MATHEMATICAL PROBLEM-SOLVING PROCESSES AND PERFORMANCE: TRANSLATION AMONG SYMBOLIC REPRESENTATIONS

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Introduction[1]

Researchers in mathematics and mathematics education and cognitive psychologists have long recognized that a very important, if not essential, component of successful problem solving is the ability to translate between different symbolic representations of information (e.g., Clement, Lochhead, & Monk, 1980; Hooper, 1981; Janvier, 1987; Kaput, 1987; Lesh, Post, & Behr, 1987; Lesh, Landau, & Hamilton, 1983; Nesher, 1982; Shavelson, 1981; Shavelson & Salomon, 1985; Silver, 1985). Problem solving often involves translating from the symbolic representation of the problem as given (typically words and numbers) to another symbolic form that more readily leads to a solution (e.g., diagram, graph, picture, algebra, words, or some combination of these). Yet, as has been demonstrated in some well known studies, students at all ages have difficulty translating from one representation to another (e.g., Clement et al., 1980; Galvin & Bel, 1977; Nesher, 1979; Paige & Simon, 1966).

Although researchers and theorists recognize the importance of being able to translate among symbolic representations, we have only a limited understanding about the exact nature of students' abilities and difficulties in making translations. Furthermore, we know little about the extent to which their patterns of performance are linked to the symbolic representations and kinds of translation used in instruction.

The main issue addressed here is that students rarely have been asked to solve problems on the same topic that systematically vary the symbolic representation of both the problem as given and the response that is required. Only a systematically varied set of problems can reveal the skills students have in dealing with different kinds of translation. That is, a comprehensive set of problems is needed to know whether it is possible to generalize students' skills in translation from one problem type to another. Furthermore, students' ability to translate across symbolic forms cannot be separated from the effects of instruction. If students can perform translations that are routinely practiced during instruction but have difficulty performing translations that are not covered in instruction, differences in performance would be attributable to instruction, not to inherent difficulties in certain kinds of translation. Systematically investigating the relationship between the kinds of translation used in instruction and students' problem solving processes and performance is an important first step in clarifying the role of instruction.

The study reported in this paper was designed to address the issues just described. We collected information about students' performance on problems varying in symbolic form and the kinds of symbolic representations and translation used in instruction. We paid particular attention to the symbolic form of the response required as well as that of the problem given. In a previous study (Shavelson, Webb, Shemesh, & Yang, 1987), the symbolic form of the response required influenced students' problem solving processes and performance more than the form of the problem given. In particular, students applied the same numerical or algebraic algorithm whenever the response required was numerical, regardless of the form of the problem as given, but the response required (numerical or verbal description) markedly influenced how students solved the problem. The present study, then, used a greater variety of symbolic forms of the response required (graph, picture, number, algebra, words).

[1] We would like to thank Linda Robertson, Russell Wada, and John Novak for their assistance in this study.

Method

Sample. The sample consisted of 29 students enrolled in an Algebra II class in an eight-week summer instructional program for minority students. All students were Black or Hispanic and most were about to enter grade 11.

Materials. For two topic areas, solving simultaneous equations in two unknowns and distance-rate-time relationships, sets of problems were developed that varied the symbolic form of the problem as given (words in a story problem, graph, diagram, algebra) and of the response required (words, graph, diagram, algebra). All other aspects of the problems (e.g., context, numbers used, complexity of the equations) were controlled to make the problems as parallel as possible except for symbolic form. Approximately half of the problems were open-ended; the remaining problems were in multiple choice form to shorten the time necessary to solve them. Even for problems in multiple choice form, however, students were encouraged to explain reasons for selecting their responses.

Data on teacher instructional methods came from the printed materials the teacher used, students' notes during the classes, and interviews of the teacher. This information was analyzed to determine the variety of symbolic forms used in instruction and kinds of translation explicitly discussed.

Results

The analyses presented focus on problems that concern the same topic but vary either the symbolic form of the problem as given, the symbolic form of the response required, or both. The two topics are solving two equations with two unknowns and issues related to distance-rate-time.

Solving Two Equations With Two Unknowns

Four problems on the individual test that concerned solving two equations with two unknowns varied both the symbolic form of the problem as given (word problem vs. algebraic equations) and the symbolic form of the response required (numerical vs. verbal). Problem 1 was a traditional word problem that required a numerical response; problem 2 presented two equations for students to solve; problem 3 presented a word problem and asked students to explain, without solving the problem, why two particular erroneous solutions were incorrect; and problem 4 presented two algebraic equations and asked students to select the word problem best described by the equations (see Figures 1 to 4 in Appendix A).

Students' responses to these problems were scored in two ways: presence of conceptual and procedural errors. Examples of conceptual errors included setting up the equations incorrectly (problem 1), trying to substitute one equation into itself (problem 2), insisting that erroneous solutions to a word problem were correct (problem 3), and selecting a word problem that did not correspond to the equations (problem 4). Procedural errors consisted of arithmetic mistakes, such as incorrectly multiplying an equation by a constant (particularly negative constants). Initially, students' responses were scored according to the severity and frequency of errors. However, since the results were nearly identical to those scoring only the presence vs. absence of a conceptual or procedural error, the latter scoring is presented here for parsimony. A score of 1 indicates no error; a score of 0 indicates an error.

