A Note on Knowledge-Based Model Construction in Educational Assessment

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A NOTE ON KNOWLEDGE-BASED MODEL CONSTRUCTION IN EDUCATIONAL ASSESSMENT[1]

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Abstract

This paper discusses the concept of "conformability" between graphical model fragments used with knowledge-based model construction in educational assessment. One fragment contains the persistent student model, or variables which are the primary targets of inference. Fragments corresponding to tasks contain one or more observable variables, which are children of one or more student model variables. These latter evidence models are attached to the student model, evidence is absorbed as observations are made, and the fragment is released. The consideration addressed here is ensuring that the boundary between the compiled student model and evidence model fragments supports coherent propagation of information. The issue is illustrated with a numerical example, and approaches for forcing comparability are suggested.

Keywords: Bayes nets, educational assessment, graphical models, knowledge-based model construction.

[1] We thank Ed Herskovitz for comments and discussions on this topic.

Introduction

Bayesian inference networks (BINs; also referred to as causal probability networks and influence diagrams) are systems for representing and carrying out probability-based inference in sets of interrelated variables (Almond, 1995; Jensen, 1996; Pearl, 1988). Starting from a directed acyclic graphical (DAG) representation of the dependence relationships among the variables, a representation of the joint probability distribution is constructed in terms of the distributions of subsets of interrelated variables (cliques) by distributions of intersecting subsets (clique intersections). A "junction tree" of cliques with the property of single-connectedness enables coherent propagation of new information throughout the network, using only calculations local to cliques and their immediate neighbors.

In this context, "knowledge based model construction" (KBMC) means using information about context and goals to BINs that suit local purposes, rather than attempting to construct an all-encompassing BIN that must apply to all contexts and goals (Breese, Goldman, & Wellman, 1994). In educational assessment, KBMC can be applied with BIN fragments that pertain to the student model, which contains the primary targets of inference, and fragments specific to tasks which provide evidence about student model variables from students' behaviors or productions (Almond & Mislevy, in press).

We assume the Lauritzen-Spiegelhalter (1988) algorithm as the basic method of propagating information, including the provision of entering likelihoods as "virtual evidence" as described by Pearl (1988, pp. 44-46). This note shows how to construct BIN fragments for the persistent student model, or SM, and attachable evidence model, or EMs, which are *conformable* in the following sense: the SM variables which form the shared boundary of the SM and any given EM appear in a clique in the junction trees of the compiled BIN fragments corresponding to both the SM and the EM.

A Running Example

The ideas will be illustrated in a small example with five persistent SM variables, A, B, C, D, and E, and three tasks, X, Y, and Z. Tasks X and Z each have one observable variable, X1 and Z1, respectively. Task Y has two observable variables, Y2 and Y3, and also has an unobserved "auxiliary" variable Y1 which has been introduced to account for a hypothetical conditional dependence between Y2 and Y3. The BIN for the full situation is shown below. The goal is to create an SM model fragment and three fragments for evidence in the tasks that can recreate the same probabilistic updating by dynamically assembling the SM with evidence models, one at a time.

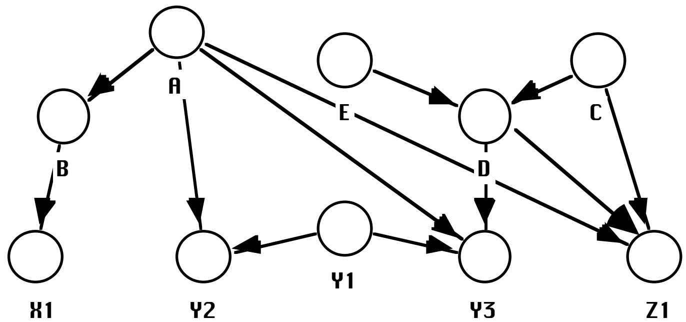

Figure 1. A BIN for all three tasks at once. We want to be able to update the SM as if we had implemented this full BIN, but do it assembling just one task's worth at a time, absorbing evidence, and moving on.

BIN Fragments

The Student Model

The SM consists of (1) persistent student model variables and (2) probability distributions and conditional distributions required to specify their joint distribution prior to observing task responses. These appear in the SM BIN fragment.

Example:

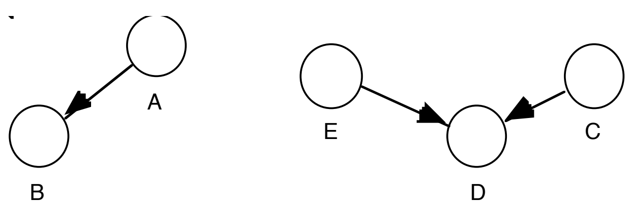

Figure 2. This is a student model with five student-model variables, A, B, C, D, and E. {A,B} is independent of {C,D,E}. Unconditional prior distributions are implied for A, E, and C.

Evidence Models

An EM consists of (1) observable variables associated with the task, (2) possibly auxiliary variables used to account for dependencies among observables beyond those explained by their SM parents,[2] (3) "stubs" indicating the SM variables that are parents of the observables, and (4) conditional distributions as required to specify the joint distribution of the EM variables given all combinations of the SM parents.

The "footprint" of an EM is the set of SM variables which are parents of its other variables (observable variables or auxiliary variables). In an EM BIN fragment, the initial distribution of the SM variable stubs is uniform over all possible combinations of values.

Examples:

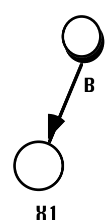

Figure 3a. EM X: One observable; footprint={B}.

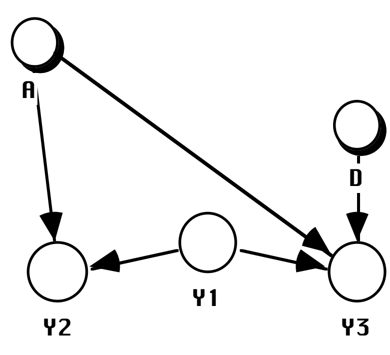

Figure 3b. EM Y: Two observables, Y2 and Y3; one auxiliary EM variable, Y1; footprint={A,D}.

