar or had forgotten a previous concept. Is it ok for me to interrupt the lesson to instruct the students on these concepts?

- Yes.



## Properties of Arithmetic

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### Overview

The lessons you will be going over with your students in the next couple of days cover the big idea of the properties of arithmetic—specifically, that multiplication can be defined as repeated addition of the same quantity; also that multiplying a quantity like  $(x + 2)$  by 3 illustrates the distributive property. You will use the ideas underlying the distributive property to help students develop a deeper understanding of how the property can be used to both simplify expressions and equations, and solve other types of problems. Developing an understanding of the idea of distribution, and not just the memorization of a process, may help students avoid common misconceptions that interfere with understanding the distributive property (e.g. incorrect distribution), and serves as a foundation for understanding other concepts (e.g. solving equations, simplifying expressions).



## Rational Number Equivalence

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### Definition

- A rational number is any real number (negative, positive, or zero) that can be expressed as a quotient of two integers.
  - 0 can be expressed as  $\frac{0}{1}$
  - 8 can be expressed as  $\frac{8}{1}$
  - 4 can be expressed as  $\frac{-4}{1}$
  - .62 can be expressed as  $\frac{62}{100}$
  - $2\frac{3}{4}$  can be expressed as  $\frac{11}{4}$
- Rational numbers can have several meanings:
  - Fraction (part-whole):  $\frac{3}{4}$  means 3 out of 4
  - Division:  $\frac{3}{4}$  means 3 divided by 4
  - Ratio (part:part):  $\frac{3}{4}$  means 3 compared to 4
  - Measure:  $\frac{3}{4}$  means three-quarters of one unit
  - Multiplicative operator:  $\frac{3}{4}$  means three-quarters of (or times) a thing

### Questions to Consider

- How deep is my student's understanding of the idea that  $x/x = 1$ ?
- How are rational numbers used in the curriculum I teach in grade x?
- How is this intended to enrich or improve my understanding of the content and of how to teach the content?

### What "Big Idea" is this designed to address?

- Equivalent rational numbers may be created through the application of the multiplicative identity (multiplying any fraction by  $x/x$ )
- The multiplicative identity states that if you multiply any number by 1, and the resulting product is the same number.
- Any number divided by its own value, or written over itself in a fraction, is 1 (except 0).
- If you multiply any number by  $x/x$  the resulting product is equivalent to the original number

### Teaching Rational Number Equivalence

- Students are often taught to find equivalent fractions through the use of a procedure, and are not provided with an explanation as to why the procedure is mathematically sound.
- One of the goals of this project is to reduce the number of such facts that students (and you) need to memorize to a minimum number of simple ideas and to build more complex ideas from a few number of simple concepts.
- Using such a structure suggests that students will have to memorize less, and you will have to teach fewer ideas in depth and have an opportunity to apply them more broadly.

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Most students struggle for years trying to understand the concept of Rational Numbers. In part, this struggle arises because of the way the concept is taught. We have taken one aspect of Rational Numbers (developing equivalent rational expressions) and tied it to a concept that most students understand well (the multiplicative identity). Making such a linkage seems to avoid common misunderstandings that interfere with understanding Rational Numbers (e.g. "whatever you do to the top, you do to the bottom."), and serves as a foundation for understanding other concepts (e.g. solving equations, simplifying Rational Expressions, and operations on Rational Numbers).



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### Overview

The lessons you will be going over with your students in the next few days explore content centered on the big idea of solving equations. In this unit, students will apply the concept of equality to solve one-step equations for an unknown. You will use the ideas underlying inverse operations and the additive identity property to help students develop a deeper understanding of how to solve equations in order to find an unknown. Eventually, students will use this concept to substitute, simplify, rewrite and otherwise transform a variety of equations (e.g. linear, quadratic, exponential, and rational) into a form that they easily solve. In each case, the understanding of why one expression is equivalent to its transformation is essential to solving the presented problem.



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What are common errors students make when asked to find equivalent fractions, and why do they make these errors?

1. **Misconception:** “Whatever you do to one side, do to the other”

This statement is mathematically incorrect. Students are often taught this adage because it does work with some simple equations involving whole numbers, it does not however, work with all equations. It should therefore not be taught as a procedure for students to follow.

**Strategies:**  
Demonstrate why this type of thinking is incorrect.

For example, when solving the equation  $\frac{x}{3} + \frac{1}{4} = 4$ , it is permissible to

multiply the first term by and the second term by  $\frac{4}{4}$ , but one need not do anything to “the other side” of the equation.

The quintessential rule when teaching solving equations is: “whatever you do to one side, do to the other.” Although this may be true for most additions, subtractions, multiplications and divisions, it is not always true nor is it required in every instance.

- For example:
- One cannot divide both sides by zero.
  - Adding or subtracting zero on one side of an equation.
  - Trying to take the absolute value of or squaring both sides of an equation could also lead to inaccurate results.

- If asked to solve the equation  $3x = 2x$ , what would you do?
- If you say it is unsolvable, look again. In fact, if you try to divide both sides by “x,” you are left with  $3 = 2$ . Clearly this can’t be true and it might reinforce the notion that the equation has no solution.
  - Is it even allowable to divide both sides by x? The adage suggests that it is; however, division by x is not allowed if  $x = 0$  (division by zero is undefined!). So you can’t “do the same thing to both sides” here if  $x = 0$ .

- Let’s try another way. The equation  $3x = 2x$  can be rewritten as  $x+x+x = x+x$  because that is what multiplication means. Now we can:
- Subtract an x from both sides (the subtraction property of equality) to get  $x+x = x$
  - Subtract another x from both sides (or subtract 2x from both sides from the start) to get  $x = 0$
  - At this point, you can see that  $x = 0$ . Indeed, if we check our work by substituting 0 for x, we find that  $3 \bullet 0 = 2 \bullet 0$  since  $0 = 0$  (the identity property).

What we encourage teachers to avoid is giving students new rules to memorize, especially when the rules might lead to misconceptions.

2. **Misconception:** Students see the variable as only representing an unknown object

**Strategies:**  
A variable can represent any number (a variable) of things.

Note that a classic barrier to understanding the algebra is that students see the variable as only representing an unknown object.

In reality, a variable need not represent a concrete thing or single unknown. They can also represent any number (a variable) of things.

For example, when asked to represent an increase in the width of a square by two units, many students first want to know the original dimensions of the square. In reality, since the side of the square could be any number (x), the width of the new square could be represented as “x+2”.

3. **Misconception:** Students solving equations through the use of “mental math”

This often leads to incorrect solutions, something that can easily be avoided through the processes of explanation of work completed and the checking of solutions.

**Strategies:**  
Explanations help students understand the mathematical principles they are using to find their solutions.

- In the discussion of the Solving Equations unit we have used worked examples (mathematical proof) and advocate its use in instruction
- Worked examples (where more and more steps are successively removed from the end of a problem) have been demonstrated to help deepen student understanding (Sweller & Cooper, 1985).
  - Similarly, asking students to show and explain the mathematical “big ideas” that justify each step in a solution, have been shown to improve student understanding (Renkl, 2002).

While students may be able to solve equations mentally, they should show work (and explain steps) to develop understanding, reduce the recall mistakes (e.g. inappropriate fact family such as  $\frac{x}{12} = 4$ , so  $x = 3$ ), and minimize calculation errors ( $100 - x = 87$ ;  $x = 23$ ).

4. **Misconception:** Students “cancel out” the variable when solving an equation involving multiplication or division.