Individual performance. Table 1 (see Appendix A) presents the means and standard deviations for conceptual understanding scores for the four problems. The data were analyzed using a two-way repeated measures analysis of variance (symbolic form of problem as given vs. symbolic form of response required). As the results in Table 1 suggest, there was no main effect for either the symbolic form of the problem as given [$F(1, 17) = 0.49$, n.s.] or the symbolic form of the response required [$F(1, 17) = 0.49$, n.s.]. The interaction between the two factors, however, was significant [$F(1, 17) = 11.33$, $p < .005$]. Student performance was highest when the symbolic form of the problem as given corresponded to that of the response required (word problem --> words; algebraic --> numerical). Student performance was significantly worse when the symbolic form of the problem as given did not correspond as closely to the symbolic form of the response required (word problem --> numerical; algebraic --> word problem). These results suggest that translation from one symbolic form to another (from problem as given to response required) added a degree of difficulty not found in the other problems.

Pairwise correlations were computed to examine whether individual performance was consistent across problems. None of the correlations were statistically significant. This result shows that individual students varied in their ability to translate across symbolic forms. For example, mean performance was similar for items 1 and 4, but individuals who did well on item 1 (word problem --> numerical response) were not necessarily the same individuals who did well on item 4 (algebraic equations --> word problem). The same interpretation applies to items 2 (algebraic equations --> numerical response) and 3 (word problem -->words). Ability to perform one kind of translation does not predict students' ability to perform another kind of translation.

Data about performance on procedural aspects of the problems (arithmetic skills) are presented in Table 1. (Item 4 did not measure procedural skills and so is not included here.) A one-way repeated analysis of variance showed no significant differences between mean procedural scores [$F(2,34) = 0.43$, $p < .66$]. Students' tendencies to make arithmetic errors did not depend on the kind of translation between symbolic forms required by the problem. The correlation for procedural performance was statistically significant ($r = .61$, $p < .005$) for at least on one pair of problems (1 and 2). This suggests that, for these two problems, an individual student's procedural performance was consistent. Conclusively, the results show more consistency of performance across problems for procedural skills than for conceptual understanding. If one is interested in measuring procedural skills, the type of problem that is given to students to solve is less critical than it is for measuring conceptual understanding.

Correlations were computed between conceptual and procedural performance for each item to measure students' consistency of conceptual understanding and procedural skills. The correlation was statistically significant ($r = .38$, $p < .05$) for only one item (2), suggesting that students' conceptual understanding and procedural skills are largely uncorrelated.

Relationship between student performance and instruction. The interpretation given of the findings in Table 1 is that translation between different symbolic forms makes problems more difficult for students than translation between symbolic forms that closely correspond. An alternative explanation is that the performance shown in Table 1 might be a reflection of instructional experience, rather than due to inherent difficulties with translation per se, with the higher performance corresponding to kinds of translation between symbolic forms that were covered and practiced extensively in the class and the lower performance corresponding to kinds of translation that were not covered or practiced in the class. To test such an interpretation, information about students' instructional experiences was collected from various sources: course syllabi, handouts, quizzes, tests, homework assignments, reading materials, students' notes taken throughout the course, and interviews with the instructor. Analysis of the materials indicated that instructional experience did not account for the results reported in Table 1. Students had considerable practice with all types of problems with the exception of problem 3 (explaining why erroneous solutions to a word problem were incorrect). Yet performance on problem 3 was near the best among the four problems. Students had the most practice solving word problems (translating between verbal presentation and numerical response) and generating word problems that corresponded to pairs of equations (translating between algebraic equations and verbal descriptions), and yet showed the worst performance on problems of these types.

Distance-Rate-Time Relationships

Two kinds of problems concerning distance-rate-time relationships appeared on the test: problems assessing whether students knew and could apply the formula D=RT (distance = rate x time) as well as substitute the correct values into the formula, and problems assessing their understanding of relative speed from graphs of time vs. speed. Each problem type is considered in turn.

Application of D=RT formula. Three problems on the test measured students' ability to apply the formula D=RT (see Figures 5 to 7). Problem 5 posed a simple word problem for students to solve. Problem 6 presented a graph of time vs. speed and problem 7 posed a similar problem as a word problem; both problems asked students to select the correct numerical expression for the distance traveled. Problems 6 and 7 were designed to be as comparable as possible to test the effects of the symbolic form of the information given (graph vs. verbal description). As was the case for the problems involving solving two equations with two unknowns, scoring for severity of errors and scoring merely for the occurrence of errors produced nearly the same results; the results of the latter scoring method are presented here for parsimony. The problems were scored in two ways: (a) a score of 1 was given if students gave or selected the correct relationship among variables (D=RT) and a score of 0 was given for giving or selecting the wrong relationship (D=T/R), and (b) a score of 1 was given if students selected the correct times and speeds and a score of 0 was given otherwise.

Student performance on the three D=RT problems is given in Table 2. A one-way repeated measures analysis of variance of the scores for applications of the D=RT formula was significant [$F(2,34) = 6.18, p < .006$]. Further analyses showed that the difference between items 6 and 7 was not significant. This result suggests that the symbolic form of the problem as given (graph vs. word problem) had little effect on mean performance, possibly because the response required (numerical expression) was the same in both cases. Interestingly, however, the correlation between problems was not significant ($r = .06$), showing that students who could correctly select the D=RT relationship for one problem could not necessarily do so on the other problem.