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[2] Examples of auxiliary variables are (1) student's degree of familiarity with the topic paragraph upon which several reading comprehension questions are based, (2) strategy employed on a multi-step problem that can be solved in different ways, and (3) task effect when several aspects of each performance are rated.

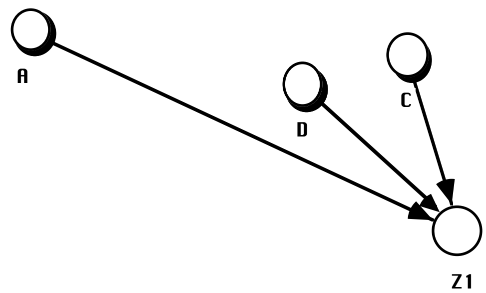

Figure 3c. EM Z: One observable; footprint={A,C,D}.

Figure 3. BIN fragments for evidence models. "Stubs" corresponding to SM variables are shadowed.

Compiled BIN Fragments

A *compiled* BIN contains a junction tree, or a tree of cliques (subsets of variables) for which probability tables are calculated, then manipulated to propagate the effects of information on one or more variables on belief about the others. Lauritzen and Spiegelhalter's algorithm for producing a junction tree starts by "moralizing" the graph represented by the original BIN; that is, adding edges to "marry parents." In Figure 2, for example, SM variables C and E are both parents of D; an edge will be added to connect them. The moralized graph is one which has married parents in this manner and dropped the directionality of the edges. Figure 4 is the moralized graph for the student model. A triangulated graph is then produced, which adds edges as necessary to break cycles of length four or greater.

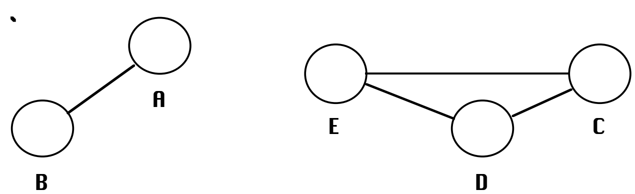

Figure 4. Moralized graph of student model.

The Student Model

A compiled SM BIN fragment contains a junction tree for the SM BIN, with additional constraints defined with respect to a designated list of EM footprints: *For each EM footprint, the junction tree must contain a clique of SM variables containing that footprint.* The potential tables imply the joint distribution of all the SM variables.

Examples:

In the following examples, a leaf is made explicit for the SM variables that represent just the shared boundary between the SM and any EMs at issue. This is for clarity, since updating could be carried out beginning with any clique that contains all the relevant variables. The ordering of variables within cliques is alphabetical.

Figure 5a. Junction tree for SM, in and of itself. The cliques are disconnected; information about A or B does not change belief about C, D, or E.

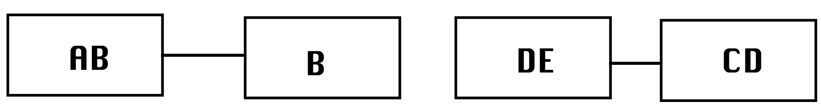

Figure 5b. Junction tree for SM, with the EM X footprint constraint enforced. A leaf for B has been added for clarity, although no essential change to the junction tree is required. Evidence from EM X can be entered through the B node.

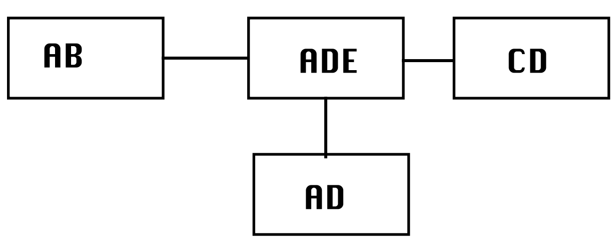

Figure 5c. Junction tree for SM, with the EM Y footprint constraint enforced. Because the footprint of EM Y is {A,D}, a clique containing this SM variables is required. This results in a connection between the formerly disconnected subsets of SM variables. Evidence from EM Y can be entered via AD node.

Figure 5d. Junction tree for SM, with just the EM Z footprint constraint enforced. Evidence from EM Z entered via ACD node.

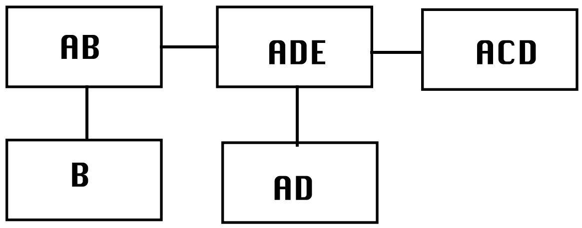

Figure 5e. Junction tree for SM, with all three footprint constraints enforced. Evidence for each EM can be entered through a conformable node, with coherent updating ensured. Compared to the original SM junction tree in Figure 5a, the junction tree with footprint constraints is connected and has larger cliques. It does have cliques, though, that are prepared to absorb or transmit information coherently to any EM.

Evidence Models

A *compiled* EM BIN fragment contains a junction tree, with an additional constraint defined with respect to its SM footprint: *The junction tree must contain a clique of the SM stubs associated with this EM.*

Examples:

For each task, two figures are shown. The first is for the junction tree of the EM BIN fragment. It is written with the SM variables isolated in a terminal node. The second is for the junction tree obtained when the EM fragment is assembled with an SM fragment that is conformable with all three tasks.

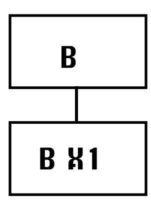

Figure 6a. Junction tree for Task X EM BIN fragment, including footprint constraint

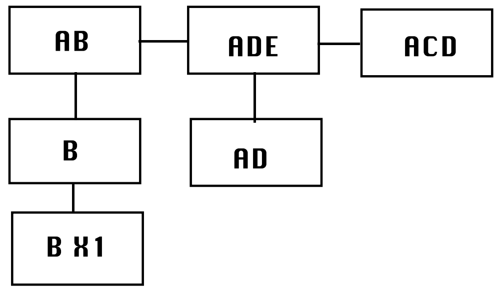

Figure 6b. Junction tree for assembled Task X EM fragment and SM BIN fragment.