This often leads students to believe that x/x becomes 0 instead of 1 since it has been “cancelled.”

- Strategies:**  
It is important to make clear that to “isolate x” when we are multiplying or dividing, we need to multiply (or divide) by one (not zero).
- For this reason, we suggest teachers avoid using the word “cancel” when simplifying a rational expression or equation with students.
  - Instead it is probably better to indicate that the coefficient of the variable is one. The multiplicative identity then allows us to “ignore” the coefficient.
  - Just as the addition of “opposites” produces the identity number for addition (0), there is a way to use multiplication to produce the identity number for multiplication (1).

How would you show that the expression  $\frac{x}{y}$  (where  $y \neq 0$ ) can be rewritten as  $x \bullet \frac{1}{y}$ ?

One possible “worked example” demonstrating  $\frac{x}{y} = x \bullet \frac{1}{y}$ :

- |        |   |  |
|--------|---|--|
| Step 1 | $\frac{x}{y}, y \neq 0$                     | Given expression   |
| Step 2 | $\frac{x}{y} \bullet 1$                     | Multiplicative identity. The value of a quantity doesn’t change when multiplied by 1 so this is equivalent to the above. |
| Step 3 | $\frac{x}{y} \bullet \frac{1}{\frac{1}{y}}$ | Any non-zero number divided by itself is one. Since y is not zero, 1/y is a real number and is non-zero                  |
| Step 4 | $x \bullet \frac{1}{y}$                     | A number times its reciprocal is 1.  |
| Step 5 | $x \bullet \frac{1}{y}$                     | Division by 1 does not change the value of the dividend.   |





## Background Survey

This short survey will provide us with information regarding the instructors that are participating in the study. The answers to these questions will give insight into teacher's attitudes towards assessments, teaching and educational experience, and technology in the classroom. **Please answer each question and return with your first set of materials.** It should only take about 10 minutes to complete the survey.

To download the survey, click [here](#)

## Implementation Surveys

This short survey will provide us with information regarding how the instructors that are participating in the study chose to implement the PowerSource materials. The answers to these questions will give insight into what Checks for Understanding forms were used, the functionality of the Teacher Handbook, and how the Lesson 2 Worksheet was implemented. **Please answer each question and return with each set of materials.** It should only take about 10 minutes to complete each survey.

Download the Rational Number Equivalence survey [here](#)

Download the Properties of Arithmetic survey [here](#)

Download the Solving Equations survey [here](#)

Download the Review and Applications survey [here](#)



**Appendix E:**  
**Transfer Measure Items and Sources**

ITEM	Item Text
1	What do you need to add to eighty-three to make one hundred?
2	Write the fraction $\frac{3}{9}$ on your answer sheet in its simplest form.
3	There were two thousand people at a concert. Nine hundred and ninety-two of them were women. How many of the people were not women?
4	Write a fraction that is less than $\frac{4}{9}$ .
5	Write a different fraction that is equivalent to three-fifths.
6	$b = 14 + a$ . When $a$ equals 7, what is the value of $b$ ?
7	If $12n = 3621$ , then $n$ equals: a) 3 b) 7 c) 36 d) 63
8	Which of the following ratios is equivalent to the ratio of 6 to 4? a) 12 to 18 b) 12 to 8 c) 8 to 6 d) 4 to 6 e) 2 to 3
9	For all numbers $k$ , $k + k + k + k + k$ can be written as a) $k + 5$ b) $5k$ c) $k5$ d) $5(k + 1)$
10	
11	
12	
13	
14	
15	
16	16. Which of the following is equal to $6(x + 6)$ ? a) $x + 12$ b) $6x + 6$ c) $6x + 12$ d) $6x + 36$ e) $6x + 66$
17	Simplify using the distributive property. $y(y - 6) =$



18	How much change will John get back from \$5.00 if he buys 2 notebooks that cost \$1.80 each? a) \$1.40 b) \$2.40 c) \$3.20 d) \$3.60
19	The perimeter of a square is 36 inches. What is the length of one side of the square? a) 4 inches b) 6 inches c) 9 inches d) 18 inches
20	Which of the following numerical expressions gives the area of the rectangle below? a) $4 \cdot 6$ b) $4 + 6$ c) $2(4 \cdot 6)$ d) $2(4 + 6)$ e) $4 + 6 + 4 + 6$
21	What is the value of x in the triangle? a) $65^\circ$ b) $82^\circ$ c) $90^\circ$ d) $92^\circ$ e) $98^\circ$
22	
23	
24	
25	If $3 + w = b$ , then $w =$ a) 39 b) $b \cdot 3$ c) $b + 3$ d) $3 - b$ e) $b - 3$
26	
27	
28	
29	
30	
31	In which list of fractions are all of the fractions equivalent? a) 12 , 24 , 46 b) 23 , 46 , 812 c) 25 , 410 , 850 d) 34 , 46 , 68

32	<p>n is a number. When n is multiplied by 7, and 6 is then added, the result is 41.</p> <p>Which of these equations represents this relation?</p> <p>a) <math>7n + 6 = 41</math>  b) <math>7n + - 6 = 41</math>  c) <math>7n \cdot 6 = 41</math>  d) <math>7(n + 6) = 41</math></p>
33	<p>The diagram shows triangle PQR.</p> <p>Work out the sizes of angles a, b, and c.</p>
	<p>What would be your answer if you were asked to multiply <math>8 \cdot (x + 34)</math> ?</p> <p>a) <math>8x + 34</math>  b) <math>8 \cdot 34x</math>  c) <math>8x + 6</math>  d) <math>x + 6</math></p>
	<p>Write a fraction that has a denominator of 100 and is equivalent to <math>\frac{7}{20}</math> .</p>
	<p>Explain why the fraction <math>\frac{1}{2} \cdot \frac{3}{4}</math> is equivalent to the fraction <math>\frac{2}{3}</math>?</p>
	<p>What value of x makes the equation true?</p> <p><math>x - 9 = 32</math></p> <p>a) 23  b) 41  c) 32  d) 9</p>
	<p>Solve: <math>6n = 36</math></p> <p>a) 12  b) 2  c) 30  d) 6</p>
	<p>What is the value of p in the equation below ?</p> <p><math>14p = 4</math></p> <p>a) <math>p = 4</math>  b) <math>p = 16</math>  c) <math>p = 4 \cdot 14</math>  d) <math>p = 3 \cdot 34</math></p>
	<p>What is the next step to solve this equation?</p> <p><math>x - 7 = 13</math></p> <p>a) Subtract 7 from both sides  b) Add x to both sides  c) Add 7 to both sides  d) Subtract 13 from both sides</p>
	<p>Sam's uncle is 21 years older than Sam. His uncle is 42. What equation could you use to solve for Sam's age, s ?</p> <p>a) <math>s + 21 = 42</math>  b) <math>42 + 21 = s</math>  c) <math>s - 21 = 42</math>  d) <math>s - 42 = 21</math></p>
	<p>Which of the following shows the distributive property being used correctly to simplify the expression: <math>3(4) + 3(2)</math></p> <p>a) <math>3(4)(2)</math>  b) <math>3(4 + 2)</math>  c) <math>4(3 + 2)</math>  d) <math>4(2) + 2(2)</math></p>

Charlie can type 32 words per minute. At this rate, how long would it take him in minutes to type 128 words? a) 1 b) 3 c) 4 d) 2
--