The superior performance of students on problem 5 suggests that students' ability to apply the D=RT relationship on a simple one-step problem does not imply that students will be able to apply the relationship in a multiple-step problem.

Students' performance on these three problems on their ability to select the correct numbers for rate and time is also given in Table 2. A one-way repeated measures analysis of variance was not significant [$F(2, 34) = 2.53, p < .10$]. Furthermore, the correlation between problems 6 and 7 was significant ($r = .54, p < .01$). (Correlations with problem 5 could not be calculated due to lack of variance for that problem.) These results suggest that students' ability to select the correct numbers for rate and time were consistent across these problems and did not depend on the symbolic form of the problem as given nor on whether the problem was one-step or multi-step.

Understanding speed from graphs. Figures 8 to 11 give the four problems that assessed students' understanding of speed from graphs of time vs. speed. The direction of translation in the four problems were the following: graph to words in problem 8, picture to graph in problem 9, words to graph in problem 10, and graph to picture in problem 11. Performance on these problems (on a 0 vs. 1 scale as for the previous items) appears in Table 2. A one-way repeated measures analysis of variance was significant [$F(3, 57) = 9.75$, $p < .001$]. Post hoc comparisons revealed that problem 8 was significantly easier than the other problems, and that problems 9 and 11 were significantly different.

In comparing the performance of students across these problems, it is reasonable to suspect that problem 8 was easier than the rest due to the following: (a) it involved car traveling on roads rather than biking uphill and downhill, and (b) it involved only two nonzero speeds. Nonetheless, we still believe that student performance would have been good had those other features been introduced. If so, then translating from a graph into words was the easiest task for students. (Of course, this problem should be revised in future studies to make it more comparable to the others.) Similarly, problem 11 may have been more difficult than the others due to the lack of a "stop" rather than the particular direction of translation (graph to picture). Even with these qualifications, the difference in performance suggests that some directions of translation are easier than others.

All of the correlations among problems 9, 10, and 11 were statistically significant (ranging from .42 to .65, $p < .03$ to $p < .002$). The correlations with problem 8 were not significant due to the lack of variability in performance on this problem (all students except one got it right). These results suggest that the order of difficulty of the kinds of translation was consistent across students.

Relationship between performance and instruction. Analysis of the course materials and interviewing the instructor revealed that students had practice with all of these types of problems. In fact, for translating between graphs and other representations, students worked on problems that were considerably more complex than those used in the current study. Therefore, differences in performance on these problems were probably not due to differential exposure to them in the course.

Discussion

This study has several implications for research and practice in mathematics education and testing. First, presenting students with only conventional symbolic representations of problems (typically numerical, algebraic, or story problems requiring numerical answers) is likely to give a limited picture of students' mathematical problem solving abilities. Students can memorize algorithms for clearly identified problem types presented in conventional ways (e.g., see Mayer, 1981) and yet be unable to solve problems involving the same concepts but presented in different symbolic forms.

Second, it is possible to understand students' difficulties in translating among symbolic representations by systematically varying the symbolic form of problem and response required. Such a test or measure can have important diagnostic value in the classroom. The data presented here suggest that the symbolic form of the response required plays a critical role in determining performance, yet this feature of problems is rarely recognized as an important source of variation in performance.

A third related point is that using alternative symbolic forms of the response required may be a good way to measure students' conceptual understanding of mathematics. Problems requiring numerical responses typically involve procedural skills as well as conceptual understanding. It is often difficult to disentangle the two, particularly on tests with multiple-choice response formats. Asking students to think through a problem requiring a different (non-numerical) representation may yield less ambiguous information about what students do and do not understand.

Fourth, the kinds of translation between symbolic forms covered during instruction did not seem to play a major role in this study. Students had practice with virtually all of the kinds of translation in the problems presented on the test, yet their performance differed markedly across different problems. It is possible that differences in performance across problems would have been accentuated still further if the instructor had covered fewer kinds of translation. To examine the role of instruction systematically, future studies should compare performance for instruction varying in kinds of translation covered.

A final word should be said about the limitations of this study and the implications for the design of future studies. The instructional program examined in this study was a special one: a summer course for promising minority students in mathematics. The students who participated in the course had been identified by previous teachers as having potential for learning mathematics and science. Furthermore, the instructors in the summer program are specially selected and have deep commitments to teaching and mathematics and science. As stated above, the features of this program may have influenced the results. Future studies should examine a range of student populations and instructional settings, with larger samples, to determine the impact of these variables on students' ability to solve mathematical problems presented in and requiring responses in different symbolic forms.

References

Clement, J., Lochhead, J., & Monk, G. (1980). Translating difficulties in learning mathematics. *American Mathematics Monthly, 88,* 286-290.

Ericsson, K.A., & Simon, H.A. (1980). Verbal reports as data. *Psychological Review, 87,* 215-251.

Galvin, W.P., & Bell, A.W. (1977, December). *Aspects of difficulties in the solution of problems involving the formation of equations.* Shell Centre for Mathematical Education, University of Nottingham, England.