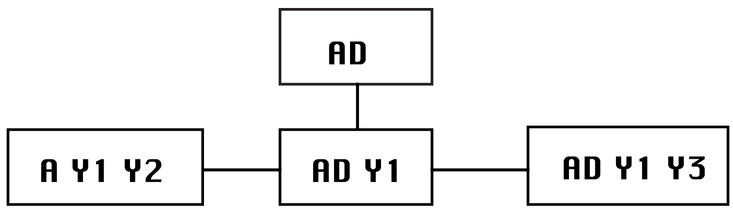

Figure 7a. Junction tree for Task Y EM BIN fragment, including footprint constraint.

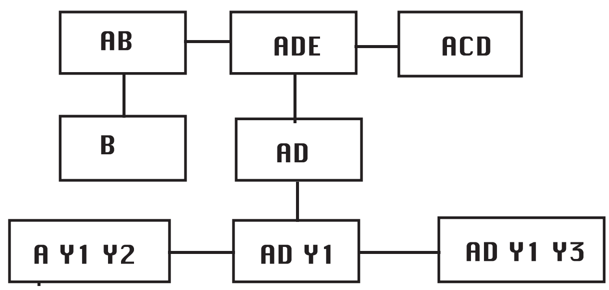

Figure 7b. Junction tree for assembled Task Y and SM BIN fragments.

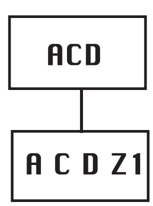

Figure 8a. Junction tree for Task Z EM BIN fragment, including footprint constraint.

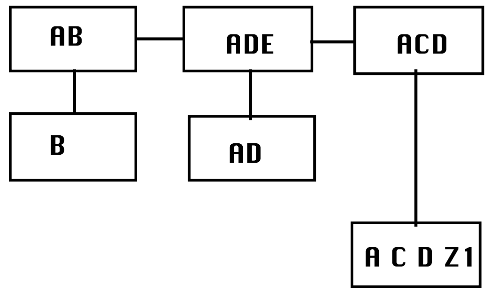

Figure 8b. Junction tree for assembled Task Z and SM BIN fragments.

Forcing Conformability in Compilation

How can the additional constraints required to produce matching footprints in SM and EM BIN fragments be forced to appear in their compiled versions?

Adding Edges by Hand

Once an SM or EM BIN fragment has been created, one can add edges to connect the SM variables which are parents of the EM observables or auxiliary variables by hand before initiating compilation. Consider Task Y as an example, with footprint {A,D}. Before compiling the SM BIN fragment, one would add an edge between A and D. Before compiling the Task Y EM BIN fragment, one would add an edge between the A and D stubs.

Although carrying this procedure correctly would induce cliques with the required SMs, leaving the process to hand opens the door to errors, and, depending on the software used, can require manual control of variable orderings. This approach might be satisfactory for experiments or small isolated applications, but would not be acceptable for a production system.

Supernodes

A "brute force" method automatic procedure to force conformability of SM and EM boundary variables when BIN fragments are compiled is to add a "supernode" containing the Cartesian product of all values of all variables so involved into the fragment in question. Doing so would assure that an EM and SM junction tree would both have a single, common supervariable that captures all information that must pass from the variables in one fragment to the variables in the other. Moreover, if the ordering of the variables and their values were made to agree in both junction trees and potential tables, standard algorithms could be used to create these conformable junctions trees. The compiled SM and a given EM would then share exactly one variable, namely the supervariable containing all possible values of the footprint variables.

Unfortunately, the computational overhead associated with supernodes can easily become overwhelming. Suppose, for example, there are six ternary variables in a SM/EM boundary. The supernode added into both the SM and the EM has 6^3, or 216 possible values. Its parents are the six variables, with the same 216 possible combinations. The potential table for this relationship is 216 by 216, but all entries in a given row and column are zero except for the single 1 that indicates which combination of parent values corresponds to which value of the supervariable. Avoidable manipulations of this magnitude must be carried out twice, in both the SM and the EM sides of the combined BIN.

Adding Constraints to Moralization

Depending on the algorithm used to produce a junction tree, it may be possible to introduce a list of constraints at the stage of moralization. Each set of footprint SM variables for designated EM motifs is specified as necessary to connect fully, as would be required if all were the parents of a given child.

Phantom Variables

If adding constraints directly is not possible, the same effect can be achieved with a bit more work by actually introducing such a child for each footprint set—a "phantom variable." This approach involves additional coding and structure before and after the compilation stage. Phantom variables are used for compilation under standard algorithms, which will produce extraneous potential tables and terminal leaves in the junction tree that can be stripped away when compilation is complete.

The idea is to include a simple fictitious variable in an SM or an EM (with only one value if allowed by the compilation environment, or two if a nondegenerate variable is required) which has as parents only and exactly the SM variables that correspond to the footprint of an EM of interest.

Examples:

Shown below are BIN fragments for the SM and the EMs which have been augmented with phantom variables so that the junction trees of the compiled fragments will have nodes that include the SM boundary sets. As it turns out, the EMs for tasks X and Z both already have BINs for which compilation will include a clique for the SM/EM boundary so task Y is most interesting among the EMs. The phantom variables are shown below for all three tasks nevertheless. Employing the phantom variable method routinely in all cases may be preferable to checking case-by-case to determine whether it is needed, in order to standardize record-keeping and expectations.

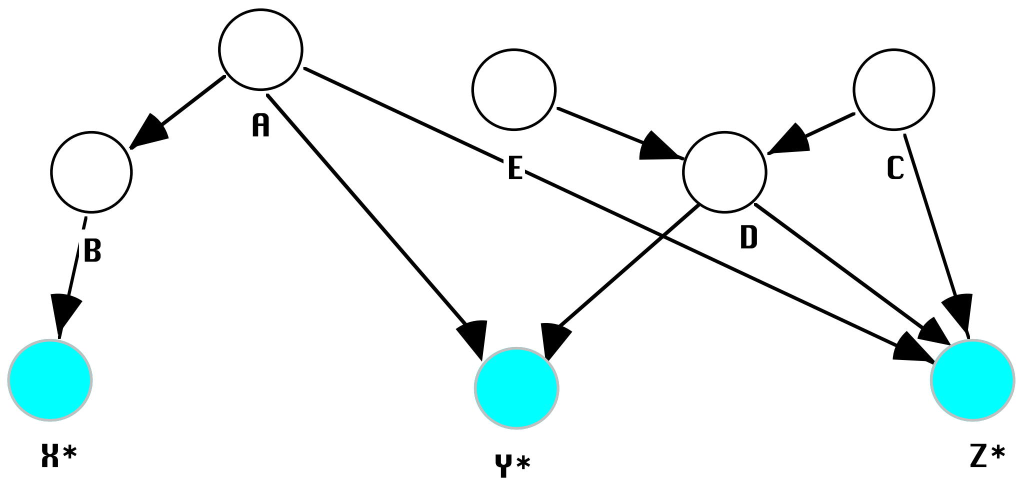

Figure 9. Augmented BIN fragment for Student Model. Phantom variables (X*, Y*, and Z*) are included for all three EM motifs. Moralization will force a junction tree that includes cliques with the boundary sets of SM variables, as discussed in 3.1.