SOURCE	CA Standard
QCA, key stage 3, p. 4, #2	NS 2.0
QCA, key stage 3, p. 4, #7	NS 2.4
QCA, key stage 3, p. 5, #19	NS 2.0; NS 2.3
TIMSS, grade 8, 2003, item number: m022012	NS 1.1
QCA, key stage 3, p. 8, #5	NS 2.1 AF 1.2
QCA, key stage 3, p. 4, #3	NS 1.3
NAEP, grade 8, 2003, #58	NS 1.2
TIMSS, grade 8, 1999, item number: p11	AF 1.0
QCA, key stage 3, tier 4-6, paper 2, p. 27, #26	
TIMSS, grade 8, 2003, item number: m032036	
QCA, key stage 3, p. 8, #13	
QCA, key stage 3, tier 4-6, paper 2, p. 27, #26	
QCA, key stage 3, tier 6-8, paper 2, p. 11, #10	
QCA, key stage 3, tier 6-8, paper 2, p. 11, #10	
NAEP, grade 8, 2005, #41	AF 1.3
QCA, key stage 3, tier 6-8, paper 2, p. 15, #14	AF 1.3

NAEP, grade 8, 2003, #7	NS 2.3
NAEP, grade 8, 2003, #10	AF 3.1
NAEP, grade 8, 2003, #34	AF 3.1
NAEP, grade 8, 2003, #32	MG 2.2
QCA, key stage 3, p. 8, #11	MG 2.2
QCA, key stage 3, tier 3-5, paper 1, p. 21, #21 Also in QCA, key stage 3, tier 4-6, paper 1, p. 15, #14	
QCA, key stage 3, tier 3-5, paper 1, p. 21, #21 Also in QCA, key stage 3, tier 4-6, paper 1, p. 15, #14	
NAEP, grade 8, 2003, #47	MR 3.3
QCA, key stage 3, tier 4-6, paper 2, p. 27, #25	
QCA, key stage 3, tier 4-6, paper 1, p. 20, #21	
TIMSS, grade 8, 1999, item number: L17	
QCA, key stage 3, tier 6-8, paper 2, p. 15, #14	
TIMSS, grade 8, 2003, item number: m022253	AF 1.3
TIMSS, grade 8, 1999, item number: N14	NS 1.1

TIMSS, grade 8, 1999, item number: B12	AF 1.0
QCA, key stage 3, tier 3-5, paper 1, p. 20, #20	MG 2.2
Adapted from PISA item	
Adapted from PISA item	
Adapted from PISA item	
Adapted from 6th Grade Benchmark Test--3-- Norwalk La Mirada--item 10	AF: 1.1
Adapted from 6th Grade Benchmark Test--3-- Norwalk La Mirada--item 11	AF: 1.1
Adapted from 6th Grade Benchmark Test--3-- Norwalk La Mirada--item 12	AF: 1.1
Adapted from 6th Grade Benchmark Test--3-- Norwalk La Mirada--item 14	AF: 1.1
Adapted from 6th Grade Benchmark Test--3-- Norwalk La Mirada--item 16	AF: 1.1
Adapted from 6th Grade Benchmark Test--3-- Norwalk La Mirada--item 24	AF: 1.3

Adapted from 6th Grade Benchmark Test--3-- Norwalk La Mirada--item 28	AF: 2.3
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**Appendix F:**  
**Transfer Measure Alignment: Standards and Focal Points**

6th Grade Standards	Domain	Checks for Understanding Item Numb		
		RNE	SE	RA
<b>Number Sense</b>				
1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:	<b>RNE, SE, RA</b>	RN-EX-12 RN-EX-15		
1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.	<b>RNE, RA</b>			
1.2 Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations ( $a/b$ , $a$ to $b$ , $a:b$ ).	<b>RA</b>			
1.3 Use proportions to solve problems (e.g., determine the value of $N$ if $4/7 = N/21$ , find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.	<b>RNE, SE, RA</b>	RN-BT-6 RN-BT-2 RN-EX-6ab RN-BT-5 RN-BT-6 RN-EX-18ab RN-BT-4	SE-BT-12 SE-FS-1 SE-BT-20	
1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.				
2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:	<b>RNE, SE, RA</b>	RN-BT-7 RN-BT-8 RN-BT-9 RN-BT-15 RN-BT-16 RN-BT-17	SE-BT-6 SE-BT-7 SE-BT-8 SE-BT-9 SE-BT-12 SE-BT-20 SE-EX-26 SE-EX-19ab SE-EX-27 SE-FS-1 SE-FS-3	
2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.	<b>RNE, SE, RA</b>	RN-EX-6ab RN-EX-13 RN-BT-15 RN-EX-17 RN-EX-18ab RN-EX-28ab	SE-EX-19ab SE-EX-26 SE-EX-27 SE-EX-28 SE-EX-29	

2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., $5/8 \div 15/16 = 5/8 \times 16/15 = 2/3$ ).	<b>RNE, SE, RA</b>			
2.3 Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.	<b>RNE, SE, RA</b>			
2.4 Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).		RN-EX-28ab		
<b>Algebra and Functions</b>				
1.0 Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:	<b>PA, SE, RA</b>		SE-WP-11abc SE-WP-12abc SE-WP-14	
1.1 Write and solve one-step linear equations in one variable.	<b>SE, RA</b>		SE-WP-11abc, SE-WP-12abc	
1.2 Write and evaluate an algebraic expression for a given situation, using up to three variables.	<b>SE, RA</b>			
1.3 Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process.	<b>PA, SE, RA</b>			
1.4 Solve problems manually by using the correct order of operations or by using a scientific calculator.				
2.0 Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:	<b>RA</b>			
2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).	<b>RA</b>			
2.2 Demonstrate an understanding that rate is a measure of one quantity per unit value of another quantity.	<b>RA</b>			
2.3 Solve problems involving rates, average speed, distance, and time.	<b>RA</b>	RN-WP-6, RN-WP-7		
3.0 Students investigate geometric patterns and describe them algebraically:	<b>RA</b>			
3.1 Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$ , $A = 1/2bh$ , $C = \pi d$ - the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).	<b>RA</b>			
3.2 Express in symbolic form simple relationships arising from geometry.				
<b>Measurement and Geometry</b>				
1.0 Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:				
1.1 Understand the concept of a constant such as $\pi$ ; know the formulas for the circumference and area of a circle.				

1.2 Know common estimates of $\pi$ (3.14; 22/7) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.				
1.3 Know and use the formulas for the volume of triangular prisms and cylinders (area of base $\times$ height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.				
2.0 Students identify and describe the properties of two-dimensional figures:	<b>SE, RA</b>			
2.1 Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.				
2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.	<b>SE, RA</b>		SE-EX-30	
2.3 Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).				
<b>Statistics, Data Analysis, and Probability</b>				
1.0 Students compute and analyze statistical measurements for data sets:				
1.1 Compute the range, mean, median, and mode of data sets.				
1.2 Understand how additional data added to data sets may affect these computations of measures of central tendency.				
1.3 Understand how the inclusion or exclusion of outliers affects measures of central tendency.				
1.4 Know why a specific measure of central tendency (mean, median) provides the most useful information in a given context.				
2.0 Students use data samples of a population and describe the characteristics and limitations of the samples:				
2.1 Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.				
2.2 Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.				
2.3 Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.				
2.4 Identify data that represent sampling errors and explain why the sample (and the display) might be biased.				
2.5 Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.				