Ginsburg, H.P. (Ed.). *The development of mathematical thinking.* New York: Academic Press.

Hooper, K. (1981). *Multiple representations within the mathematical domain.* Unpublished paper, University of California, Santa Cruz.

Janvier, C. (1987). Translation processes in mathematics education. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 27-32). Hillsdale, NJ: Erlbaum.

Kaput, J.J. (1987). Representation systems and mathematics. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 19-26). Hillsdale, NJ: Erlbaum.

Lesh, R. (1981). Applied mathematical problem solving. *Educational Studies in Mathematics, 12,* 235-264.

Lesh, R., Landau, M., & Hamilton, E. (1983). Conceptual models in applied mathematical problem solving research. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematical concepts and processes* (pp. 263-343). New York: Academic Press.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33-40). Hillsdale, NJ: Erlbaum.

Mayer, R.E. (1981). Frequency norms and structural analysis of algebraic story problems into families, categories, and templates. *Instructional Science, 10,* 135-175.

Nesher, P. (1982). Levels of description in the analysis of addition and subtraction word problems. In T.P. Carpenter, J.M. Moser, and T.A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective.* Hillsdale, NJ: Erlbaum.

Nisbett, R.E., & Wilson, T.D. (1977). Telling more than we can know: Verbal reports on mental processes. *Psychological Review, 84,* 231-259.

Paige, J., & Simon, H. (1966). Cognitive processes in solving algebra word problems. In B. Kleinmutz (Ed.), *Problem solving research, method, and theory.* New York: John Wiley & Sons.

Post, T. (1987). *Teaching elementary mathematics: Research-based methods.* Boston: Allyn & Bacon.

Schoenfeld, A.H. (1985). *Mathematical problem solving.* New York: Academic Press.

Shavelson, R.J. (1981). Teaching mathematics: Contributions of cognitive research. *Educational Psychologist, 16,* 23-44.

Shavelson, R.J., & Salomon, G. (1985). Information technology: Tool and teacher of the mind. *Educational Researcher, 14,* 4.

Shavelson, R.J., Webb, N.M., Shemesh, M., & Yang, J-W. (1987). *Translation among symbolic representations in problem solving: Results of preliminary study* (CSE Tech. Rep. No. 270). Los Angeles: UCLA Center for the Study of Evaluation.

Silver, E.A. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E.A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 247-266). Hillsdale, NJ: Erlbaum.

Appendix A

Table 1

Performance on Problems Involving Two Equations and Two Unknowns

Symbolic Form of Problem as Given	Symbolic Form of Response Required: Numerical M	Numerical SD	Words M	Words SD
	CONCEPTUAL UNDERSTANDING			
Words	0.56	0.51	0.75	0.44
Algebraic Equations	0.86	0.36	0.52	0.51
	ARITHMETIC SKILLS			
Words	0.83	0.38	0.75	0.44
Algebraic Equations	0.71	0.46	N.A.	N.A.

Note: Words --> Numerical = Problem 1 (Figure 1)
Algebraic Equations --> Numerical = Problem 2 (Figure 2)
Words --> Words = Problem 3 (Figure 3)
Algebraic Equations --> Words = Problem 4 (Figure 4)

Table 2

Performance on Problems Involving Distance-Rate-Time Relationships

Problem	M	SD
APPLICATION OF D=RT FORMULA		
5 (One-step word problem)	1.00	0.00
6 (Graph --> Numerical (multi-step))	0.71	0.46
7 (Words --> Numerical (multi-step))	0.56	0.51
NUMERICAL SUBSTITUTION INTO D=RT FORMULA		
5 (One-step word problem)	1.00	0.00
6 (Graph --> Numerical (multi-step))	0.81	0.40
7 (Words --> Numerical (multi-step))	0.94	0.24
TIME VS. SPEED RELATIONSHIP FROM GRAPH		
8 (Graph --> Words)	0.95	0.22
9 (Picture --> Graph)	0.62	0.50
10 (Words --> Graph)	0.50	0.51
11 (Graph --> Picture)	0.38	0.50

Figure 1

Two Equations and Two Unknowns: Words --> Numerical Response

John needs 120 yards of wooden planks to build a staircase. He has $420 to spend. Oak is expensive, costing $4 per yard, pine costs $3 per yard. Since he cannot afford to make an all-oak staircase he would like to use as much oak as possible. This means he must spend all $420.

How much wood should he buy of each type?

Figure 2

Two Equations and Two Unknowns: Algebraic Equations --> Numerical Response

Solve the system of two equations and two unknowns.

$$x + 2y = 35$$
$$5x + y = 40$$

Figure 3

Two Equations and Two Unknowns: Words --> Verbal Response

Tickets for a baseball game cost $10 for box seats and $5 for regular seats. 100 people came to watch the game and they paid a total of $750.

<u>Without solving the problem</u>, explain why each of the following statements <u>cannot</u> be true.

a. 90 tickets for box seats were sold.

b. 60 box seats and 30 regular seats were sold.