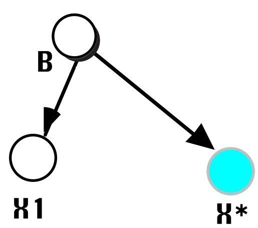

Figure 10a. Augmented BIN fragment for Evidence Model X. Moralization will force a junction tree that includes a clique with the footprint set of SM variables (although in this case the fact that just the SM stub B is a parents of the observable variable X1 would have produced a clique with the footprint anyway).

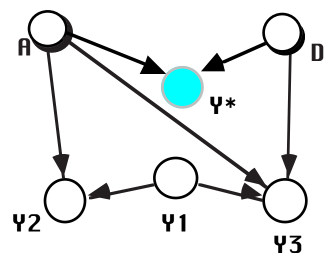

Figure 10b. Augmented BIN fragment for Evidence Model Y. Including the phantom variable Y* forces an edge between A and D during moralization, which would not be required otherwise. The junction tree will thus include a cliques with A and D.

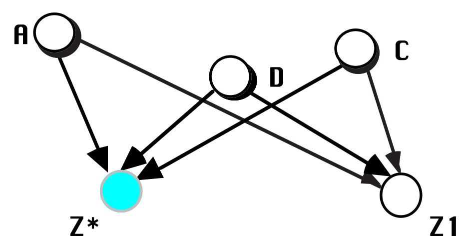

Figure 10c. Augmented BIN fragment for Evidence Model Z. Moralization will force a junction tree that includes a cliques with the footprint set of SM variables (although again in this case the fact that all SM stubs A, C, and D are parents of the observable variable Z1 would have forced edges among them anyway).

A Numerical Example

Tables 1-12 use the structure in the preceding example to compare updating with a full BIN, as in Figure 1, with updating achieved by attaching EM BIN fragments to the SM BIN fragment sequentially. All nodes are binary, with SM variables A-E and Y1 having High/Low values and observables X1, Y2, Y3, and Z1 having Right/Wrong values.

Calculations were carried out using the ERGO 1.52 computer program (Noetic Systems, 1997), using the supernode approach described in Section 4.2 because it can be implemented with the existing code. ERGO run-time files are given in the Appendix. ERGO's propagation algorithm, based on Lauritzen-Spiegelhalter, was used to propagate information within BINs and BIN fragments, while the "virtual evidence" feature was used to pass information across SM/EM boundaries.

The observations X1=Right, Y2=Right, Z1=Wrong will be entered. In each case, tables are given that show the status of the relevant BINs before and after each observation:

Table 1	Prediction of X1 in the full network, from initial status
Table 2	Prediction of X1 from initial status, using BIN fragments
Table 3	Absorbing X1=Right from initial status, using full network
Table 4	Absorbing X1=Right from initial status, using BIN fragments
Table 5	Prediction about Y2, after observing X1=Right, using full network
Table 6	Prediction about Y2, after observing X1=Right, using BIN fragments
Table 7	Absorbing Y2=Right, after X1=Right, using full network
Table 8	Absorbing Y2=Right, after X1=Right, using BIN fragments
Table 9	Prediction about Z1, after X1=Right and Y2=Right, using full network
Table 10	Prediction about Z1, after X1=Right and Y2=Right, using BIN fragments
Table 11	Absorbing Z1=Wrong, after X1=Right and Y2=Right, using full network
Table 12	Absorbing Z1=Wrong, after X1=Right and Y2=Right, using BIN fragments

Table 1

Prediction of X1 in the Full Network, From Initial Status

(1)

[B]

	B
H	.46
L	.54

Initial distribution of B in full network

(2)

$[BX_1]$

B	X_1=R	X_1=W
H	.414	.046
L	.108	.432

Initial joint distribution of B and X_1; obtained from (1) and $P(X_1 \mid B)$

(3)

$[X_1]$

	X_1
R	.522
W	.478

Initial predictive distribution for X_1 in full network; obtained by marginalizing over (2)

Table 2

Prediction of X1 From Initial Status, Using BIN Fragments

Initial status of fragments

<-SM | EM-X->

(0)

[B]

	B
H	.46
L	.54

Initial distribution of B in SM fragment

(1)

[B]

	B
H	.50
L	.50

Initial *uniform* distribution of B in EM-X fragment

(2)

[BX_1]

B	X_1=R	X_1=W
H	.45	.05
L	.10	.40

Initial joint distribution of B and X_1 in EM-X; obtained from (1) and $P(X_1 | B)$

(3)

[X_1]

	X_1
R	.55
W	.45

Initial distribution for X_1 in EM-X; obtained by marginalizing over (2)

Result of updating EM with distribution for parent distribution from SM

EM-X —>

(4)

[B]

	B
H	.46
L	.54

"Virtual evidence" procedure used to update B distribution in EM-X fragment (1) with B distribution in SM fragment (0)

(5)

[BX_1]

B	X_1=R	X_1=W
H	.414	.046
L	.108	.432

Updated joint distribution of B and X_1 in EM-X; obtained by propagating new evidence from (4)

(6)

[X_1]

	X_1
R	.522
W	.478

Updated predictive distribution for X_1 in EM-X; obtained by marginalizing over (5)

Table 3

Absorbing X1=Right From Initial Status, Using Full Network

(1)

[B]

	B
H	.46
L	.54

Initial distribution of B in full network

(2)