3.0 Students determine theoretical and experimental probabilities and use these to make predictions about events:				
3.1 Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.				
3.2 Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).				
3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, 1- P is the probability of an event not occurring.				
3.4 Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.				
3.5 Understand the difference between independent and dependent events.				
<b>Mathematical Reasoning</b>				
1.0 Students make decisions about how to approach problems:	<b>RNE, PA, SE, RA</b>		SE-WP-11abc SE-WP-12abc SE-FS-1 SE-FS-3	
1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.	<b>RNE, PA, SE, RA</b>	RN-EX-12 RN-EX-15		
1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.	<b>RNE, PA, SE, RA</b>			
1.3 Determine when and how to break a problem into simpler parts.	<b>RNE, PA, SE, RA</b>			
2.0 Students use strategies, skills, and concepts in finding solutions:	<b>RNE, PA, SE, RA</b>			
2.1 Use estimation to verify the reasonableness of calculated results.				
2.2 Apply strategies and results from simpler problems to more complex problems.	<b>RNE, PA, SE, RA</b>			
2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.	<b>SE, RA</b>			

2.4 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.	<b>RNE, PA, SE, RA</b>	RN-EX-1ab RN-EX-6ab RN-EX-12 RN-EX-13 RN-EX-15 RN-EX-16ab RN-EX-17 RN-EX-18ab	SE-EX-19ab SE-EX-23ab SE-WP-11abc SE-WP-12abc SE-FS-1 SE-FS-3	
2.5 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.	<b>RNE, PA, SE, RA</b>	RN-EX-16ab	SE-EX-19ab SE-EX-23ab SE-EX-26 SE-EX-28 SE-WP-11abc SE-WP-12abc	
2.6 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.				
2.7 Make precise calculations and check the validity of the results from the context of the problem.	<b>RNE, PA, SE, RA</b>			
3.0 Students move beyond a particular problem by generalizing to other situations:	<b>RNE, PA, SE, RA</b>			
3.1 Evaluate the reasonableness of the solution in the context of the original situation.	<b>RNE, PA, SE, RA</b>	RN-EX-6ab RN-EX-13 RN-EX-17 RN-EX-18ab	SE-EX-26 SE-EX-27 SE-EX-28 SE-EX-29	
3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.	<b>RNE, PA, SE, RA</b>			
3.3 Develop generalizations of the results obtained and the strategies used and apply them in new problem situations.	<b>RNE, PA, SE, RA</b>			
<b>7th Grade Standards</b>	<b>Domain</b>			
<b>Number Sense</b>				
1.0 Students know the properties of, and compute with, rational numbers expressed in a variety of forms:	<b>RNE, SE, RA</b>			

1.1 Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.				
1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.	<b>RNE, RA</b>			
1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.	<b>RNE, RA</b>			
1.4 Differentiate between rational and irrational numbers.	<b>RNE, RA</b>			
1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.	<b>RNE, RA</b>			
1.6 Calculate the percentage of increases and decreases of a quantity.	<b>SE, RA</b>			
1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.				
2.0 Students use exponents, powers, and roots and use exponents in working with fractions:	<b>RNE, PA, RA</b>			
2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.				
2.2 Add and subtract fractions by using factoring to find common denominators.	<b>RNE, PA, RA</b>			
2.3 Multiply, divide, and simplify rational numbers by using exponent rules.				
2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.				
2.5 Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.	<b>SE, RA</b>			
<b>Algebra and Functions</b>				
1.0 Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:	<b>PA, SE, RA</b>			
1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).	<b>SE, RA</b>			
1.2 Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$ .	<b>SE, RA</b>			
1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.	<b>PA, RA</b>			
1.4 Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.	<b>SE, RA</b>			

1.5 Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.				
2.0 Students interpret and evaluate expressions involving integer powers and simple roots:	<b>RNE, PA, RA</b>			
2.1 Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.	<b>RNE, PA, RA</b>			
2.2 Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.				
3.0 Students graph and interpret linear and some nonlinear functions:				
3.1 Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.				
3.2 Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).				
3.3 Graph linear functions, noting that the vertical change (change in y- value) per unit of horizontal change (change in x- value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.				
3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.				
4.0 Students solve simple linear equations and inequalities over the rational numbers:	<b>SE, RA</b>			
4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.	<b>SE, RA</b>			
4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation.	<b>SE, RA</b>			
<b>Measurement and Geometry</b>				
1.0 Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:				
1.1 Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).				
1.2 Construct and read drawings and models made to scale.				



1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.				
2.0 Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:	<b>RNE, SE, RA</b>			
2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.	<b>SE, RA</b>			
2.2 Estimate and compute the area of more complex or irregular two-and three-dimensional figures by breaking the figures down into more basic geometric objects.				
2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.	<b>SE, RA</b>			
2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or $[1 \text{ ft}^2] = [144 \text{ in}^2]$ , 1 cubic inch is approximately 16.38 cubic centimeters or $[1 \text{ in}^3] = [16.38 \text{ cm}^3]$ ).	<b>RNE, RA</b>			
3.0 Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:				
3.1 Identify and construct basic elements of geometric figures (e.g., altitudes, mid-points, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.				
3.2 Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.				
3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.				
3.4 Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.				
3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.				

3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).				
<b>Statistics, Data Analysis, and Probability</b>				
1.0 Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:				
1.1 Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.				
1.2 Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).				
1.3 Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.				
<b>Mathematical Reasoning</b>				
1.0 Students make decisions about how to approach problems:	<b>RNE, PA, SE, RA</b>			
1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.	<b>RNE, PA, SE, RA</b>			
1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.	<b>RNE, PA, SE, RA</b>			
1.3 Determine when and how to break a problem into simpler parts.	<b>RNE, PA, SE, RA</b>			
2.0 Students use strategies, skills, and concepts in finding solutions:	<b>RNE, PA, SE, RA</b>			
2.1 Use estimation to verify the reasonableness of calculated results.				
2.2 Apply strategies and results from simpler problems to more complex problems.	<b>RNE, PA, SE, RA</b>			
2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.				
2.4 Make and test conjectures by using both inductive and deductive reasoning.				
2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.	<b>RNE, PA, SE, RA</b>			

2.6 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.	RNE, PA, SE, RA			
2.7 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.				
2.8 Make precise calculations and check the validity of the results from the context of the problem.	RNE, PA, SE, RA			
3.0 Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:	RNE, PA, SE, RA			
3.1 Evaluate the reasonableness of the solution in the context of the original situation.	RNE, PA, SE, RA			
3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.	RNE, PA, SE, RA			
3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.	RNE, PA, SE, RA			
<b>8th Grade Standards</b>	<b>Domain</b>			
<b>Algebra I</b>				
1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:				
1.1 Students use properties of numbers to demonstrate whether assertions are true or false.	PA, RA			
2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.	RNE, RA			
3.0 Students solve equations and inequalities involving absolute values.	SE, RA			
4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$ .	PA, SE, RA			
5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.	PA, SE, RA			
6.0 Students graph a linear equation and compute the x- and y- intercepts (e.g., graph $2x + 6y = 4$ ). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$ ).				

7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.	<b>SE, RA</b>			
8.0 Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.				
9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.				
10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.	<b>RNE, SE, RA</b>			
11.0 Students apply basic factoring techniques to second-and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.	<b>RNE, RA</b>			
12.0 Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.	<b>RNE, RA</b>			
13.0 Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.	<b>RNE, SE, RA</b>			
14.0 Students solve a quadratic equation by factoring or completing the square.				
15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.	<b>SE, RA</b>			
16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.				
17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.				
18.0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.				
19.0 Students know the quadratic formula and are familiar with its proof by completing the square.				
20.0 Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.				
21.0 Students graph quadratic functions and know that their roots are the x-intercepts.				