Figure 4

Two Equations and Two Unknowns: Algebraic Equations --> Words

Which word problem is best described by the two equations:

$$x + y = 15$$

$$2x + 3y = 40.$$

a. Jim knows two pieces of gum and three licorice whips costs forty cents; while any two pieces of candy together cost fifteen cents. How much does each kind of candy cost?

b. Kate has two buckets. She knows that two small buckets of water and three large buckets of water contain a total of forty gallons. It takes her fifteen minutes to fill the small and large bucket at the water pump. How long does it take to fill the small bucket at the water pump?

c. Paul has fifteen balloons. The red balloons cost $2 apiece and the silver balloons cost $3 apiece. If all the balloons are either red or silver and Paul sells all his balloons for $40, how many silver balloons did he have?

d. Two horse shoes and three cowboy hats cost forty dollars. I want to buy a total of fifteen shoes and hats. How much is each cowboy hat?

Figure 5

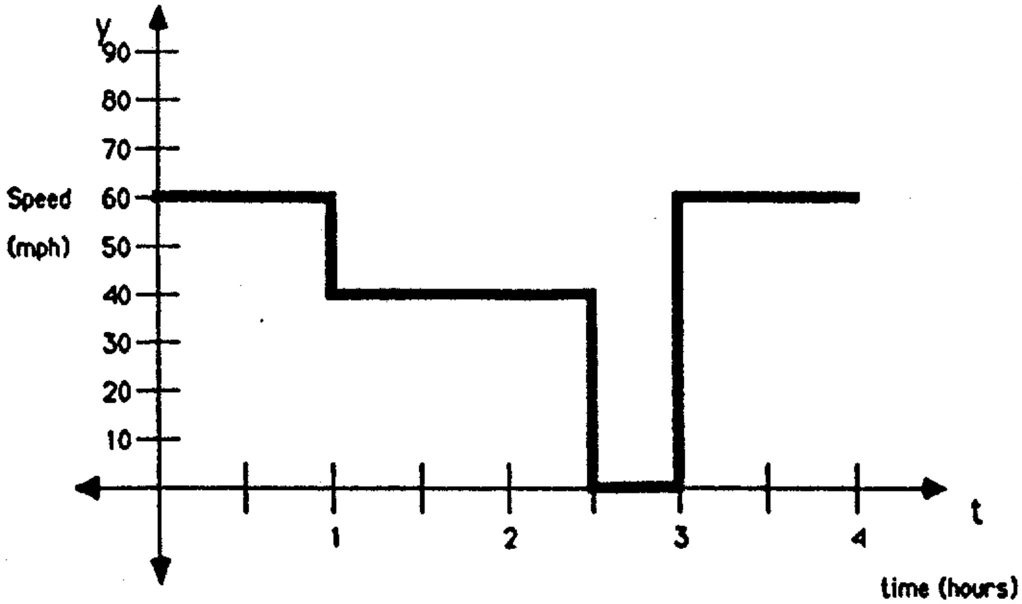
D=RT: One-Step Problem

Paul can skip six miles per hour. How far a distance can he skip in four hours?

Figure 6

D=RT: Graph --> Numerical Response

The graph below describes Mark's car trip.



Which expression(s) best estimates the distance traveled?
(More than one answer may be correct.)

a. (2 x 60) + (1.5 x 40) + (.5 x 0)

b. (60/2) + (40/1.5) + (0/.5)

c. (60/1) + (40/1.5) + (0/.5) + (60/1)

d. (1 x 60) + (2 x 40) + (1 x 0) + (1 x 60)

e. (1 x 60) + (1.5 x 40) + (0 x .5) + (1 x 60)

Figure 7

D=RT: Words --> Numerical Response

On her motorcycle trip, Jennifer rode for two hours at forty miles per hour, stopped for lunch for a half hour, rode one and a half hours at sixty miles per hour, and rode for one hour at forty miles per hour. Which expression best estimates the distance traveled on her trip?

a. (40/2) + (0/.5) + (60/.5) + (40/1)

b. (3 x 40) + (.5 x 0) + (1 x 60) + (1 x 40)

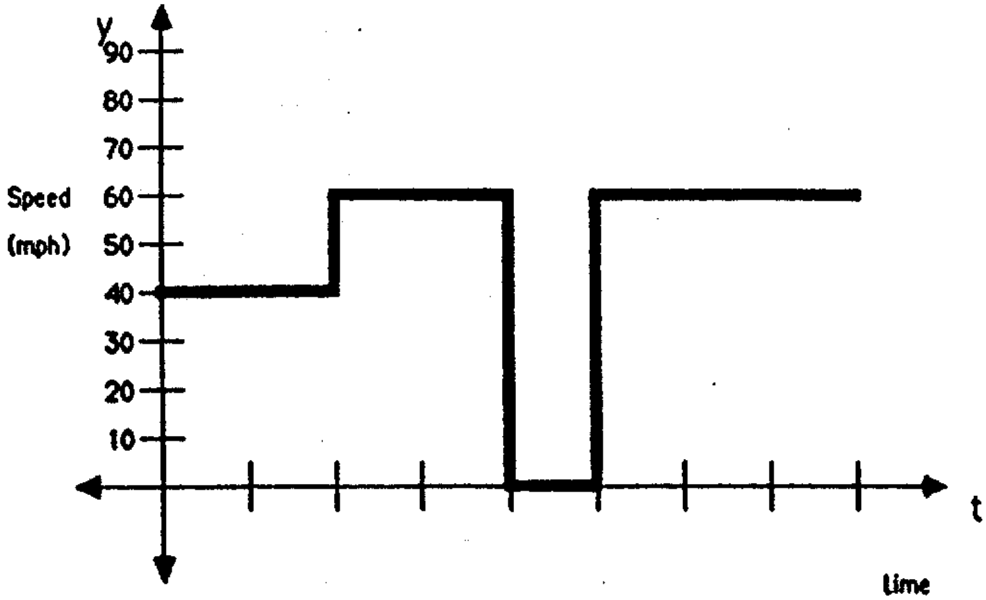
c. (2 x 40) + (.5 x 0) + (1.5 x 60) + (1 x 40)

d. (2/40) + (.5/0) + (.5/60) + (1/40)