$[BX_1]$

B	X_1=R	X_1=W
H	.414	.046
L	.108	.432

Initial joint distribution of B and X_1--obtained from (1) and $P(X_1 \mid B)$

(3)

$[X_1]$

	X_1
R	.522
W	.478

Initial predictive distribution for X_1 in full network--obtained by marginalizing over (2)

(4)

$[X_1]$

	X_1
R	.522
W	-

Observe X_1=R, and zero out W row in (3)

(5)

$[BX_1]$

B	X_1=R	X_1=W
H	.414	-
L	.108	-

Updated joint distribution for B and X_1: from (4), zero out cells for X_1=W

(6)

[B]

	B
H	.793
L	.207

Updated distribution for B: obtained by marginalizng from (5) and normalizing

Table 4

Absorbing X1=Right From Initial Status, Using BIN Fragments

Initial status of fragments (from Table 2)

<-SM | EM-X->

(0) [B]

	B
H	.46
L	.54

Initial distribution of B in SM fragment

(1) [B]

	B
H	.50
L	.50

Initial *uniform* distribution of B in EM-X fragment

(2) [BX_1]

B	X_1=R	X_1=W
H	.45	.05
L	.10	.40

Initial joint distribution of B and X_1 in EM-X; obtained from (1) and $P(X_1 \mid B)$

(3) [X_1]

	X_1
R	.55
W	.45

Initial distribution for X_1 in EM-X; obtained by marginalizing over (2)

Result of updating EM with distribution for X1=Right and injecting resulting likelihood over B into corresponding B distribution in SM

<-SM | EM-X->

(7) [B]

	B
H	.793
L	.207

Updated distribution for B in SM: obtained by entering likelihood for B from EM-X (6) as virtual evidence

(6) [B]

	B
H	.8182
L	.1818

Updated distribution for B: obtained by marginalizng from (5) and normalizing

(5) [BX_1]

B	X_1=R	X_1=W
H	.45	-
L	.10	-

Updated table for B and X_1: from (4), zero out cells for X_1=W

(4) [X_1]

	X_1
R	.55
W	-

Observe X_1=R, and zero out W row in (3)

Table 5

Prediction About Y2, After Observing X1=Right, Using Full Network

Status after having absorbed evidence from X1=Right

(1) [AD]

AD	
HH	.2697
HL	.2591
LH	.2403
LL	.2309

Distribution for AD after X1=Right

(2) [ADY1]

AD	Y1=H	Y1=L
HH	.1348	.1348
HL	.1259	.1259
LH	.1202	.1202
LL	.1155	.1155

Joint distribution for A, D, and Y1 after X1=Right

(3) [AY1]

A	Y1=H	Y1=L
H	.2643	.2643
L	.2357	.2357

Joint distribution for A, D, and Y1 after X1=Right; obtained by marginalizing over (2)

(4) [AY1Y2]

AY1	Y2=R	Y2=W
HH	.2115	.0529
HL	.1851	.0793
LH	.1649	.0707
LL	.1414	.0943

Joint distribution for A, Y1, and Y2 after X1=Right; obtained from (3) and P(Y2 | A,Y1)

(5) [Y2]

Y2	
R	.7029
W	.2971

Predictive distribution for Y2 after X1=Right; obtained by marginalizing over (4)

Table 6

Prediction About Y2, After Observing X1=Right, Using BIN Fragments

Status of BIN fragments after having absorbed evidence from X1=Right into SM only

<-SM | EM-Y->

(0) [AD]

	AD
HH	.2697
HL	.2591
LH	.2403
LL	.2309

Distribution for AD in SM after X1=Right

(1) [AD]

	AD
HH	.25
HL	.25
LH	.25
LL	.25

Uniform prior distribution for AD in EM-Y

(2) [ADY1]

AD	Y1=H	Y1=L
HH	.125	.125
HL	.125	.125
LH	.125	.125
LL	.125	.125

Initial joint distribution of A, D, and Y1 in EM-Y; obtained from (1) and P(Y1 | AD)

(3) [AY1]

A	Y1=H	Y1=L
H	.25	.25
L	.25	.25

Joint distribution of A and Y1 in EM-Y; obtained by marginalizing over (2)

(4) [AY1Y2]

AD	Y1=H	Y1=L
HH	.200	.050
HL	.175	.075
LH	.175	.075
LL	.150	.100

Joint distribution of A,Y1, and Y2 in EM-Y; obtained from (3) and P(Y2 | A, Y1)

(5) [Y2]

	Y2
R	.7
W	.3

Distribution for Y2 in EM-Y, with uniform prior on AD

Result of updating EM-Y with distribution for parent distribution from SM

EM-Y—>

(6) [AD]

	AD
HH	.2697
HL	.2591
LH	.2403
LL	.2309

Virtual evidence procedure used to update AD distribution in EM-Y fragment (1) with AD distribution in SM fragment (0)

(7) [ADY1]

AD	Y1=H	Y1=L
HH	.1348	.1348
HL	.1259	.1259
LH	.1202	.1202
LL	.1155	.1155

Updated joint distribution of A, D, and Y1 in EM-Y; obtained by propagating new information from (6)

(8) [AY1]

A	Y1=H	Y1=L
H	.2643	.2643
L	.2357	.2357

Joint distribution of A and Y1 in EM-Y; obtained by marginalizing over (7)

(9) [AY1Y2]

AY1	Y2=R	Y2=W
HH	.2115	.0529
HL	.1851	.0793
LH	.1649	.0707
LL	.1414	.0943

Joint distribution of A,Y1, and Y2 in EM-Y; obtained by propagating from (8)

(10) [Y2]

	Y2
R	.7029
W	.2971

Predictive distribution for Y2 in EM-Y; by marginalizing over (9)

Table 7

Absorbing Y2=Right, After X1=Right, Using Full Network

Status given X1=Right (copy of Table 5)

(1) [AD]

	AD
HH	.2697
HL	.2591
LH	.2403
LL	.2309

Distribution for AD after X1=Right

(2) [ADY1]

AD	Y1=H	Y1=L
HH	.1348	.1348
HL	.1259	.1259
LH	.1202	.1202
LL	.1155	.1155

Joint distribution for A, D, and Y1 after X1=Right

(3) [AY1]