22.0 Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.				
23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.				
24.0 Students use and know simple aspects of a logical argument:				
24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.				
24.2 Students identify the hypothesis and conclusion in logical deduction. 24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.				
25.0 Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:	<b>PA, RA</b>			
25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.	<b>PA, RA</b>			
25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.	<b>PA, RA</b>			
25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.				

er	Transfer Measure Item Number	NCTM Focal Points	Mathematical Standard for the Algebra Readiness
PA			
		NO1	
	#4, #31		X
	#8	NO2	
,	#7	NO2	
			X
	#1, #3	NO1, NO2	X
	#5	NO1	X

## 6th Grade California Math Standards

		NO1	<b>X</b>
	#3, #18	NO1	
	#2		
PA-BT-32 PA-BT-33 PA-WP-2 PA-WP-3 PA-EX-8	#9, #32	Alg1	<b>X</b>
		Alg1	<b>X</b>
	#6		
PA-FS-1, PA-EX-11ab, PA-EX-12ab, PA-WP-2, PA-WP-3, PA-FS-2	#16, #17, #30	Alg2	
		NO2	
		NO2	
	#19, #20		
		Geo	

## 6th Grade California Math Standards

[illegible]



6th Grade California Math Standards

PA-FS-1, PA-FS-2,			<b>X</b>
PA-BT-1, PA-BT-13,PA-BT-20, PA-BT-28, PA-BT-31,			<b>X</b>
			<b>X</b>
RN-BT-8, RN-BT-9, RN-BT-16, RN-BT-17			<b>X</b>
	#10 (maybe), #12, #13(maybe)		<b>X</b>
			<b>X</b>
			<b>X</b>
RN-EX-1ab, RN-EX-16ab			<b>X</b>

6th Grade California Math Standards

PA-BT-24 PA-BT-27 PA-BT-32 PA-BT-33 PA-EX-8 PA-EX-11ab PA-EX-12ab PA-WP-2 PA-WP-3		Alg1	X
PA-BT-24 PA-BT-27 PA-EX-8 RN-EX-1ab,			X
			X
			X
PA-WP-2 PA-WP-3 PA-EX-8			X
			X
			X
	#25		X
			<b>Mathematical Standard for the Algebra Readiness Program</b>
	#3		

	#3		X
			X
			X
			X
	#17		
			X
	#30		
			X

	#14, #15		X
	#14, #15		
			X
			X
	#10		
	#23, #24		X
			X
			X

[illegible]

## 6th Grade California Math Standards

[illegible]

			X
			X
			X
			X
			X
			X
			X
			Mathematical Standard for the Algebra Readiness Program
			X
	#17		X
	#26, #27, #28		X

[illegible]



	#26		



**Appendix G:**  
**Teacher Surveys**

# Teacher Background Survey Questions

Teacher ID: \_\_\_\_\_

1. What is your age? (in years) \_\_\_\_\_
2. Are you female or male? (Circle one)  
Female                      Male
3. Please select the **one** title that best describes your current position.  
☐ Classroom teacher (1-2 years math teaching experience)  
☐ Classroom teacher (3 or more years math teaching experience)  
☐ Out-of-classroom position such as math coach or district math personnel  
☐ Other, please specify \_\_\_\_\_
4. As of the end of the 2006-2007 school year, how many years will you have been teaching?

Total number of years teaching (any subject) \_\_\_\_\_  
Total number of years teaching math \_\_\_\_\_  
Total number of years teaching middle school math \_\_\_\_\_

5. Do you have National Board Certification (please circle one)  
Yes                      No

If yes, in what area? \_\_\_\_\_

6. Have you completed a Bachelor's degree? (circle one)  
Yes                      No

If yes, in what fields(s). Please specify major(s) and minor(s)

\_\_\_\_\_

7. Have you completed a Master's degree? (circle one)  
Yes                      No

If yes, in what fields(s). If yes, in what field(s)? Please specify specialization(s). For example, a Master's degree in education might be in curriculum/instruction or administration.

\_\_\_\_\_

8. What type of teaching credentials(s) do you hold? (Check all that apply)

- ☐ Clear Credential
- ☐ Preliminary Credential
- ☐ Emergency Credential
- ☐ Multiple Subjects
- ☐ Single Subject (math)
- ☐ Single subject (other than math)
- ☐ Currently in a program to obtain my teaching credential
- ☐ None
- ☐ Other, please specify \_\_\_\_\_

9. How many undergraduate or graduate courses have you taken at a college or university in the following areas?

Mathematics courses \_\_\_\_\_

Methods of teaching mathematics courses \_\_\_\_\_

Education courses \_\_\_\_\_

10. Considering your training and experience in both mathematics content and instruction, how prepared do you feel to teach these topics?  
(Please check one box for each topic)

	<b>1 Not Prepared</b>	<b>2 Prepared</b>	<b>3 Very Prepared</b>
Additive identity			
Distributive property			
Equivalent fractions			
Multiplicative identity			
Proportions			
Rational numbers			
Simplifying			
Solving equations (one variable)			
Solving equations (two variables)			

11. Compared to the average math teacher at the grade level you teach/taught, how would you rate your level of knowledge of the following math content areas? (Please check one box for each content area.)

	<b>1 Very Low</b>	<b>2 Low</b>	<b>3 Average</b>	<b>4 High</b>	<b>5 Very High</b>
Equivalence of rational numbers					
Distributive property					
Principles for solving equations					

12. Please rate your level of expertise in each of the items below. For each item check ONE (none, novice, adequate, good, expert)

	<b>1 none</b>	<b>2 novice</b>	<b>3 adequate</b>	<b>4 good</b>	<b>5 expert</b>
Justify the distributive property for natural numbers by using multiplication as repeated addition					
Explain why when adding fractions with common denominators, the numerators are added and the denominators stay the same.					
Explain why any non-zero number divided by it self is 1, and how this can be used with the multiplicative identity to scale ratios.					
Explain how the distributive law is used to simplify the sum of many linear, single variable equations.					
Explain how to solve a simplified, linear, single variable equation by using inverses of arithmetic operations and equivalence operations on both sides.					
Solving an equation means finding all possible values of given variables that make the equation true.					
Multiplying any number by 1, the multiplicative identity, results in a product that is the original number.					
Equivalence is a fundamental property of rational numbers: equivalent fractions, percents, and decimals all name the same relationship between two values.					

13. How often do you have the following types of interactions with other teachers? (Fill in check on box for each topic.)

	<b>1 Never or almost never</b>	<b>2 A few times per month</b>	<b>3 A few times per week</b>	<b>4 Daily or almost daily</b>
Discussion about how to teach a particular concept				
Working on preparing instructional materials				
Visits to another teacher's classroom to observe his/her teaching				
Informal observations of my classroom by another teacher				

14. In the past three years, have you participated in any of the following general professional development trainings? (Please check all that apply)

- ☐ Mathematics content
- ☐ Mathematics pedagogy/instruction
- ☐ Mathematics curriculum
- ☐ Integrating information technology into mathematics
- ☐ Improving students' critical thinking or problem solving skills
- ☐ None
- ☐ Other, please specify \_\_\_\_\_

15. In the past three years, have you participated in any of the following professional development trainings about assessment? (Please check all that apply)

- ☐ Using student assessment to evaluate instructional effectiveness
- ☐ Using student assessment to evaluate student learning
- ☐ Using student assessment to tailor instruction to students' skill level
- ☐ Learning about standards-based assessment
- ☐ None
- ☐ Other, please specify \_\_\_\_\_

16. How many **total hours** have you spent on in-service/professional development education in the 2006-2007 school years? Include attendance at such things as professional meetings, workshops, and conferences as well as any formal courses for which you have received college credit.