Figure 8

Time vs. Speed: Graph --> Words

The graph below describes the speed of a car on a trip.



Which of the choices below best describes the journey?

a. Traveled on the local roads, got onto the highway, stopped for lunch and got back onto the highway.

b. Traveled on local roads, got onto the highway, stopped for lunch and got back onto local roads.

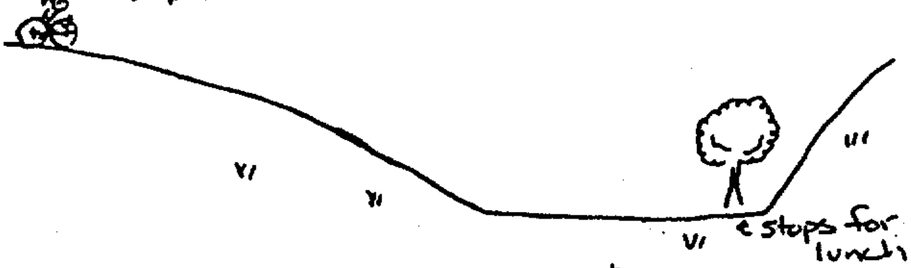
c. Traveled on the highway, stopped for lunch, got back on the highway, traveled on local roads.

d. Traveled on the highway, stopped for lunch, traveled on local roads, and got back onto the highway.

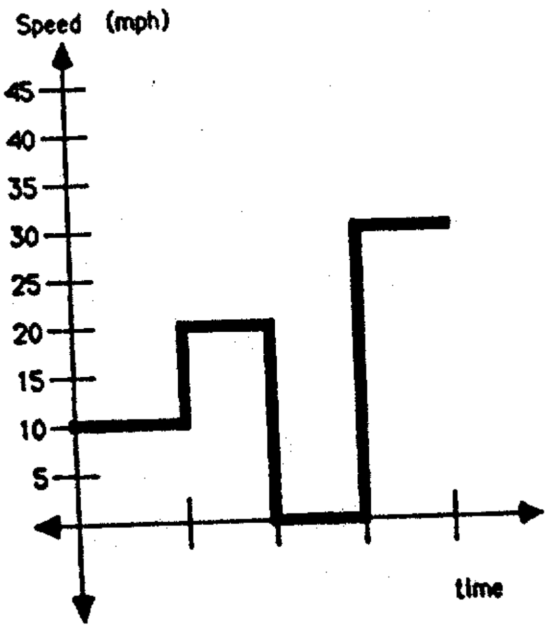
Figure 9

Time vs. Speed: Picture –> Graph

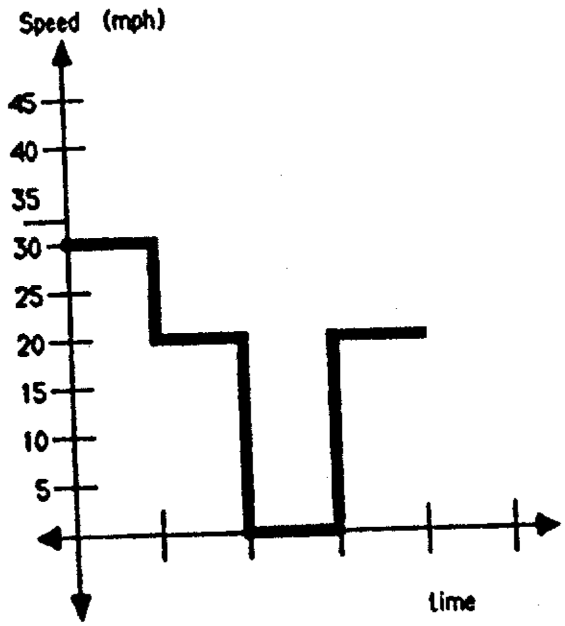
Given the picture below, which graph best describes Sue's bike trip?



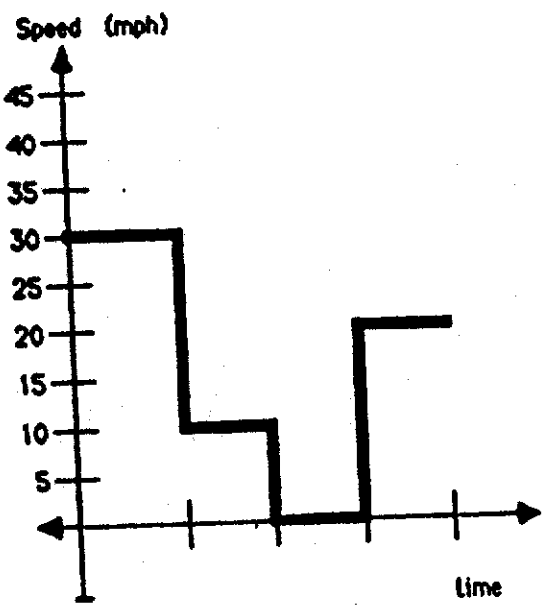
a.



b.



c.



d.