A	Y1=H	Y1=L
H	.2643	.2643
L	.2357	.2357

Joint distribution for A, D, and Y1 after X1=Right; obtained by marginalizing over (2)

(4) [AY1Y2]

AY1	Y2=R	Y2=W
HH	.2115	.0529
HL	.1851	.0793
LH	.1649	.0707
LL	.1414	.0943

Joint distribution for A, Y1, and Y2 after X1=Right; obtained from (3) and P(Y2 | A,Y1)

(5) [Y2]

	Y2
R	.7029
W	.2971

Predictive distribution for Y2 after X1=Right; obtained by marginalizing over (4)

Result of absorbing evidence from Y2=Right

(10) [AD]

	AD
HH	.2877
HL	.2765
LH	.2223
LL	.2135

Distribution for AD after X1=R & Y2=R; by marginalizing (9)

(9) [ADY1]

AD	Y1=H	Y1=L
HH	.1535	.1343
HL	.1474	.1290
LH	.1197	.1026
LL	.1150	.0986

Joint distribution for A, D, and Y1 after X1=R & Y2=R; by absorbing (8) into (2)

(8) [AY1]

A	Y1=H	Y1=L
H	.3009	.2633
L	.2346	.2012

Joint distribution for A and Y1 after X1=R & Y2=R; obtained from (7) and normalizing

(7) [AY1Y2]

AY1	Y2=R	Y2=W
HH	.2115	-
HL	.1851	-
LH	.1649	-
LL	.1414	-

Updated table for A, Y1, and Y2 after Y2=Right; by absorbing from (6)

(6) [Y2]

	Y2
R	.7029
W	-

Observe Y2=R, and zero out W row in (5)

Table 8

Absorbing Y2=Right, After X1=Right, Using BIN Fragments

BIN fragments after absorbing evidence from X1=Right into SM only (copy of first half of Table 6)

(0) [AD] <-SM | EM-Y->

	AD
HH	.2697
HL	.2591
LH	.2403
LL	.2309

Distribution for AD in SM after X1=Right

(1) [AD]

	AD
HH	.25
HL	.25
LH	.25
LL	.25

Uniform prior distribution for AD in EM-Y

(2) [ADY1]

AD	Y1=H	Y1=L
HH	.125	.125
HL	.125	.125
LH	.125	.125
LL	.125	.125

Initial joint distribution of A, D, and Y1 in EM-Y; obtained from (1) and P(Y1 | AD)

(3) [AY1]

A	Y1=H	Y1=L
H	.25	.25
L	.25	.25

Joint distribution of A and Y1 in EM-Y; obtained by marginalizing over (2)

(4) [AY1Y2]

AD	Y1=H	Y1=L
HH	.200	.050
HL	.175	.075
LH	.175	.075
LL	.150	.100

Joint distribution of A, Y1, and Y2 in EM-Y; obtained from (3) and P(Y2 | A, Y1)

(5) [Y2]

	Y2
R	.7
W	.3

Distribu-tion for Y2 in EM-Y, with uniform prior on AD

Result of updating EM-Y with Y2=Right, then injecting resulting likelihood into SM

(11) [AD] <-SM | EM-Y->

	AD
HH	.2877
HL	.2765
LH	.2223
LL	.2135

Updated distribution for AD in SM, for X1=R & Y2=R: by entering likelihood for AD from EM-Y (10) into (0) as virtual evidence

(10) [AD]

	AD
HH	.2679
HL	.2679
LH	.2321
LL	.2321

Updated distri-bution of A and D in EM-Y for Y2=Right; obtained by marginalizing over (9)

(9) [ADY1]

AD	Y1=H	Y1=L
HH	.1429	.1250
HL	.1429	.1250
LH	.1250	.1071
LL	.1250	.1071

Updated joint distribution of A, D, and Y1 in EM-Y; obtained by absorbing (8) into (2)

(8) [AY1]

A	Y1=H	Y1=L
H	.2857	.2500
L	.2500	.2143

Updated joint distribution of A and Y1 in EM-Y; obtained by marginalizing over (7)

(7) [AY1Y2]

AD	Y1=H	Y1=L
HH	.200	-
HL	.175	-
LH	.175	-
LL	.150	-

Updated table for A,Y1, and Y2 in EM-Y, given Y2=R; zero out cells in (4) for Y2=W

(6) [Y2]

	Y2
R	.7
W	-

Observe Y2=R, and zero out W row in (5)

Table 9

Prediction About Z1, After X1=Right and Y2=Right, Using Full Network

Status after having absorbed evidence from X1=Right and Y2=Right

	(1) [ACD] ACD		(2) [ACDZ1] Z1=R	Z1=W		(3) [Z1] Z1
		ACD				
HHH	.1828	HHH	.1462	.0366	R	.3096
HHL	.0429	HHL	.0086	.0343		
HLH	.1049	HLH	.0210	.0840	W	.6903
HLL	.2336	HLL	.0467	.1869		
LHH	.1412	LHH	.0282	.1130		
LHL	.0331	LHL	.0066	.0265		
LLH	.0811	LLH	.0162	.0648		
LLL	.1804	LLL	.0361	.1443		

(1) Joint distribution for ACD after X1=Right & Y2=Right

(2) Joint distribution for A, C, D, and Z1 after X1=R & Y2=R

(3) Predictive distribution for Z1 after X1=R & Y2=R; obtained by marginalizing over (2)

Table 10

Prediction About Z1, After X1=Right and Y2=Right, Using BIN Fragments

Fragment status after having absorbed evidence from X1=Right and Y2=Right into SM only

	(0) [ACD] ACD	<-SM | EM-Z->		**(1)** [ACD] ACD	ACD	**(2)** [ACDZ1] Z1=R	Z1=W		**(3)** [Z1] Z1
HHH	.1828		HHH	.125	HHH	.100	.025	R	.275
HHL	.0429		HHL	.125	HHL	.025	.100		
HLH	.1049		HLH	.125	HLH	.025	.100	W	.725
HLL	.2336		HLL	.125	HLL	.025	.100		
LHH	.1412		LHH	.125	LHH	.025	.100		
LHL	.0331		LHL	.125	LHL	.025	.100		
LLH	.0811		LLH	.125	LLH	.025	.100		
LLL	.1804		LLL	.125	LLL	.025	.100		