17. Please use 8 hours as the standard for a full day in-service and 40 hours as the standard for a week-long all day in-service or training.

Math content in-service hours \_\_\_\_\_  
 Methods for teaching math in- service hours \_\_\_\_\_  
 Other math topics in-service \_\_\_\_\_

18. To what extent do you agree or disagree with each of the following statements?  
 (Please check one box for each statement)

	<b>1 Disagree a lot</b>	<b>2 Disagree</b>	<b>3 Agree</b>	<b>4 Agree a lot</b>
More than one representation (picture, concrete material, symbols, etc.) must be used in teaching a mathematics topic				
Mathematics should be learned as sets of algorithms or rules that cover all possibilities				
Solving mathematics problems often involves hypothesizing, estimating, testing, and modifying findings				
Learning mathematics mainly involves memorizing				
There are different ways to solve most mathematical problems				
Few new discoveries in mathematics are being made				
Modeling real-world problems is essential to teaching mathematics				



19. Do you use a textbook(s) with your math class? (Circle one)

Yes

No

If yes, please write 1) publisher and 2) the title of each book you use. If you use more than one textbook, please write the primary textbook first.

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20. How do you use a textbook (s) in teaching mathematics to you math class (Please check one)

☐ As the primary basis for my lessons

☐ As supplementary resources

☐ Other, please specify \_\_\_\_\_

21. How often do your students do the following activities?

(Please check one box for each statement)

	<b>1 Never</b>	<b>2 Rarely</b>	<b>3 Weekly</b>	<b>4 Daily</b>
Reviewing/grading homework				
Listening to lecture-style presentations				
Working problems with your guidance				
Working problems on their own without your guidance				
Listening to you re-teach and clarify content/procedures				
Taking tests or quizzes				
Working in groups to solve problems				

22. How often do you include the following types of questions in your mathematics tests of examinations? (Please check one box for each statement)

	<b>1</b> <b>Always/ almost always</b>	<b>2</b> <b>Sometimes</b>	<b>3</b> <b>Never or almost never</b>
Questions involving application of mathematical procedures			
Questions involving searching for patterns and relationships			
Questions requiring written explanation or justifications			

23. Please indicate your opinion about each of the statements below relating to the kinds of things that may create difficulties for teachers in their school activities. (Please check one box for each statement)

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
	<b>Not at all</b>		<b>Very Little</b>		<b>Some Influence</b>		<b>Quite a Bit</b>		<b>A Great Deal</b>
1. To what extent can you use a variety of assessment strategies?									
2. To what extent can you provide an alternative explanation or example when students are confused?									
3. To what extent can you craft good questions for your students?									
4. How well can you implement alternative strategies in your classroom?									
5. How well can you respond to difficult questions from your students?									

6. How much can you do to adjust your lessons to the proper level for individual students?									
7. To what extent can you gauge student comprehension of what you have taught?									
8. How well can you provide appropriate challenges for very capable students?									
17. How much can you do to get students to believe they can do well in schoolwork?									
18. How much can you do to help your students' value learning?									
19. How much can you do to motivate students who show low interest in schoolwork?									
20. How much can you assist families in helping their children do well in school?									
21. How much can you do to improve the understanding of a student who is failing?									
22. How much can you do to help your students think critically?									
23. How much can you do to foster student creativity?									
24. How much can you do to get through to the most difficult students?									

24. Other comments/questions

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Thank you for completing the survey!

# Teacher Background Survey Questions

Teacher ID: \_\_\_\_\_

1. Considering your training and experience in both mathematics content and instruction, how prepared do you feel to teach these topics?  
(Please check one box for each topic)

	<b>1 Not Prepared</b>	<b>2 Prepared</b>	<b>3 Very Prepared</b>
Additive identity			
Distributive property			
Equivalent fractions			
Multiplicative identity			
Proportions			
Rational numbers			
Simplifying			
Solving equations (one variable)			
Solving equations (two variables)			

2. Compared to the average math teacher at the grade level you teach/taught, how would you rate your level of knowledge of the following math content areas? (Please check one box for each content area.)

	<b>1 Very Low</b>	<b>2 Low</b>	<b>3 Average</b>	<b>4 High</b>	<b>5 Very High</b>
Equivalence of rational numbers					
Distributive property					
Principles for solving equations					

3. Please rate your level of expertise in each of the items below. For each item check ONE (none, novice, adequate, good, expert)

	<b>1 none</b>	<b>2 novice</b>	<b>3 adequate</b>	<b>4 good</b>	<b>5 expert</b>
Justify the distributive property for natural numbers by using multiplication as repeated addition					
Explain why when adding fractions with common denominators, the numerators are added and the denominators stay the same.					
Explain why any non-zero number divided by it self is 1, and how this can be used with the multiplicative identity to scale ratios.					
Explain how the distributive law is used to simplify the sum of many linear, single variable equations.					
Explain how to solve a simplified, linear, single variable equation by using inverses of arithmetic operations and equivalence operations on both sides.					
Solving an equation means finding all possible values of given variables that make the equation true.					
Multiplying any number by 1, the multiplicative identity, results in a product that is the original number.					
Equivalence is a fundamental property of rational numbers: equivalent fractions, percents, and decimals all name the same relationship between two values.					

4. How often do you have the following types of interactions with other teachers? (Fill in check on box for each topic.)

	<b>1 Never or almost never</b>	<b>2 A few times per month</b>	<b>3 A few times per week</b>	<b>4 Daily or almost daily</b>
Discussion about how to teach a particular concept				
Working on preparing instructional materials				
Visits to another teacher's classroom to observe his/her teaching				
Informal observations of my classroom by another teacher				

5. To what extent do you agree or disagree with each of the following statements? (Please check one box for each statement)

	<b>1 Disagree a lot</b>	<b>2 Disagree</b>	<b>3 Agree</b>	<b>4 Agree a lot</b>
More than one representation (picture, concrete material, symbols, etc.) must be used in teaching a mathematics topic				
Mathematics should be learned as sets of algorithms or rules that cover all possibilities				
Solving mathematics problems often involves hypothesizing, estimating, testing, and modifying findings				
Learning mathematics mainly involves memorizing				
There are different ways to solve most mathematical problems				

Few new discoveries in mathematics are being made				
Modeling real-world problems is essential to teaching mathematics				

6. How often do your students do the following activities?  
(Please check one box for each statement)

	<b>1 Never</b>	<b>2 Rarely</b>	<b>3 Weekly</b>	<b>4 Daily</b>
Reviewing/grading homework				
Listening to lecture-style presentations				
Working problems with your guidance				
Working problems on their own without your guidance				
Listening to you re-teach and clarify content/procedures				
Taking tests or quizzes				
Working in groups to solve problems				

7. How often do you include the following types of questions in your mathematics tests of examinations? (Please check one box for each statement)

	<b>1 Always/ almost always</b>	<b>2 Sometimes</b>	<b>3 Never or almost never</b>
Questions involving application of mathematical procedures			
Questions involving searching for patterns and relationships			



Questions requiring written explanation or justifications			
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8. Please indicate your opinion about each of the statements below relating to the kinds of things that may create difficulties for teachers in their school activities. (Please check one box for each statement)

	1	2	3	4	5	6	7	8	9
	Not at all		Very Little		Some Influence		Quite a Bit		A Great Deal
1. To what extent can you use a variety of assessment strategies?									
2. To what extent can you provide an alternative explanation or example when students are confused?									
3. To what extent can you craft good questions for your students?									
4. How well can you implement alternative strategies in your classroom?									
5. How well can you respond to difficult questions from your students?									
6. How much can you do to adjust your lessons to the proper level for individual students?									
7. To what extent can you gauge student comprehension of what you have taught?									
8. How well can you provide appropriate challenges for very capable students?									