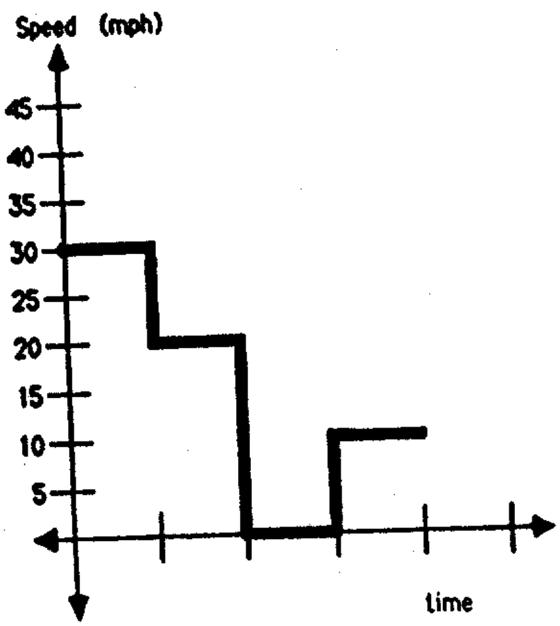


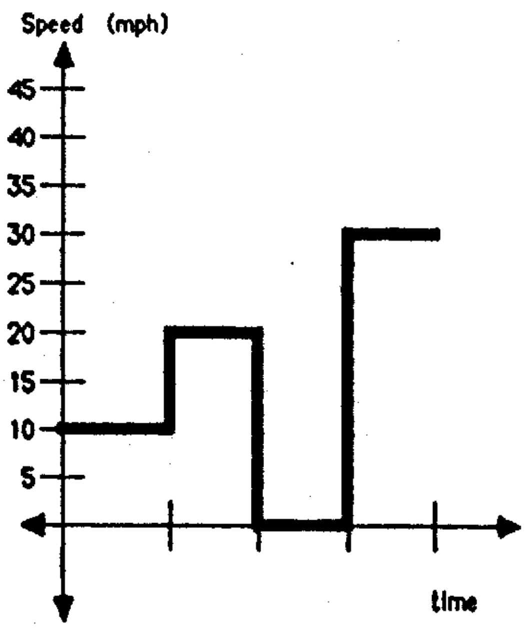
Figure 10

Time vs. Speed: Words --> Graph

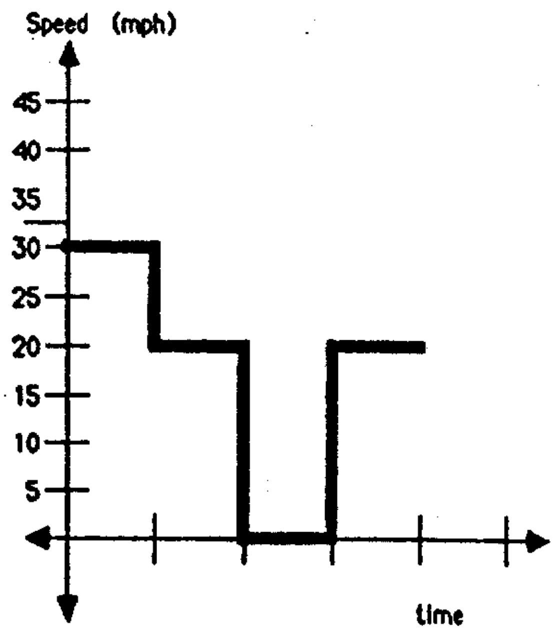
Jill lives at the top of Mt. M^{c}Goo. She bikes down the mountain, over a flat lake bed, stops to change a flat tire and bikes up a short hill to Jack's house.

Which graph best describes her journey?

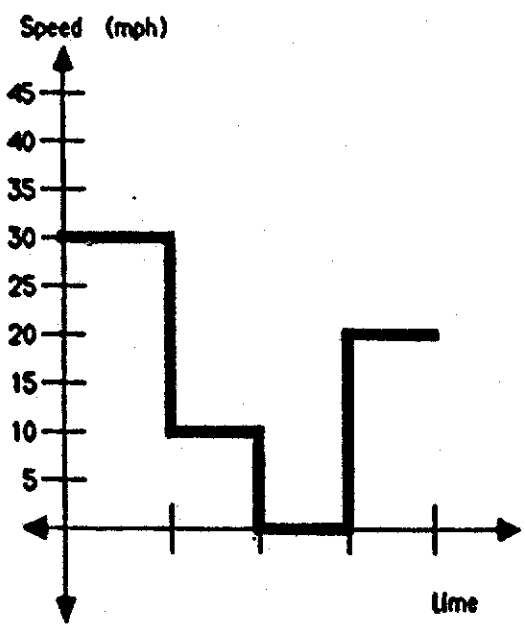
a.



b.



c.



d.