(0) Distribution. for ACD in SM after absorbing X1=R & Y2=R

(1) Initial uniform distribution for ACD in EM-Z

(2) Initial joint distribution for A, C, D, and Z1 in EM-Z

(3) Initial predictive distribution for Z1 in EM-Z; obtained by marginalizing over (2)

(table continues)

Table 10 (continued)

Result of updating EM-Z with distribution for parent distribution from SM

EM-Z—>

	(4) [ACD]		**(5)** [ACDZ1]			**(6)** [Z1]
	ACD	ACD	Z1=R	Z1=W		Z1
HHH	.1828	HHH	.1462	.0366	R	.3096
HHL	.0429	HHL	.0086	.0343		
HLH	.1049	HLH	.0210	.0840	W	.6903
HLL	.2336	HLL	.0467	.1869		
LHH	.1412	LHH	.0282	.1130		
LHL	.0331	LHL	.0066	.0265		
LLH	.0811	LLH	.0162	.0648		
LLL	.1804	LLL	.0361	.1443		

(4) Virtual evidence procedure used to update ACD distribution in EM-Z fragment (1) with ACD distribution in SM fragment (0)

(5) Updated joint distribution of A, C, D, and Z1 in EM-Z; obtained by propagating new information from (4)

(6) Predictive distribution for Z1 in EM-Z, given X1=R & Y2=R; obtained by marginalizing over (5)

Table 11

Absorbing Z1=Wrong, After X1=Right and Y2=Right, Using Full Network

Status given X1=Right and Y2=Right (copy of Table 9)

	(1) [ACD] ACD
HHH	.1828
HHL	.0429
HLH	.1049
HLL	.2336
LHH	.1412
LHL	.0331
LLH	.0811
LLL	.1804

Joint distribution for ACD after X1=Right & Y2=Right

ACD	**(2)** [ACDZ1] Z1=R	Z1=W
HHH	.1462	.0366
HHL	.0086	.0343
HLH	.0210	.0840
HLL	.0467	.1869
LHH	.0282	.1130
LHL	.0066	.0265
LLH	.0162	.0648
LLL	.0361	.1443

Joint distribution for A, C, D, and Z1 after X1=R & Y2=R

	(3) [Z1] Z1
R	.3096
W	.6903

Predictive distribution for Z1 after X1=R & Y2=R; obtained by marginalizing over (2)

(table continues)

Table 11 (continued)

Result of absorbing evidence from Z1=Wrong

	(6) [ACD] ACD
HHH	.0530
HHL	.0497
HLH	.1216
HLL	.2707
LHH	.1636
LHL	.0384
LLH	.0939
LLL	.2091

Distribution for ACD after X1=Right, Y2=Right, & Z1= Wrong; from (5), then normalizing

ACD	**(5)** [ACDZ1] Z1=R	Z1=W
HHH	-	.0366
HHL	-	.0343
HLH	-	.0840
HLL	-	.1869
LHH	-	.1130
LHL	-	.0265
LLH	-	.0648
LLL	-	.1443

Updated table for A, C, D, and Z1 after Z1=Wrong; by absorbing (4) into (2)

	(4) [Z1] Z1
R	-
W	.6903

Predictive distribution for Z1 after X1=R & Y2=R; obtained by marginalizing over (2)

Table 12

Absorbing Z1=Wrong, After X1=Right and Y2=Right, Using BIN Fragments

Fragment status after X1=Right and Y2=Right in SM only (same as first half of Table 10)

<-SM | EM-Z->

(0)

	[ACD] ACD
HHH	.1828
HHL	.0429
HLH	.1049
HLL	.2336
LHH	.1412
LHL	.0331
LLH	.0811
LLL	.1804

Distribution. for ACD in SM after absorbing X1=R & Y2=R

(1)

	[ACD] ACD
HHH	.125
HHL	.125
HLH	.125
HLL	.125
LHH	.125
LHL	.125
LLH	.125
LLL	.125

Initial uniform distribution for ACD in EM-Z

(2)

ACD	[ACDZ1] Z1=R	Z1=W
HHH	.100	.025
HHL	.025	.100
HLH	.025	.100
HLL	.025	.100
LHH	.025	.100
LHL	.025	.100
LLH	.025	.100
LLL	.025	.100

Initial joint distribution for A, C, D, and Z1 in EM-Z

(3)

	[Z1] Z1
R	.275
W	.725

Initial predictive distribution for Z1 in EM-Z; obtained by marginalizing over (2)

(table continues)

Table 12 (continued)

Result of updating EM-Z with Z1=Wrong, then injecting resulting likelihood into SM

<-SM | EM-Z->

(7)
[ACD]

	ACD
HHH	.0530
HHL	.0497
HLH	.1216
HLL	.2707
LHH	.1636
LHL	.0384
LLH	.0939
LLL	.2091

Updated distribution for ACD in SM for X1=R Y2=R & Z1=W: by entering likelihood for ACD from EM-Z (6) into (0) as virtual evidence

(6)
[ACD]

	ACD
HHH	.0345
HHL	.1379
HLH	.1379
HLL	.1379
LHH	.1379
LHL	.1379
LLH	.1379
LLL	.1379

Updated joint distribution of A, C, and D in EM-Z; obtained by marginalizing over (5)

(5)
[ACDZ1]

ACD	Z1=R	Z1=W
HHH	-	.025
HHL	-	.100
HLH	-	.100
HLL	-	.100
LHH	-	.100
LHL	-	.100
LLH	-	.100
LLL	-	.100

Updated table for A, C, D, and Z1 in EM-Z, given Z1=W; zero out cells in (2) for Z1=R

(4)
[Z1]

	Z1
R	-
W	.725

Observe Z1=W, and zero out R row in (3)