17. How much can you do to get students to believe they can do well in schoolwork?									
18. How much can you do to help your students' value learning?									
19. How much can you do to motivate students who show low interest in schoolwork?									
20. How much can you assist families in helping their children do well in school?									
21. How much can you do to improve the understanding of a student who is failing?									
22. How much can you do to help your students think critically?									
23. How much can you do to foster student creativity?									
24. How much can you do to get through to the most difficult students?									

9. Other comments/questions

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Thank you for completing the survey!



Teacher Survey-Rational Number Equivalence  
Group 1

1. Teacher ID: \_\_\_\_\_
  
2. When you gave the first *Checks for Understanding* to your students, had you already covered any of the concepts addressed within (in your regular school curriculum)?  
Yes                      No
  
3. Briefly explain (if applicable) how you changed your instruction on rational number equivalence after administering, and reviewing the *Checks for Understanding*:  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
  
4. Did you feel that you had enough time to complete all that was asked of you?  
Yes                      No
  
5. Did you use lesson 1 in the *Teacher Handbook*?    Yes    No
  
6. If yes, please rate how closely you adhered to the sample lesson presented in the Teacher Handbook:  

Not at all		Somewhat		Very closely
1	2	3	4	5

7. Did you use lesson 2 in the *Teacher Handbook*?      Yes   No

8. If you used the lessons, approximately how long did each one of them take? (in minutes)

Lesson 1: \_\_\_\_\_

Lesson 2: \_\_\_\_\_

9. If applicable, please, briefly, describe how you used Lesson 2 in the *Teacher Handbook*.

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10. What, if, any difficulties did you encounter in using either the *Checks for Understanding* or the lessons in the *Teacher Handbook*? (please describe)

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11. If you used them, did you feel as if the lessons in the *Teacher Handbook* increased your students' understanding of the topic?

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12. Do you have any suggestions about how any of the materials (assessments, Lessons, teacher instructions) could be improved?

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13. Having taught rational number equivalence, how confident are you that your students understand the concepts?

Not at all		Somewhat		Very
1	2	3	4	5

THANK YOU FOR COMPLETING THIS SURVEY!

If you have any comments about the questions, please write them in the space below.

Teacher Survey-Properties of Arithmetic  
Group 1

1. Teacher ID: \_\_\_\_\_
  
2. When you gave the first *Checks for Understanding* to your students, had you already covered any of the concepts addressed within (in your regular school curriculum)?  
Yes                      No
  
3. Briefly explain (if applicable) how you changed your instruction on the properties of arithmetic—focus on the distributive property— after administering, and reviewing the *Checks for Understanding*:  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
  
4. Did you feel that you had enough time to complete all that was asked of you?  
Yes                      No
  
5. Did you use lesson 1 in the *Teacher Handbook*?    Yes    No
  
6. If yes, please rate how closely you adhered to the sample lesson presented in the Teacher Handbook:  

Not at all		Somewhat		Very closely
1	2	3	4	5

7. Did you use lesson 2 in the *Teacher Handbook*?      Yes    No

8. If you used the lessons, approximately how long did each one of them take? (in minutes)

Lesson 1: \_\_\_\_\_

Lesson 2: \_\_\_\_\_

9. If applicable, please, briefly, describe how you used Lesson 2 in the *Teacher Handbook*.

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10. What, if, any difficulties did you encounter in using either the *Checks for Understanding* or the lessons in the *Teacher Handbook*? (please describe)

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11. If you used them, did you feel as if the lessons in the *Teacher Handbook* increased your students' understanding of the topic?

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12. Do you have any suggestions about how any of the materials (assessments, Lessons, teacher instructions) could be improved?

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13. Having taught the distributive property, how confident are you that your students understand the concepts?

Not at all		Somewhat		Very
1	2	3	4	5

THANK YOU FOR COMPLETING THIS SURVEY!

If you have any comments about the questions, please write them in the space below.

Teacher Survey-Solving Equations  
Group 1

1. Teacher ID: \_\_\_\_\_
  
2. When you gave the first *Checks for Understanding* to your students, had you already covered any of the concepts addressed within (in your regular school curriculum)?  
Yes                      No
  
3. Briefly explain (if applicable) how you changed your instruction on the properties of arithmetic—focus on the distributive property— after administering, and reviewing the *Checks for Understanding*:  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
  
4. Did you feel that you had enough time to complete all that was asked of you?  
Yes                      No
  
5. Did you use lesson 1 in the *Teacher Handbook*?    Yes    No
  
6. If yes, please rate how closely you adhered to the sample lesson presented in the Teacher Handbook:  

Not at all		Somewhat		Very closely
1	2	3	4	5

7. Did you use lesson 2 in the *Teacher Handbook*?      Yes    No

8. If you used the lessons, approximately how long did each one of them take? (in minutes)

Lesson 1: \_\_\_\_\_

Lesson 2: \_\_\_\_\_

9. If applicable, please, briefly, describe how you used Lesson 2 in the *Teacher Handbook*.

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10. What, if, any difficulties did you encounter in using either the *Checks for Understanding* or the lessons in the *Teacher Handbook*? (please describe)

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11. If you used them, did you feel as if the lessons in the *Teacher Handbook* increased your students' understanding of the topic?

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12. Do you have any suggestions about how any of the materials (assessments, Lessons, teacher instructions) could be improved?

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13. Having taught the distributive property, how confident are you that your students understand the concepts?

Not at all		Somewhat		Very
1	2	3	4	5

THANK YOU FOR COMPLETING THIS SURVEY!

If you have any comments about the questions, please write them in the space below.

Teacher Survey - Review & Applications  
Group 1

1. Teacher ID: \_\_\_\_\_
  
2. When you gave the first *Checks for Understanding* to your students, had you already covered any of the concepts addressed within (in your regular school curriculum)?  
Yes                      No
  
3. Briefly explain (if applicable) how you changed your instruction on the concepts found within the review and applications handbook after administering, and reviewing the *Checks for Understanding*:  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
  
4. Did you feel that you had enough time to complete all that was asked of you?  
Yes                      No
  
5. Did you use lesson 1 in the *Teacher Handbook*?    Yes    No
  
6. If yes, please rate how closely you adhered to the sample lesson presented in the Teacher Handbook:  

Not at all		Somewhat		Very closely
1	2	3	4	5

7. Did you use lesson 2 in the *Teacher Handbook*?      Yes    No

8. If you used the lessons, approximately how long did each one of them take? (in minutes)

Lesson 1: \_\_\_\_\_

Lesson 2: \_\_\_\_\_

9. If applicable, please, briefly, describe how you used Lesson 2 in the *Teacher Handbook*.

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10. What, if, any difficulties did you encounter in using either the *Checks for Understanding* or the lessons in the *Teacher Handbook*? (please describe)

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11. If you used them, did you feel as if the lessons in the *Teacher Handbook* increased your students' understanding of the topic?