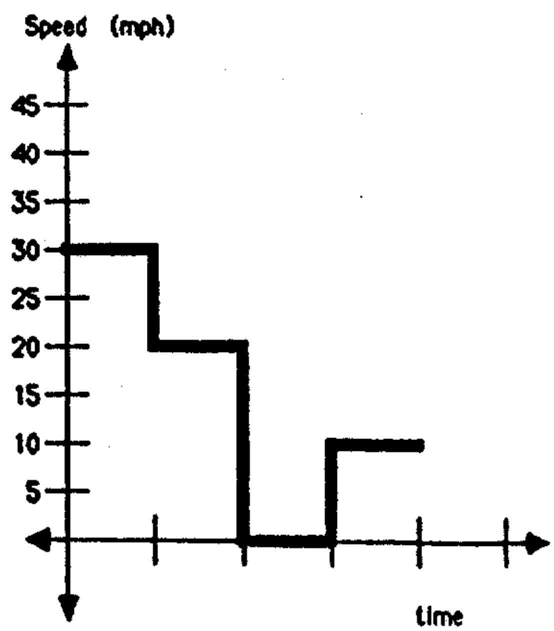
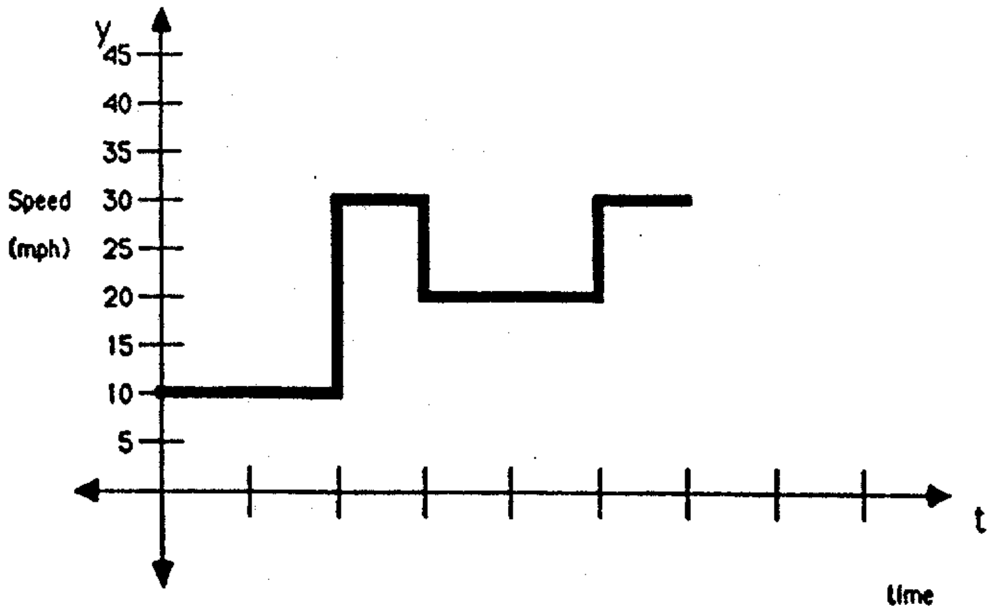


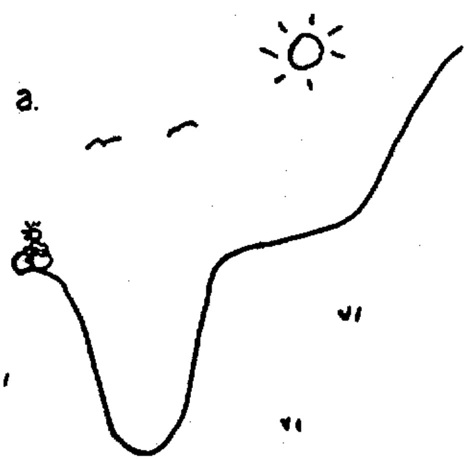
Figure 11

Time vs. Speed: Graph --> Picture

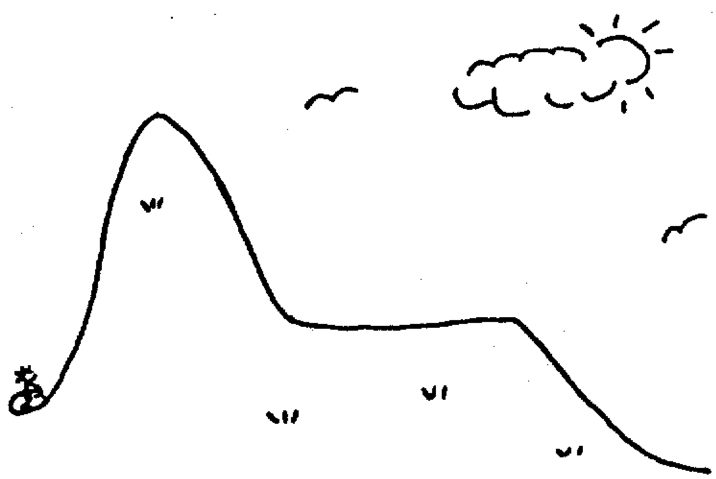
Linda went for a bike ride. The graph below describes her trip. Which picture best describes her bike ride?



a.



b.



c.

d.