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APPENDIX

ERGO Run-Time File for Full Network

85227296 10 6 4 1 3 10 4 7 9 8 5 2 6 1 9 2 4 8 10 5 7 6 3 2 2 2 2 2 2 2 2 2 2 0 3 0 0 4 16 2 4 5 1 2 3 4 1.280000e-01 3.200000e-02 3.200000e-02 1.280000e-01 4.800000e-02 4.800000e-02 1.920000e-01 1.920000e-01 4.800000e-02 4.800000e-02 1.920000e-01 1.920000e-01 7.200000e-02 7.200000e-02 2.880000e-01 2.880000e-01 1 1 2 4 4 16 3 1 4 1 4 5 6 2.500000e-01 2.500000e-01 2.000000e-01 3.000000e-01 2.000000e-01 3.000000e-01 1.500000e-01 3.500000e-01 2.000000e-01 3.000000e-01 1.500000e-01 3.500000e-01 1.000000e-01 4.000000e-01 5.000000e-02 4.500000e-01 2 0 2 4 3 8 1 5 1 5 7 8.000000e-01 2.000000e-01 7.000000e-01 3.000000e-01 7.000000e-01 3.000000e-01 6.000000e-01 4.000000e-01 1 0 2 4 3 8 2 4 2 4 8 6.300000e-01 1.800000e-01 7.000000e-02 1.200000e-01 2.800000e-01 3.000000e-02 4.200000e-01 2.700000e-01 1 1 1 2 2 4 6 1 1 9 7.000000e-01 3.000000e-01 3.000000e-01 7.000000e-01 5 0 1 2 2 4 9 9 10 9.000000e-01 1.000000e-01 2.000000e-01 8.000000e-01 A B C D E X1 Y1 Y2 Y3 Z1 h i lo hi lo hi lo hi lo hi lo hi lo right wrong hi lo right wrong right wrong right wrong

ERGO Run-Time File for Full Network, with Supernodes and Nodes to Track Joint Distributions

85227296 16 9 5 1 9 2 16 10 13 4 15 3 14 6 11 5 7 8 12 1 3 9 7 13 11 14 15 2 5 12 16 6 10 8 4 2 8 16 4 8 8 2 4 2 2 2 2 2 2 2 2 0 4 0 0 5 128 2 4 6 8 1 2 3 4 5 1.280000e-01 0.000000e+00 3.200000e-02 0.000000e+00 0.000000e+00 3.200000e-02 0.000000e+00 1.280000e-01 0.000000e+00 4.800000e-02 0.000000e+00 1.920000e-01 0.000000e+00 0.000000e+00 4.800000e-02 0.000000e+00 1.920000e-01 0.000000e+00 4.800000e-02 0.000000e+00 1.920000e-01 0.000000e+00 0.000000e+00 4.800000e-02 0.000000e+00 1.920000e-01 0.000000e+00

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14 15 16 9.000000e-01 0.000000e+00 0.000000e+00 1.000000e-01 0.000000e+00 0.000000e+00
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Y3 Z1 hi lo HHH HHL HLH HLL LHH LHL LLH LLL HHHR HHHW HHLR HHLW HLHR

HLHW HLLR HLLW LHHR LHHW LHLR LHLW LLHR LLHW LLLR LLLW HH HL LH LL HHH HHL HLH HLL LHH LHL LLH LLL HHR HHW HLR HLW LHR LHW LLR LLW hi lo HR HW LR LW hi lo hi lo hi lo right wrong hi lo right wrong right wrong right wrong

ERGO Run-Time File for Student Model, with Supernodes

85227296 7 4 4 1 5 2 6 3 7 4 1 3 5 7 2 4 6 2 8 4 2 2 2 2 0 3 0 0 4 64 2 3 4 1 2 3 4 1.600000e-01 0.000000e+00 0.000000e+00 1.600000e-01 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 2.400000e-01 0.000000e+00 0.000000e+00 2.400000e-01 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 2.400000e-01 0.000000e+00 0.000000e+00 2.400000e-01 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 3.600000e-01 0.000000e+00 0.000000e+00 3.600000e-01 1 0 2 4 3 16 1 4 1 4 5 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 1 0 2 4 3 8 2 4 2 4 6 6.300000e-01 1.800000e-01 7.000000e-02 1.200000e-01 2.800000e-01 3.000000e-02 4.200000e-01 2.700000e-01 1 0 1 2 2 4 1 1 7 7.000000e-01 3.000000e-01 3.000000e-01 7.000000e-01 A ACD AD B C D E hi lo HHH HHL HLH HLL LHH LHL LLH LLL HH HL LH LL hi lo hi lo hi lo hi lo

ERGO Run-Time File for Evidence Models

85227296 8 4 3 3 4 6 2 5 7 8 1 8 4 1 2 5 3 6 7 8 4 2 2 2 2 2 2 2 0 0 0 0 2 4 1 2 4.500000e-01 5.000000e-02 1.000000e-01 4.000000e-01 0 1 0 0 3 16 3 3 4 5 1.000000e-01 8.750000e-02 1.000000e-01 8.750000e-02 8.750000e-02 7.500000e-02 8.750000e-02 7.500000e-02 2.500000e-02 3.750000e-02 2.500000e-02 3.750000e-02 3.750000e-02 5.000000e-02 3.750000e-02 5.000000e-02 2 0 2 8 3 16 4 5 4 5 6 4.000000e-01 6.000000e-01 5.000000e-01 5.000000e-01 3.000000e-01 7.000000e-01 5.000000e-01 5.000000e-01 3.000000e-01 7.000000e-01 5.000000e-01 5.000000e-01 1.000000e-01 9.000000e-01 5.000000e-01 5.000000e-01 0 0 0 0 2 16 7 8 1.000000e-01 2.500000e-02 2.500000e-02 2.500000e-02 2.500000e-02 2.500000e-02 2.500000e-02 2.500000e-02 2.500000e-02 1.000000e-01 1.000000e-01 1.000000e-01 1.000000e-01 1.000000e-01 1.000000e-01 1.000000e-01 ACD AD B X1 Y1 Y2 Y3 Z1 HHH HHL HLH HLL LHH LHL LLH LLL HH HL LH LL hi lo right wrong hi lo right wrong right wrong right wrong

ERGO Run-Time File for Evidence Models, with Supernodes and Nodes to Track Joint Distributions

85227296 12 5 4 6 7 8 10 4 5 3 9 11 12 2 1 12 11 7 5 6 1 2 3 8 4 9 10 8 16 4 8 8 2 4 2 2 2 2 2 0 0 0 0 3 16 1 2 3
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