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12. Do you have any suggestions about how any of the materials (assessments, Lessons, teacher instructions) could be improved?

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13. Having taught the material in the review and applications handbook, how confident are you that your students understand the concepts?

Not at all		Somewhat		Very
1	2	3	4	5

THANK YOU FOR COMPLETING THIS SURVEY!

If you have any comments about the questions, please write them in the space below.





**Appendix H:**  
**Interview and Observation Measures**

## Teacher Interview Protocol 2007/2008

Interviewer:
Date of Interview:
Big Idea Unit:
Teacher Name:
Teacher ID:
Grade level:
Name of School:
School District:
Start Time:
End Time:
Class Size:

**Introductory Remarks:**

Thank you for taking the time to participate in the classroom observation and teacher interview study. This interview will take approximately 20-30 minutes to complete. I just want to remind you again that this interview is being recorded and I want to make sure that this is still okay with you.

As you may have already heard, we are conducting these interviews with teachers who participated in the UCLA PowerSource/CRESST Study. The information we gather will help members of the research team improve professional development trainings and materials for the 2008-2009 school year. Your responses will be kept confidential and will be used for this purpose. Do you have any questions before we begin?

## Teacher Classroom Observation

I. Implementation of Big Ideas/Curriculum			Teacher Activities						
Content Covered <small>Check for each time period content area is addressed at 5 minute intervals</small>	Example	Domain	Frequency (in minutes)						Comments
			5	15	25	35	45	55	
"It (a number or variable) goes away"		SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
"Whatever you do to one side..."		SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Additive Identity	$a + 0 = a$	SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Additive Inverse	$a + -a = 0$	SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Commutative Properties	$a + b = b + a$	SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Inverse Operations	e.g. addition is inverse of subtraction	SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Properties of Equality (addition, multiplication)		SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Any non-zero number divided by itself is 1	$a/a = 1$ ( $a \neq 0$ )	RNE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Cross Multiply		RNE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Multiply the "top & bottom" by the same number	$\frac{1}{2} \times 2 = 2/4$	RNE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Area/Array model	*** *** ***	PA	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Distribute multiplier to each addend		PA	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Repeated Addition model	$3 \times 4 = 4 + 4 + 4 +$ $4$	PA	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
"Cancel"		RNE/SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Multiplicative Identity	$x \bullet 1 = x$ , $x \neq 0$	RNE/SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Multiplicative Inverse		RNE/SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Multiply by a number		RNE/SE	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Division is the same as multiplication by the reciprocal (invert & multiply)		RNE/SE/ PA	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Other			<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
TOTAL									

## Teacher Classroom Observation

II. Teacher Behaviors							
Please indicate if activity is occurring (at 5 minute intervals)	Frequency (in minutes)						Comments
	5	15	25	35	45	55	
Teacher discusses misconceptions based on PowerSource results	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Teacher demonstrates correct solution methods for a problem	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Teacher demonstrates incorrect solution methods for a problem	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Teacher provides procedural feedback (i.e., answer is right or wrong)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Teacher provides procedural feedback plus correction (i.e., answer is right or wrong and correct answer)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Teacher questions for understanding (i.e., teacher uses questioning to help better understand reasons for a student's answer)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Teacher uses guided problem solving (i.e., uses questions to help students find the correct responses themselves)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Teacher specifically references Checks for Understanding results	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Teacher specifically references other test/quiz results	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Teacher specifically references homework results	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
<b>TOTAL</b>							

## Teacher Classroom Observation

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III. Lesson Structure		
Please estimate percentage of time spent on the following activities	Percentage (%)	Comments
Individual work (students working independently)	%	
Small groups (3+ students)	%	
Pairs	%	
Whole group/teacher lectures to class	%	
Whole group/teacher leads class in discussion	%	
Whole group/teacher questions students	%	
Whole group/teacher provides feedback to students	%	
Other (list)	%	

## Teacher Classroom Observation

4

IV. Use of PowerSource and other materials		
Please indicate if PowerSource of other materials were utilized	Yes (check)	Comments
Teacher used lesson from handbook directly as written	<input type="checkbox"/>	
Teacher integrated handbook lesson with other materials (e.g., self-designed, other curriculum)	<input type="checkbox"/>	
Teacher did not use lesson from handbook	<input type="checkbox"/>	
Did the teacher cover all concepts in the handbook?	<input type="checkbox"/>	
Did the teacher cover the concepts in the same order as presented in the handbook?	<input type="checkbox"/>	
Which of the following materials were used?		
- Lesson 2 Worksheet	<input type="checkbox"/>	
- Tally sheets	<input type="checkbox"/>	
- Powerpoint slides (PowerSource)/overheads	<input type="checkbox"/>	
- Powerpoint slides (non-PowerSource)/overheads	<input type="checkbox"/>	
- Manipulatives	<input type="checkbox"/>	
- Computers/handhelds	<input type="checkbox"/>	
- Other (please describe)	<input type="checkbox"/>	

IV. Additional Comments
Additional Comments





## Formative Assessments

1(a). How did you use the results from the 1<sup>st</sup> Checks for Understanding to plan your instruction for Lesson 1?

1(b). How did you use the results of the 2<sup>nd</sup> Checks for Understanding to plan Lesson 2?

2(a). What student strengths emerged from (understanding skills and knowledge) the 1<sup>st</sup> Check for Understanding?

2(b). Are there other sources of data (such as other quizzes tests other formative assessments homework or reports from tutors) you use to determine where your students' strengths lie?

3(a). What learning challenges emerged from the results of the 2<sup>nd</sup> Check for Understanding?

3(b). Are there other sources of data (such as other quizzes tests other formative assessments homework or reports from tutors) you use to determine where your students' areas of difficulty lie?

4. What strategies do you think helps student's grasp Big ideas? (instruction of area/array, repeated addition model, other procedural or conceptual knowledge)?

## Instructional Plan Development

(5). What specifically do you use to develop your instruction plan?

(6). Do you find the PowerSource materials easy/hard to use? (Handbooks assessments overheads)? (interviewer include prompts to help teacher expand and understand nature of the questions)

7. How much time did you use preparing for this PowerSource lesson?

8. How much time do you use to develop an average (non-PowerSource) lesson?

## Professional Development

9. What makes PowerSource materials used in the professional development trainings easy (hard) to use? (PD content, format, lesson organization, handbook examples, overheads, forms)?

10. Do you have any specific recommendations for improving/modifying the PowerSource materials?

11. How well do you feel PowerSource materials align with existing district curricula?

12. What knowledge from Professional Development trainings helped you implement Big Ideas in the classroom? (research, instructional methods for Big Ideas, scoring rubrics, common student errors, comparable items on Checks for Understanding).

13. What information from the Professional Development trainings helped you use assessment information in a formative way? (discussion of student responses, frequencies, percents, common student errors, scoring rubrics, instructional methods for Big Ideas, research behind Big Ideas, comparable items on Checks for Understanding).

14. What do you feel would improve the Professional Development Trainings?

15. What or how well does Professional Development content align(s) with existing district curricula?

16. How well prepared do you feel implementing PowerSource and why? What contributes to your preparedness/lack of preparedness?

**Additional Comments**

17. Is there anything else that you would like to add or mention that you did not get a chance to say?